Preface

This book aims to provide a thorough introduction to smooth and nonsmooth (convex and nonconvex) optimization theory on finite dimensional normed vector spaces ($\mathbb{R}^p$ spaces with $p \in \mathbb{N} \setminus \{0\}$). We present several important achievements of nonlinear analysis that motivate optimization problems and offer deeper insights into the further developments. Many of the results in this book hold in more general normed vector spaces, but the fundamental ideas of the theory are similar in every setting. We chose the framework of finite dimensional vector spaces since it offers the possibility to simplify some of the proofs and permits an intuitive understanding of the main ideas.

This book is intended to support courses of optimization and/or nonlinear analysis for undergraduate students, but we hope that graduate students in pure and applied mathematics, researchers and engineers could also benefit from it. We base our hopes on the following five facts:

- This book is largely self-contained: the prerequisites are the main facts of the classical differential calculus for functions of several real variables and linear algebra. They are recalled in the first chapter, so that this book can be then used without other references.
- We give necessary (and sometimes sufficient) optimality conditions in several cases of regularity for each problems' data: in the case of smooth functions, convex nonsmooth functions, and locally Lipschitz nonconvex functions in order to cover a large part of optimization theory.
- We present many deep results of nonlinear analysis under natural assumptions and proofs.
- Basic theoretical algorithms and their effective implementation in Matlab, together with the results of these numerical simulations are presented, and this shows both the power and practical applicability of the theory.
- An extended chapter of problems and their solutions gives the reader the possibility to solidify theoretical facts and to have a better understanding on various aspects of the main results in this book.

It should be clearly stated that we do not claim any originality in this monograph, but the selection and the organization of the material reflects our point of view on optimization theory. Based on our scientific and teaching criteria, the material of this book is organized into seven chapters which we briefly describe here.

The first chapter is introductory and fixes the general framework, the notations and the prerequisites.

The second chapter contains several concepts and results of nonlinear analysis which are essential to the rest of the book. Convex sets and functions, cones, the Bouligand tangent cone to a set at a point are studied, and we give complete proofs of fundamental results, among which Farkas Lemma, Banach Principle of fixed point and Graves Theorem.

The third chapter is the first one fully dedicated to optimization problems; it presents in detail the main aspects of the theory for the case of smooth data. We present general necessary and sufficient optimality conditions of first and second-order for problems with differentiable cost functions and with geometrical or smooth functional constraints. We arrive at the famous Karush-Kuhn-Tucker optimality conditions and we investigate several qualification conditions needed in this celebrated result.

The fourth chapter concerns the case of convex nonsmooth optimization problems. We introduce here, in compensation for the missing differentiability, the concept of the subgradient and we deduce, in this setting, necessary optimality conditions in Fritz John and Karush-Kuhn-Tucker forms.

The fifth chapter generalizes the theory. We work with functions that are neither differentiable nor convex, but are locally Lipschitz. This is a good setting to present Clarke and Mordukhovich generalized differentiation calculus, which finally allows us to arrive, once again, at optimality conditions with similar formulations as in the previous two chapters.

The sixth chapter is dedicated to the presentation of some basic algorithms for smooth optimization problems. We show Matlab code that accurately approximate the solutions of some optimization problems or related nonlinear equations.

The seventh chapter contains more than one hundred exercises and problems which are organized according to main themes of the book: nonlinear analysis, smooth optimization, nonsmooth optimization.

In our presentation we used several important monographs as follows: for theoretical expositions we mainly used (Zălinescu 1998; Pachpatte 2005; Nocedal and Wright 2006; Niculescu and Persson 2006; Rădulescu et al. 2009; Mordukhovich 2006; Hiriart-Urruty 2008; Clarke 2013; Cârjă 2003;), while for examples, problems and exercises we used (Pedregal 2004; Nocedal and Write, 2006; Isaacson and Keller 1966; Hiriart-Urruty 2009; Hestenes 1975; Forsgren et al. 2002; Clarke, 1983) and (Bazaraa et al. 2006). Finally, for the Matlab numerical simulations we used (Quarteroni and Saleri 2006).

In the case of the Ekeland Variational Principle, which was obtained in 1974 in the framework of general metric spaces, and whose original proof was based on an iteration procedure, the simpler proof for the case of finite dimensional vector spaces we present here was obtained in 1983 by J.-B. Hiriart-Urruty in (Hiriart-Urruty 1983). The very simple and natural proof of Farkas Lemma that is given in this book is based on the paper of D. Bartl (Bartl 2012) and on a personal communication to the authors from C. Zălinescu. The Graves theorem is taken from (Cârjă 2003).

Of course, the main reference for convex analysis is the celebrated R. T. Rockafellar monograph (Rockafellar 1970), but we also used the books (Niculescu and Persson 2006; Zălinescu 1998) and (Zălinescu 2002). For the section concerning the fixed points for function of real variable, we used several problems presented in (Radulescu et al. 2009).

For the section dedicated to the generalized Clarke calculus, we used the monographs (Clarke 1983; Clarke 2013; Rockafellar and Wets 1998). Theorem 5.1.32 is taken from (Rockafellar 1985). For the second part of Chapter 5, dedicated to Mordukhovich calculus, we mainly used (Mordukhovich 2006). The calculus rules for the Fréchet subdifferential of difference of functions, as well as the chain rule for the Fréchet subdifferential were taken from (Mordukhovich et al. 2006).

Many of the optimization problems given as exercises are taken from (Hiriart-Urruty 2008) and (Pedregal 2004), but (Hiriart-Urruty 2008) was used as well for some other theoretical examples such as the second problem from Section 3.4 or the Kantorovich inequality. The rather complicated proof of the fact that the Mangasarian Fromovitz condition is a qualification condition is taken from (Nocedal and Write 2006) which is used as well for presenting the sufficient optimality conditions of second order in Section 3.3. The Hardy and Carleman inequalities correspond to material in (Pachpatte 2005).

Chapter 6 is dedicated to numerical algorithms. We used the monographs: (Isaacson and Keller 1966) (for the convergence of Picard iterations and the Aitken methods), (Nocedal and Write 2006) (for the Newton and the SQP methods). For the presentation of the barrier method, we used (Forsgren et al. 2002).

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