Karpman–Washimi Ponderomotive Magnetization and Radiated Power: Streaming and Resonant Effects

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The electron streaming and resonance effects on the non-stationary Karpman–Washimi nonlinear ponderomotive magnetization and radiated power are investigated in a quantum plasma. The ponderomotive Karpman–Washimi magnetization and radiation power due to the ponderomotive force are obtained as functions of the electron streaming velocity, Fermi velocity, wave frequency, and wave number. In small wave numbers, it is found that the electron streaming effect enhances the Karpman–Washimi ponderomotive magnetization. It is found that the electron streaming effect shifts the resonant wave number to the smaller wave number domain. It is also found that the quantum effect reduces the electron streaming velocity for the ponderomotive magnetization near the resonant wave number. In addition, the wave frequency for the resonant Karpman–Washimi radiated power is found to be increased with increasing wave number.

\textbf{Key words:} Streaming and Resonant Interaction; Karpman–Washimi Nonlinear Magnetization; Ponderomotive Force; Induced Radiated Power; Quantum Plasma.

1. Introduction

Recently, there has been a great interest in various physical processes in dense quantum plasmas [1 – 7] composed of low-temperature and high-density electrons and ions since the quantum plasmas have been found in modern sciences and technologies such as laser-produced plasmas, nano-electronic devices, quantum dots, quantum wells, and semiconductor devices and as well as in various dense astrophysical systems such as the compact objects, i.e., white dwarfs and neutron stars. Since the Langmuir oscillations in quantum plasmas has been found to be propagated due to the quantum effect caused by the Bohm potential term [8], the physical properties of quantum plasmas have been extensively investigated using the linearized quantum hydrodynamic equation including the influence of the Bohm effect [9]. It has been known that the ponderomotive force in plasmas would be either caused by the inhomogeneity of the plasma medium or by the inhomogeneity of the field configuration. It is also shown that the spectral information on plasma parameters can be obtained by the spatial and temporal ponderomotive forces which are proportional to the intensity of the field amplitude in plasmas. Further, it is found that the Karpman–Washimi procedure is one of the main mechanisms for the generation of the slowly varying magnetic field by the ponderomotive force in laser heating of plasmas [10]. Since then, the propagation and trapping of the wave in plasmas have been extensively explored by using the nonlinear Karpman–Washimi ponderomotive interaction between the electromagnetic wave and the plasma system [11 – 17]. The stability of the plasma system encompassing the streaming charge component has been considerably investigated in various plasmas [18]. Very recently, the dispersion property for a streaming quantum plasma has been obtained by the Fermi–Dirac distribution with the appropriate Doppler shift [19, 20]. Hence, the Karpman–Washimi ponderomotive magnetization and radiation power in a streaming quantum plasma would be quite different from those in a stationary quantum plasma. However, the physical property of the Karpman–Washimi ponderomotive magnetization and radiation power in a streaming quantum plasma caused by the time-variation of the field in-
tensity has not been investigated as yet. Thus, in this paper, we have investigated the influence of the electron streaming and resonance on the non-stationary Karpman–Washimi ponderomotive magnetization in a cold streaming quantum plasma with the appropriate Doppler shift.

In Section 2, we discuss the Karpman–Washimi procedure due to the ponderomotive interaction. In Section 3, we obtain the ponderomotive magnetization and radiated power in a streaming quantum plasmas and also discuss the dispersion property of the quantum plasma. In Section 4, we obtain the influence of the streaming and resonant phenomena on the ponderomotive Karpman–Washimi magnetization and radiated power. Finally, the conclusion is given in Section 5.

