AMBIGUITY AND INFORMATION IN THE CONTEXT OF 'NATURAL COMMUNICATION': AN OBSTACLE- ORIENTED GALOISIAN STANDPOINT ELIAS ZAFIRIS

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THE 'OBSTACLE-ORIENTED' APPROACH TO COMPLEXITY AND COMMUNICATION

The present essay arose out of the curiosity to delve deeper into the nature of obstacles or obstructions that prevent a single, uniform, and linear approach to dealing with systems, entities or, more generally, beings, and their communication. The current motivation comes from a certain degree of dissatisfaction with the overflow of scientific production on what is called 'science of complexity' and the qualification of what accounts as 'information of complex systems' based exclusively on specific 'target-oriented approaches'.

The thread of this amphiboly starts from questioning what is considered to be 'complex' in contradistinction to what is considered to be 'simple.' Usually these terms are implicitly pre-loaded with an ontological meaning, which essentially identifies complexity with certain aggregations of elementary sharply distinguishable constituents developing emergent properties that are behaviourally observed under specific interactions or conditions. This approach is assumed to be valid irrespectively of scale, depending on what is axiomatically baptized as an elementary constituent, so that certain statistical patterns can be applied upon them targeting the simulation of their behaviour. A natural set of questions in this setting is the following: What makes an entity a constituent, how can an entity be characterized as elementary, and most important, how can an entity be sharply distinguishable?

The afterthought of these considerations is that the epithet 'complex' emanates from the contradistinction to what is called 'simple', where what is 'simple' is identified with the 'axiomatic elementary' in the context of the vast majority of these 'target-oriented approaches'.

Notwithstanding these scientific tendencies of the present, this is not actually the way that humankind came to terms with complexity. The targets were not predefined, but rather always emerged out of necessity to cope with obstacles in communication of every particular sort. In this respect, the meaning of complexity is altered dramatically if theorized from an 'obstacle-oriented' standpoint instead of a 'target-oriented' one. More precisely, the ability to locate obstacles or obstructions forcing a deviation from some condition of inertiality becomes the primordial task. The localization of an obstacle necessitates a process of metaphora around this obstacle, i.e. a temporal flow around it through another level of hypostasis. The metaphora is effective if the flow is communicative, i.e. if it establishes bidirectional bridges connecting these two levels. In this sense, an 'obstacle-oriented' approach is not directly a 'problem-solving' approach, but an indirectly 'problem-embracing' one. The different levels are not in any relation of hypotaxis of one to another, but only in relation of parataxis, and what really matters is the connectivity in the passage from one level to another in a

bidirectional way via the bridges. Most significantly, connectivity does not pre-suppose sharply distinguishable entities, but is deformation invariant given the localization of an obstacle. This is quite intuitive to conceive, since otherwise it would defeat the purpose of metaphora in relation to communication.

In this state of affairs, if the history of humankind is viewed from the standpoint of the 'obstacle-oriented approach' to complexity, pertaining to locating and embracing obstructions of any particular type, a naturally arising problematics refers to delineating the most persistent and covariant forms of this stance. The character of temporal persistence and covariance is meant to serve as a criterion for the 'ousia ontos ousa' of history. Surprisingly enough, this criterion filters out only two kinds of entities, namely myths and theorems. At a first encounter with this provocative claim, there seems to be a contradiction, but this is only apparent, caused mainly from a certain type of pre-occupation with 'target-oriented' methods instead of 'obstacle-oriented ones'. Indeed, both myths and theorems become noematic if they are obstacle-embracing: They are actually different types of metaphora in relation to communication. Myths and narratives constitute an embodied symbolic process of metaphora around a localized obstacle. Mathematical theorems constitute an abstract symbolic process of metaphora around a localized obstacle. Although mythos and logos deceptively seem to occupy opposite sides of the linear spectrum forced by 'analytic reason', they are, in fact, antipodally inter-related and topologically glued together in the projective geometric rooting of this linear spectrum, centred around the notion of an obstacle.

