

# Contents

Preface — vii

## 1 Preliminaries — 1

- 1.1  $\mathbb{R}^p$  Space — 1
- 1.2 Limits of Functions and Continuity — 5
- 1.3 Differentiability — 10
- 1.4 The Riemann Integral — 15

## 2 Nonlinear Analysis Fundamentals — 18

- 2.1 Convex Sets and Cones — 18
- 2.2 Convex Functions — 30
  - 2.2.1 General Results — 30
  - 2.2.2 Convex Functions of One Variable — 38
  - 2.2.3 Inequalities — 42
- 2.3 Banach Fixed Point Principle — 52
  - 2.3.1 Contractions and Fixed Points — 53
  - 2.3.2 The Case of One Variable Functions — 62
- 2.4 Graves Theorem — 71
- 2.5 Semicontinuous Functions — 73

## 3 The Study of Smooth Optimization Problems — 78

- 3.1 General Optimality Conditions — 78
- 3.2 Functional Restrictions — 91
  - 3.2.1 Fritz John Optimality Conditions — 92
  - 3.2.2 Karush-Kuhn-Tucker Conditions — 95
  - 3.2.3 Qualification Conditions — 102
- 3.3 Second-order Conditions — 109
- 3.4 Motivations for Scientific Computations — 113

## 4 Convex Nonsmooth Optimization — 117

- 4.1 Further Properties and Separation of Convex Sets — 117
- 4.2 The Subdifferential of a Convex Function — 120
- 4.3 Optimality Conditions — 125

## 5 Lipschitz Nonsmooth Optimization — 131

- 5.1 Clarke Generalized Calculus — 131
  - 5.1.1 Clarke Subdifferential — 131
  - 5.1.2 Clarke Tangent and Normal Cones — 148
  - 5.1.3 Optimality Conditions in Lipschitz Optimization — 156
- 5.2 Mordukhovich Generalized Calculus — 159

- 5.2.1 Fréchet and Mordukhovich Normal Cones — 160
- 5.2.2 Fréchet and Mordukhovich Subdifferentials — 167
- 5.2.3 The Extremal Principle — 177
- 5.2.4 Calculus Rules — 181
- 5.2.5 Optimality Conditions — 193

**6 Basic Algorithms — 196**

- 6.1 Algorithms for Nonlinear Equations — 197
  - 6.1.1 Picard's Algorithm — 197
  - 6.1.2 Newton's Method — 203
- 6.2 Algorithms for Optimization Problems — 206
  - 6.2.1 The Case of Unconstrained Problems — 206
  - 6.2.2 The Case of Constraint Problems — 213
- 6.3 Scientific Calculus Implementations — 223

**7 Exercises and Problems, and their Solutions — 240**

- 7.1 Analysis of Real Functions of One Variable — 240
- 7.2 Nonlinear Analysis — 252
- 7.3 Smooth Optimization — 263
- 7.4 Nonsmooth Optimization — 291

**Bibliography — 313**

**List of Notations — 315**

**Index — 317**