2. Karpman–Washimi Mechanism

In the Karpman–Washimi mechanism [10, 11], the total ponderomotive force \( F_{\text{Tot}}(r,t) \) of the electromagnetic field \( E(r,t) \)
\[ F_{\text{Tot}}(r,t) = F_{s,P}(r,t) + F_{t,P}(r,t), \] (1)
where \( E(r,t) \) represents the envelope of the electromagnetic wave at the position \( r \) and time \( t \), \( k \) is the wave vector, and \( \omega(k) \) is the frequency of the wave propagating in the plasma medium. The notation ‘s’ stands for the complex conjugate. Here, \( F_{s,P}(r,t) \) and \( F_{t,P}(r,t) \) are, respectively, the ponderomotive forces related to the space-variation of the electromagnetic field, i.e., \( \nabla |E_0(r,t)|^2 \):
\[ F_{s,P}(r,t) = \frac{1}{16\pi} [\epsilon(\omega,k) - 1] \nabla |E_0(r,t)|^2, \] (2)
and the time-variation of \( |E_0(r,t)|^2 \):
\[ F_{t,P}(r,t) = \frac{1}{16\pi} \omega^2 \frac{\partial}{\partial \omega} \left[ \omega^2(\epsilon(\omega,k) - 1) \right] \nabla \frac{\partial}{\partial t} \left[ k|E_0(r,t)|^2 \right], \] (3)
where \( \epsilon(\omega,k) \) is the permittivity of the plasma system. The slowly varying electric field \( E_S(r,t) = F_{\text{Tot}}(r,t)/n_0e \) balanced by the total ponderomotive force per unit volume \( F_{\text{Tot}}(r,t) \) would be then obtained by the force balance condition [11]:
\[ E_S(r,t) = \frac{1}{16\pi n_0 e} \left\{ \nabla \left[ (\epsilon(\omega,k) - 1)|E(r,t)|^2 \right] \right. \]
\[ + \frac{1}{e} \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial \omega} \right] \nabla \frac{\partial}{\partial \omega} \left. \left[ (\omega^2(\epsilon(\omega,k) - 1)) k|E(r,t)|^2 \right] \right\}, \] (4)
where \( n_0 \) is the average electron density, \( e \) is the magnitude of the electron charge, and \( c \) is the speed of light. Since the curl of \( F_{s,P}(r,t) \) is zero, the nonlinear field [21] part \( E_{NL} \left\{ \nabla \left[ (\epsilon - 1)|E|^2 \right] \right\} \) in the first term on the right-hand side of (4) would be conservative in a plasma. Hence, the corresponding ponderomotive vector potential \( A_P(r,t) \) and ponderomotive magnetic field \( B_P(r,t) \) generated by the current of the direction \( k \) would be obtained by the following forms:
\[ A_P(r,t) = -\frac{1}{16\pi n_0 e} k|E_0(r,t)|^2 \frac{\partial}{\partial \omega^2 \omega} \] \[ \left. \left[ \omega^2(\epsilon(\omega,k) - 1) \right] \right), \] (5)
\[ B_P(r,t) = -\frac{1}{16\pi n_0 e} \omega^2 \frac{\partial}{\partial \omega} \left[ n_0(\omega(\epsilon(\omega,k) - 1)) \right] \] \[ \left. \nabla \times \left[ k|E_0(r,t)|^2 \right] \right). \] (6)

3. Magnetization in Streaming Quantum Plasmas

In recent years, quantum plasmas have been usually explored by the Wigner–Poisson and Schrödinger–Poisson equations in the mean-field formulations with suitable boundary and initial conditions [20]. Recently, the plasma permittivity function [19, 20] \( \epsilon_{q}(\omega,k) \) in unmagnetized quantum plasmas composed of the streaming electrons and the motionless ions including the quantum density fluctuations caused by the Bohm potential term as well as the Fermi pressure effect has been obtained by using the quantum hydrodynamic model:
\[ \epsilon_{q}(\omega,k) = 1 - \frac{\omega_{pe}^2}{(\omega - k \cdot U_0)^2 - k^2 V_F^2 - \hbar^2 k^4/4m_e^2}, \] (7)
since the frequency \( \omega' \) in a frame of reference moving with the streaming electrons is replaced by \( \omega' = \omega - k \cdot U_0 \), i.e., the Doppler shift, where \( U_0 \) is the streaming velocity of the electrons, \( \omega_{pe}[=(4\pi n_e e^2/m_e)^{1/2}] \) is the electron plasma frequency, \( n_e \) is the electron number density, \( m_e \) is the mass of the electron, \( V_F[= ...
(2k_B T_e/m_e)^{1/2} is the electron Fermi velocity, k_B is the Boltzmann constant, T_e is the electron Fermi temperature, \( \hbar \) is the rationalized Planck constant, and the term proportional to \( \hbar^2 \) stands for the electron quantum diffraction effect. In magnetized plasmas, the time derivatives of the field amplitude should be taken into account. Then, we consider the time derivatives of the field as \( \partial |E_0|^2 / \partial t \sim |E_0|^2 / L \), where \( L \) stands for the typical scale length of \( |E_0|^2 \). Hence, the strength of the Karpman–Washimi ponderomotive magnetic field \( B_{KW} \) in a streaming quantum plasma is found to be

