

Preface

Historically, most of the turbulence studies were concerned with nonconductive fluids, described by the Navier–Stokes equations. This is because most fluids present on Earth are nonconductive. In the context of a larger Cosmos, this situation is not a rule but rather an exception. Indeed, space is filled with ionizing radiation and only the protection of our atmosphere, which is very dense by astronomical standards, allows us to have a big volumes of insulating fluids, such as the atmosphere and the oceans. In contrast, most of the ordinary matter in the universe is ionised, that is, in a state of plasma. The description of ionized, well-conductive fluids must include magnetic Lorentz force and the induction equation for the magnetic field. As it turned out, turbulent conductive fluids tend to quickly generate their own magnetic fields in the process known as dynamo. On the other hand, the presence of the dynamically important magnetic field could be considered an observational fact. In spiral galaxies, the magnetic field has a regular component, usually along the arms and a random turbulent component of the same order. The value of the magnetic field, around $5 \mu\text{G}$, roughly suggests equipartition between magnetic and kinetic forces. The excitation of magnetic turbulence is thought to be the main cause of accretion, because hydrodynamic thin disks are stable. Accretion onto black holes is estimated to be the most potent source of energy in Cosmos, exceeding thermonuclear burning in stars. Closely related to this problem are widely observed astrophysical jets, which are highly collimated flows perpendicular to the accretion disks in which the magnetic field plays an important role in collimation and energy transfer. Understanding shear-driven dynamo and its nonlinear evolution is of high astrophysical importance. Direct observation of magnetic turbulence is the interstellar medium, galaxy clusters, solar wind have confirmed earlier claims that turbulence is inevitable in high-Reynolds' number flows, even though astrophysical flows are characterized by fairly small number densities. Through the book we mention various examples of astrophysical turbulence as well as turbulence in our solar system, the heliosphere, while the observational techniques to study, specifically, astrophysical turbulence are reviewed in the last chapter.

Observations of magnetized turbulence in the interstellar medium, galaxy clusters and the solar wind have confirmed that turbulence is indeed ubiquitous in astrophysical flows and has been detected in almost all astrophysical and space environments; see, e. g., [173, 11, 76]. The Reynolds' numbers of astrophysical turbulence are, typically, very high, owing to astrophysical scales which are enormous compared to dissipative scales. Recent years have been marked by a new understanding of the key role that turbulence plays in a number of astrophysical processes [90, 129]. Most notably, turbulence has drastically changed the paradigms of interstellar medium and molecular cloud evolution [408, 343, 431]; see also [311]. While small scale, kinetic turbulence has been probed by a variety of approaches such as gyrokinetics, Hall MHD and electron MHD [196, 381, 84], this book is mostly concerned with so-called mag-

netohydrodynamics of MHD, which is a simple one-fluid description, similar to the Navier–Stokes equation. It could be thought as a 2nd Newton’s law for conductive fluid and also could be derived systematically from kinetic theory. The conventional condition of the applicability of the fluid approach, namely that a mean-free path of the particle due to particle-particle collisions must be much smaller than gradient scales of the problem (the so-called Knudsen number much smaller than unity) seems to be too restrictive in most plasmas, especially astrophysical plasmas. Indeed, plasmas are often collisionless, i. e., the effects of collisions could be neglected compared to collective effects, as charged particles are capable of long-range interactions. In many cases, such as the star’s interiors, plasma could be considered highly collisional. In other environments, such as solar wind or galaxy clusters, pair collisions will be inefficient and collective interactions must play their own dissipative role. The observations of inertial range fluctuations way below pair collision mean-free path in the solar wind support this idea. Although this book is mostly focused on standard nonrelativistic MHD, in Chapter 5 we mention some relativistic MHD studies, which are relevant for relativistic astrophysical sources such as gamma-ray bursts and active galactic nuclei (AGN) jets. Relativistic force-free MHD is an important limit applicable to relativistic electron-positron jets close to the central engine of AGN, where the energy density of matter is negligible compared to electromagnetic energy density.

Turbulence is a common phenomenon, a time-dependent, quasi-stochastic flow, associated with nonlinearities present in fluid equations. Dissipation in fluids is often associated with microscopic phenomena and dissipation scales are typically much smaller than the scales of the problem. Physically, the development of turbulence could be seen as exciting many degrees of freedom present in a fluid system. Turbulence has been observed since long time ago, as evidenced by Leonardo sketches. A major role in turbulence research is played by invariants of the “ideal” equations free of dissipation, examples of which are energy conservation or the circulation of velocity frozen into a fluid. The way these invariants are broken by turbulence could be insightful for understanding observable physical phenomena. Some of the conserved quantities form a “cascade” through scales, which could be understood quantitatively by scaling arguments. In the context of magnetized fluids turbulence breaks an ideal frozen-in condition for magnetic field lines and facilitates magnetic reconnection. As stochastic phenomenon turbulence is best studied with statistical methods, the quantities averaged over ensemble play a major role. These include power spectra and structure functions. Turbulence theory produced a number of analytically derived relations for certain structure functions, which allows for cross-checks for consistency with observation and experiment.