In order to articulate this claim, we will consider the dual pair of concepts consisting of ambiguity and information in relation to communication. On the side of the myth, we will scrutinize the narrative referring to the genesis of Goddess Aphrodite according to Hesiod's Theogony. On the side of the logos, we will scrutinize the theorem referring to the genesis of the roots of a polynomial equation by radicals according to Galois' Theory. The remarkable conclusion is that in both cases information emerges as 'anadyomene' from another level of hypostasis, being in communication with the level where the initial obstruction is located via bidirectional bridges. Before embarking on this task, it is instructive to include a basic familiarization with the model of 'natural communication', which constitutes the 'nuts and bolts' of the 'obstacle-oriented approach' to complexity.

THE MODEL OF 'NATURAL COMMUNICATION'

The main initial motivation behind the set-up of the model of 'natural communication' has been the transcendence of the current, 'target-oriented' complexity paradigm into a new one, envisioning and

implementing an architectonics of communication, encapsulating the concepts of category theory, sheaf theory, and quantum mechanics. The primal realization is that an architectonics of relations based on communication is ultimately necessitated in all cases, where direct accessibility to sharply distinguishable domains of objects and their behaviour is not feasible due to obstacles or obstructions of any particular type. In these domains, objects are intrinsically shaped according to foamy or cloudy patterns, and they are characterized by topological plasticity, emergent properties and generically probabilistic attributes. The application of pre-specific, readily-tailored design ontologies to these domains, based on the notion of sharply-distinguishable elementary constituency, not only distorts the architectonics of their non-trivial connectivity patterns, but limits and restricts, even inadvertently, their potential computational capacities.

The basic idea to address and utilize the architectonic modelling of these domains is the following: Instead of analysing not-directly-accessible domains in terms of set-based elements and their hypothetical absolute relations, adjoin to them other adequately-understood or directly-accessible domains, which can provide pointers and open up communication channels with the former ones. The process of adjoining should not be ad hoc and should not depend on artificial choices, meaning that it should be qualified as structurally-respecting, at least, locally or partially. This refers to the ‘naturalness requirement’ of the model, a term which is exemplified in the context of category theory. Technically, this adjunction process is described in terms of a pair of adjoint functors between the involved categorical domains.

In practice, the process of adjoining a controllable domain to another one, initially not directly accessible, or obstacle-laden categorical domain, amounts to viewing these domains as different categorical levels in a stratified universe of discourse, which are bi-directionally connected by means of opposite-oriented bridges. The architectonics of communication targets precisely the conception and explicit construction of these bridges, once suitable probing domains have been structurally delineated for adjunction to the directly inaccessible domain. The bridge directed to the controllable domain plays the role of an encoding bridge, whereas its inverse plays the role of a decoding bridge. These connecting bridges affect the communication between the involved domains, in the sense that they naturally establish universal bidirectional communication channels through which a schema of metaphora is accomplished, based on the notion of an ‘obstacle-encircling’ temporal flow. Topologically thinking, the initially inaccessible categorical domain is resolved cyclically by a process of unfolding with respect to the probing domain that has been adjoined to it. In this manner, the invariants emerging by the process of unfolding depict the invariant characteristics of the reciprocal communication flow between these

domains. Consequently, the complexity of the not-directly-accessible, or obstacle-laden, domain is not specified constitutionally on the basis of a pre-assumed or axiomatic elementary ontology, but relationally and functorially, in terms of the invariants emerging in the bidirectional communication flow established with some appropriate probing domain. From an algebraic viewpoint, these invariants can be qualified in terms of structural group-type ciphers for the symbolic encryption/decryption of the induced flow.

In a nutshell, the obstacle-embracing schema of metaphora giving rise to this cyclic flow, called 'logical conjugation', is conceptually based on a legitimate logical manoeuvre through a controllable or a directly accessible domain that is adjoined naturally to an obstacle-laden domain, furnishing markers or pointers or gauges of specifying the latter indirectly through bidirectional communication bridges. The logical conjugation method should be thought of operationally in terms of a 'motivic key' that bears the capacity to unlock harmonically the complexity of the inaccessible domain depending on the nature and type of the obstacle. The creative art of the 'obstacle-oriented approach' in the context of the model of 'natural communication' consists in the innovation of motivic keys suited to the nature and type of an encountered obstacle. A motivic key always gives rise to a partition spectrum of the obstacle-laden domain. Each cell of this spectrum is characterized completely by the pair of encoding/decoding bridges utilized for communicating an aspect of the directly inaccessible domain with respect to the probing domain.