\[
B_{KW}(\omega, k, U_0, V_F) = \frac{e}{8\pi n_0 \sigma_e} \left[ \frac{\hbar^2 k^4}{4m_e^2} - k^2 (U_0^2 - V_F^2) + k U_0 \omega \right] \left[ (\omega - k U_0)^2 - k^2 V_F^2 - \frac{\hbar^2 k^4}{4m_e^2} \right]^{-2} \frac{k}{L} |E_0|^2.
\]

(8)

As shown, the streaming effects on the Karpman–Washimi ponderomotive magnetic field \( B_{KW} \) are explicitly indicated in (8). Then, the induced cyclotron frequency \( \omega_{\text{ind}}(\equiv \omega_{\text{ind}}(\omega_{pe}) \right) \) associated with the induced Karpman–Washimi magnetic field \( B_{KW} \) due to the non-stationary ponderomotive interaction related to the time-variation of the intensity of the field in a streaming quantum plasma is obtained by

\[
\omega_{\text{ind}}(\omega, k, U_0, V_F) = \frac{\omega_{pe}}{8\pi m_e n_0 \sigma_e} \left[ \frac{\hbar^2 k^4}{4m_e^2} - k^2 (U_0^2 - V_F^2) + k U_0 \omega \right] \left[ (\omega - k U_0)^2 - k^2 V_F^2 - \frac{\hbar^2 k^4}{4m_e^2} \right]^{-2} \frac{k}{L} |E_0|^2.
\]

(9)

\[
M_{KW}(\omega, k, U_0, V_F) = \frac{\pi}{4} \left[ \frac{\hbar}{k} \left( \sqrt{k^2 - \omega^2} \right)^2 - \frac{\hbar}{k} \left( \sqrt{k^2 - \omega^2} \right)^2 \right]^{-1}.
\]

(10)

\( \tilde{u}_0 \equiv e |E_0| / m_e \omega_{pe} \lambda_0 \). As shown in this equation, the streaming and resonant effects provide crucial influence on the induced magnetization in quantum plasmas. This expression of the Karpman–Washimi ponderomotive magnetization \( M_{KW}(\omega, k, U_0, V_F) \) would be then the key parameter for investigating the physical characteristics of the quantum density fluctuations in a streaming cold quantum plasma. In quantum plasmas, when the density and temperature are given by \( n_0 = 3.2 \cdot 10^{18} \text{ cm}^{-3} \) and \( T_e = 348 \text{ K} \), the Fermi wave length \( \lambda_F = (k_B T_e / 2\pi n_0 \sigma_e)^{1/2} \) and standard Debye length \( \lambda_D \) are, respectively, given by \( \lambda_F = 5.0 \cdot 10^{-8} \text{ cm} \) and \( \lambda_D = 7.2 \cdot 10^{-8} \text{ cm} \). For a cold quantum plasma such as \( n_0 = 3.2 \cdot 10^{18} \text{ cm}^{-3} \) and \( T_F = 30 \text{ K} \), the Fermi wave length and Debye length are also given by \( \lambda_F = 1.5 \cdot 10^{-8} \text{ cm} \) and \( \lambda_D = 2.1 \cdot 10^{-8} \text{ cm} \). Then, we have found that the Debye length is usually greater than the Fermi wave length in quantum plasmas. Hence, the influence of the streaming electrons on the Karpman–Washimi magnetization can be investigated by the variation of the resonant behaviour of the ponderomotive magnetization function \( \partial M_{KW}(\omega, k, U_0, V_F) / \partial \omega \):