Lately, the theory and experiment has been complemented by a new method, numerical simulations that we often call “numerics” in this book. One example of the type of numerical simulations, direct numerical simulations (DNS), are prominently presented in this book. DNS refers to a “fully resolved” numerical experiment, where numerics faithfully reproduce properties of the original equations of fluid dynamics.

An opposite of DNS, Implicit Large Eddy Simulations (ILES), are also very common in astrophysics. These are not attempting to archive high accuracy at all scales, but instead aim to reproduce large-scale and intermediate-scale dynamics correctly. Numerics stand in between theory and experiment. On one hand, numerics solve MHD equations directly, without regard to doubt whether MHD is applicable. The simulation setup is often abstract, such as using a periodic box, attempting to emulate infinite space and/or statistically homogeneous turbulence. In nature, e. g., in the solar wind, homogeneity is rather an exception. In this aspect, numerics are close to theory. On the other hand, numerics are often called a “numerical experiment,” as it measures the relevant physical quantities of the phenomena without invoking much of the theoretical assumptions or prerequisites. Compared to a real-life experiment, in numerics it is easier to study statistically homogeneous or statistically stationary state cases and it is possible to measure statistical quantities very well, in principle with arbitrary precision. Progress in computing allowed us to use grid sizes of around 4000^3 nowadays, and Reynolds’ numbers in these experiments started approaching those encountered in space physics.

Chapter 2 overviews the origin of astrophysical magnetic fields, a problem which is known as turbulent dynamo. This problem can be roughly subdivided into the so-called large-scale dynamo, a generation of magnetic fields on scales larger than the outer scale of turbulence and small-scale dynamo, which generates small-scale turbulent magnetic perturbations. On some level, MHD equations seem to suggest that magnetic field is “frozen” into well-conducting fluids, and as it is being carried around by the fluid magnetic field it is amplified by stretching and folding. This process has been extensively studied since the 1960s. In this book, we decided that it would be impossible to cover such a broad topic as turbulent dynamo; instead, we wrote Chapter 2 intending to demonstrate that turbulent dynamo is generic, i. e., given kinetic motions dominate over magnetic stress the dynamo will continue to increase magnetic energy. We restrict ourselves to small-scale dynamo, which is more generic. Also we discuss mostly “nonlinear dynamo,” the situation in which magnetic forces cannot be ignored, albeit often on fairly small scales, which is universally applicable to most observed objects.

Similar to hydrodynamic turbulence, most of the theoretical progress on MHD turbulence has been made in the so-called incompressible limit, which is covered in Chapter 3. The physical justification of this limit in hydrodynamics was the fact that sound waves could often be ignored in turbulent dynamics and the solenoidal turbulent motions have their own dynamics, which are well-described by incompressible equations, at least on small scales. Similar notion is used in MHD where it is often possible to assume that the MHD fast mode splits from an incompressible cascade of Alfvénic and slow modes. It turns out that another split is possible due to the extreme anisotropy in the inertial range of MHD turbulence when the slow mode becomes passive and does not contribute to essential nonlinearity, its cascade being slaved to the

Alfvénic mode. The cascade of Alfvénic mode is often called Alfvénic turbulence or reduced MHD and contains all necessary statistical properties of small-scale MHD turbulence. It turns out that this MHD turbulence in a strong mean field (strong local B_0 compared to perturbations) is essentially three-dimensional, and not two-dimensional as was thought before.

Iroshnikov and Kraichnan first pointed out that the magnetic field cannot be excluded by the choice of reference frame, therefore, in every parcel of the fluid a mean magnetic field will remain, which would be much stronger than small-scale perturbations. This naturally led to further studies which took into account anisotropy of MHD turbulence in a strong mean field namely the work of Goldreich and Sridhar which used an uncertainty relation between the wave frequency and the cascade timescale to formulate the so-called “critical balance.” Later we show how this picture is connected to the modern view of MHD turbulence. Chapter 3 describes a special case of MHD turbulence without cross-helicity. Physically, this means that the amount of perturbations propagating in one direction along mean magnetic field is perfectly statistically balanced by perturbations moving in the opposite direction; this case being called “balanced turbulence.” However, in real systems, MHD turbulence is often imbalanced, because there are sources and sinks of energy and these are rarely homogeneous in space. Also, due to fluctuations of all quantities in time, conceptually, one will have to understand the most general imbalanced case in order to create a theory for the specific balanced case. Unlike hydrodynamics, where the introduction of fluctuations of dissipation rate ϵ in the theory was rather obvious, e. g., Kolmogorov–Obukhov extension of the Kolmogorov model or the She–Leveque model, in MHD the introduction of imbalance significantly complexifies theory. In Chapter 4, we discuss some new ideas which lead to creation of imbalanced turbulence models and their numerical testing.

Chapter 5 is devoted to compressible MHD turbulence and various approaches in the weakly compressible, as well as supersonic limit. In MHD, the decomposition into four basic linear modes has been insightful for describing strongly compressible turbulence. The study of supersonic ISM turbulence is important for understanding the structure of molecular clouds and subsequent star formation. In this respect, the studies of density scalings and thermal instability in DNS have become commonplace. The solar wind, a MHD flow emitted from the sun at speeds 400–800 km/s is also compressible and its properties and the transition to dispersive regime at small scales is a large part of MHD turbulence now, due to availability of in-situ measurements.