For all practical purposes, the model of 'natural communication' can be implemented algorithmically as follows: We consider a problem in the context of a domain whose object and relations are directly inaccessible. It is instructive to think of this domain as a particular level in a broad 'universe of discourse', which can engulf other possible levels as well. First, we move out of the context of the problem, formulated at the level of the inaccessible domain, by adjoining to it another controllable domain, assuming existence within the same universe. In order to accomplish this, we have to set up an encoding bridge from the level of the inaccessible domain to the level of the accessible domain. Once we have succeeded to set up this bridge, we are able to mirror the initial problem at the level of the controllable domain, where we have the means to solve it effectively. The process is completed by setting up an inverse decoding bridge from the level of the controllable domain to the level of the inaccessible one.

In this way, the solution established at the controllable domain can be lifted at the initial context of the problem, and thus the problem is resolved in the context of its initial formulation. The reciprocal encoding and decoding bridges constitute the means of a novel architectonics of communication. This architectonics gives rise to a schema

of metaphora that resolves an inaccessible domain by a communicative bidirectional and cyclic process of embracing the obstacle causing the direct inaccessibility. It has to be stressed that the schema of metaphora can be iterated by the involvement of more than one controllable domains adjoined jointly to the inaccessible domain. The skeleton of this algorithmic procedure of resolving inaccessible domains via motivic keys remains invariant under the adjunctions of deeper levels, and most importantly, always gives rise to a partition spectrum of the inaccessible domain, whose cells are indexed by the respective pairs of encoding/decoding bridges. Conclusively, the spectral resolution conducted by setting up a motivic key to an inaccessible domain for embracing the type of obstacle it is laden with constitutes the essence of logical conjugation, whence the communicating domains, represented by levels connected by the encoding/decoding bridges, are called conjugate domains.

THE GALOISIAN CONCEPTION OF AMBIGUITY AND INFORMATION

Galois' memoir is entitled *On the conditions for the solvability of equations by radicals*. In this work he manages to resolve completely the problem of algebraic solvability of polynomial equations, i.e. equations of the form $p(x) = 0$. In particular, he demonstrated that a general formula for the roots of a fifth (or any higher) degree polynomial equation does not exist by utilizing only the usual algebraic operations of addition, subtraction, multiplication, division, and applying radicals, i.e. roots of n th degree.

The first novelty of Galois' method consists in the introduction of the group concept, which he devised as a structure G to encode the permutations of the roots of a polynomial $p(x)$ in $F[x]$, which map each root onto a conjugate of it, where F is a field, called the ground field of coefficients. The second novelty of Galois' method consists in employing the structure of the above group G together and in relation to the structure of its invariant subgroups under conjugation, i.e. its self-conjugate or normal subgroups. The underlying reason is that the notion of structural divisibility of a Galois group becomes possible only with respect to its normal subgroups. The third novelty of Galois' method consists in the notion of group solvability. The triumph of Galois' theory is based on the theorem that a polynomial equation is solvable by radicals if and only if the corresponding Galois group of the equation is solvable. A solvable group is a group whose derived series terminates at the trivial subgroup in finitely many steps. Intuitively, the derived series is set up by stratification into group levels together with a descending staircase among these strata formed by identifying each subgroup in the descending series with a normal subgroup of the previous one such that the induced factor group is Abelian.

In this state of affairs, the Galois group G corresponding to a polynomial $p(x)$ in $F[x]$, where F is the ground field, can be thought of as a non-numerical, but structural measure of complexity of this polynomial. In turn, the iterative process involved in setting up the derived series of G , by means of normal subgroups, can be thought of as a process of complexity reduction until the trivial subgroup of G is eventually reached. It is important in this frame of thinking to clarify what we mean by a structural measure of complexity as this is encoded in the Galois group of a polynomial $p(x)$ in $F[x]$. The general notion of a group is associated with a criterion of symmetry under its action. What is the implicated notion of symmetry in this case, and how can it be associated with a measure of complexity?

The answer comes from noticing that the Galois group G , being the group of permutations of the roots of a polynomial $p(x)$ in $F[x]$, which maps each root onto a conjugate of it, actually encodes these F -transformations of the roots of $p(x)$ that cannot be distinguished from the resolution capacity afforded by the ground field F . In other words, the Galois group G of $p(x)$ is an ambiguity group with respect to F , and this ambiguity structure amounts to a measure of complexity of $p(x)$ in relation to the spectral resolving means afforded by F . Henceforth, the symmetry encapsulated by the Galois group G of $p(x)$ in relation to F can be reduced only by possible successive symmetry breakings until the total symmetry constituting the structural measure of F -complexity is completely reduced.