\[
\partial M_{KW}(\omega, k, U_0, V_F) = \left( \frac{\pi k^3}{4m_e} \left( \frac{k^2 - \omega^2}{U_0^2 - V_F^2} + k U_0 \frac{\omega}{k} \right) \left( \frac{\omega^2}{k^2} - (U_0^2 - V_F^2) \right)^{-1} \right) \left( \frac{\omega^2}{k^2} + (U_0^2 - V_F^2) \right)^{-1}.
\]

(11)

Since the electron Larmor process produces the cyclotron emission spectrum, the Karpman–Washimi radiated power \( P_{KW}(\omega, k, U_0, V_F) \) due to the ponderomotive interaction in a streaming quantum plasma is represented by

\[
P_{KW}(\omega, k, U_0, V_F) = \frac{\pi^4 P_0}{2} \left( \frac{k^8}{U_0^2 - V_F^2 + k U_0 \omega} \right) \left( \frac{\hbar^2 k^4}{4m_e^2} - k^2 V_F^2 - \frac{\hbar^2 k^4}{4m_e^2} \right)^{-1}.
\]

(12)

where \( P_0 \equiv e^2 U_0^2 \lambda_0 / 3^{3/2} L^4 \) and \( r_L \) is the Larmor radius for the cyclotron motion.
4. Streaming and Resonant Effects

It is interesting to note that the investigation of the Karpman–Washimi ponderomotive magnetization $M_{KW}$ is essential to explore the physical consequences of the streaming and resonant phenomena in quantum plasmas. Figure 1 shows the Karpman–Washimi ponderomotive magnetization $M_{KW}$ as a function of the scaled wave number $\bar{k}$ for various values of the scaled streaming electron velocity $\bar{U}_0$. As shown in this figure, the influence of the electron streaming in cold quantum plasmas enhances the ponderomotive magnetization $M_{KW}$ in small wave numbers, i.e., $M_{KW}$ for $\bar{U}_0/\bar{V}_F > 1$ is greater than $M_{KW}$ for $\bar{U}_0/\bar{V}_F < 1$. In the small wave number domain, it is then expected that the Karpman–Washimi ponderomotive magnetizations in streaming quantum plasmas are always greater than those in stationary quantum plasmas. Figure 2 shows the resonant behaviour of the Karpman–Washimi ponderomotive magnetization $M_{KW}$ as a function of the scaled wave number $\bar{k}$ for various values of the scaled streaming electron velocity $\bar{U}_0$. As it is seen, the streaming effect shifts the resonant wave number of the Karpman–Washimi ponderomotive magnetization to the smaller wave number domain. It is interesting to note that the resonant wave number of the Karpman–Washimi ponderomotive magnetization for $\bar{U}_0/\bar{V}_F = 1$ is found to be smaller than that for $\bar{U}_0/\bar{V}_F < 1$, i.e., almost stationary quantum plasmas. Hence, it is also found that the quantum effect plays an important role in the ponderomotive magnetization in streaming quantum plasmas. Figure 3 represents the surface plot of the variation of the ponderomotive magnetization $\partial M_{KW}$ near the resonant wave number region as a function of the scaled streaming electron velocity $\bar{U}_0$ and scaled electron Fermi velocity $\bar{V}_F$. From this figure, it is found that the electron streaming velocity $\bar{U}_0$ for the resonant domain of the ponderomotive magnetization decreases with an increase of the electron Fermi velocity $\bar{V}_F$. Hence, we have found that the quantum effect reduces the electron streaming velocity for the ponderomotive magnetization.
tization near the resonant wave number. Figure 4 represents the surface plot of the Karpman–Washimi radiated power $P_{KW}$ due to the ponderomotive force in a streaming quantum plasma as a function of the scaled wave number $\bar{k}$ and scaled wave frequency $\bar{\omega}$. As we can see in Figure 4, it is found that the wave frequency for the resonant Karpman–Washimi radiated power increases with an increase of the wave number. In addition, it is found that the relation for the wave number and wave frequency shows a nonlinear character since the radiated power $P_{KW}$ contains the electron quantum diffraction effect $k^4$, i.e., $\hbar^2 k^4 / 4m_e^2$, in (12). Hence, the influence of the electron streaming on the ponderomotive magnetization in quantum plasmas would be expressed by the characteristic function $F(\bar{\omega}, \bar{k}, \bar{U}_0, \bar{V}_F)$:

$$F(\bar{\omega}, \bar{k}, \bar{U}_0, \bar{V}_F) = \left( \frac{k^4 - k^2(\bar{U}_0^2 - \bar{V}_F^2) + \bar{k}U_0\bar{\omega}}{k^4 + k^2\bar{V}_F^2} \right)^4 \left( \frac{\bar{\omega}^2 - \bar{k}^2\bar{V}_F^2 - \bar{k}^4}{(\bar{\omega} - k\bar{U}_0)^2 - k^2\bar{V}_F^2 - k^4} \right)^8$$

(13)

It has been also known that the non-ponderomotive nonlinear force proportional to the plasma collision frequency, i.e., the Stamper force [21], in addition to the Karpman–Washimi ponderomotive force would be caused by the electrodynamic interaction of the electromagnetic wave with the plasma in a collisional plasma. Recently, excellent investigations are given for the ponderomotive acceleration of electrons and self-focusing and self-channelling processes [22, 23]. Hence, the investigation of the Stamper force, ponderomotive acceleration, and self-focusing effects on the radiated power in quantum plasmas will be treated elsewhere. Very recently, excellent investigations [24, 25] on the ponderomotive acceleration of electrons by a short laser pulse undergoing relativistic self-focusing and the relativistic self-distortion of a Gaussian laser pulse have been investigated in plasmas. Then, the investigation of the plasma screening effect on the ponderomotive acceleration in quantum plasmas will also be treated elsewhere.

5. Conclusion

In this paper, we have investigated the influence of the electron streaming and resonance on the non-stationary Karpman–Washimi ponderomotive magnetization and radiated power in a streaming quantum plasma. The induced Karpman–Washimi magnetization and radiation power due to the ponderomotive force are obtained as functions of the electron streaming velocity, Fermi velocity, wave frequency, and wave number. In this study, we have found that the electron streaming effect enhances the Karpman–Washimi ponderomotive magnetization for small wave numbers. It is also found that the electron streaming effect shifts the resonant wave number to the smaller wave number domain and, however, the quantum effect reduces the electron streaming velocity for the ponderomotive magnetization near the resonant wave number. In addition, we have found that the wave frequency for the resonant Karpman–Washimi radiated power increases with increasing wave number. Hence, from this work, we have found that the streaming and resonant effects play crucial roles in the Karpman–Washimi ponderomotive magnetization and radiated power in a streaming cold quantum plasma. In this work, we can found that the Karpman–Washimi magnetization for a quantum plasma with $n_0 = 3.2 \times 10^{18}$ cm$^{-3}$ and $T_F = 348$ K is found to be greater than that for $n_0 = 3.2 \times 10^{18}$ cm$^{-3}$ and $T_F = 348$ K due to the temperature effects on the Karpman–Washimi ponderomotive magnetization. Hence, it would be useful to investigate the plasma temperature using the non-stationary Karpman–Washimi magnetization procedure in quantum plasmas. In addition, we can explore the streaming velocity in quantum plasmas using the Karpman–Washimi radiated power $P_{KW}(\bar{\omega}, \bar{k}, \bar{U}_0, \bar{V}_F)$. These results would provide useful information on the Karpman–Washimi magnetization and radiated power.

Fig. 4 (colour online). Surface plot of the Karpman–Washimi radiated power $P_{KW}$ as a function of the scaled wave number $\bar{k}$ and scaled wave frequency $\bar{\omega}$ when $\bar{U}_0 = 2$ and $\bar{V}_F = 1$. 

power due to the non-stationary ponderomotive interaction in streaming dense quantum plasmas containing low-temperature and high-density electrons and ions.

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