Reciprocally, the annihilation of the ambiguity encoded in the Galois group G of $p(x)$ in relation to F —at the level of groups—is decoded by a gradual process of successive field extensions of the ground field—at the ‘obstacle-laden’ level of fields—to be thought of as an unfolding process of F to larger or more refined fields that increase the resolution capacity afforded by the ground field itself, until eventually all roots can become spectrally distinguishable. More precisely, given a polynomial $p(x)$ in $F[x]$, of degree n , it can be shown that there exists a minimal extension of F , called the splitting field F_p of $p(x)$, in which all n roots of $p(x)$ can be distinguished. The splitting field F_p of $p(x)$, constitutes in this way the smallest total field extension of the ground field F over which the polynomial $p(x)$ splits or decomposes into linear factors. Spectral distinguishability of the roots of $p(x)$ constitutes the pre-condition for qualifying each of these roots as a bearer of information, by means of an actual spectral distinction in the context of the splitting field F_p of $p(x)$.

In this manner, ambiguity—as a group-theoretic measure of complexity—, and information—as a spectral measure of actual distinctions—, form a Galoisian complementary pair, whence the level of groups and the level of fields constitute conjugate domains. The descending series of ambiguity groups starting from the Galois group until

the trivial group is eventually reached, represents all the successive stages of symmetry breaking, i.e. all the successive stages of complexity or ambiguity reduction with respect to the ground field until all the roots of $p(x) = 0$ become spectrally distinguishable in the inversely proportional ascending series of field extensions from the ground field to the splitting field of $p(x)$. Therefore, information in Galois' theory emerges via a cyclic metaphorical process from the 'obstacle-laden' level of fields to the level of groups and back in the form of 'anadyomene' through the foam of spectral ambiguity, embracing eventually the roots-obstacles.

THE MYTH OF APHRODITE ANADYOMENE REVISITED

Following the narrative of Hesiod's *Theogony*, the genesis of Goddess Aphrodite took place in an extraordinary manner. She emerged in the form of a fully-grown female figure rising out of the sea foam (aphros). Her name Aphrodite, or Aphrogeneia, according to Hesiod, originates from the fact that she grew and formed amid the sea foam. The epithet 'anadyomene' uncovers the metaphor process of Aphrodite's genesis. This is a process of circulation between the Heavenly level (Uranus), and the Earthly level (Gaia), which is meant to embrace the obstacle of heterogeneous, chimerical and intrinsically incongruent constituency of these two levels. The initiator of this obstacle-embracing communicative process is the Titan Chronos, who stands for the personification of Time as a unitary circular flow bridging reciprocally these two levels into a germinal syzygy. According to the myth, Chronos forced the transposition of the sperm of Uranus into the sea water, and out of this syzygy a white foam spread around the locus of germination in the sea from the immortal flesh. In this 'ambiguity foam' there started to grow the maiden Aphrodite virtualized, to emerge eventually as a fully grown in-formed spectral figure by inverse transposition to the level of Gods, completing in this way the temporal circulatory metaphorical process of anadyomene initiated by Chronos. It is important to highlight that the two levels of the myth are not hierarchically ordered, meaning that there is no relation of hypotaxis or subordination of one level to the other. On the contrary, the two levels exist autonomously in parataxis and only the unitary circular flow of Chronos binds them in communication through reciprocal bridges. The bridge from the Heavenly to the Earthly level is the germination bridge, whereas the reciprocal, or conceptually inverse one, is the virtual growth bridge. The latter is characterized as virtual because the growth of the figure to emerge as Aphrodite is always surrounded by the sea foam, and thus, the growing figure is indistinguishable and inseparable from the foam. This is crucial for conceiving both the symmetry implicated by the circular unitary flow-action of Chronos and the emergence of Aphrodite

as a fully grown in-formed figure only after gradually breaking the symmetry of this inter-level communicative flow, and thus, separated by the sea foam and distinguished or discerned spectrally from it at the level of Gods.