4 Microwaves and their Interactions with Materials

The earth has three important fields associated with it. Two of these fields are obvious and taken for granted, the other is important, but not so obvious to the casual observer. The obvious fields are: the gravitational field and the magnetic field. The not so obvious one is the electrical field that exists between the surface of the earth and the ionosphere, which is high in the atmosphere. Interestingly, the gravitational field is the weakest of all field forces. The main reason that gravitational fields are so obvious is because planets and stars are so massive and therefore manifest large gravitational fields. The magnetic field is strong and protects the earth by deflecting charged particles away from the earth and provides orientation for many creatures including humanity; however magnetic fields are much weaker than an electrical field.

Experiments since very early times demonstrate that there are two types of matter. When two particles of the same type of matter are brought into close proximity to each other, a force grows between them that push them apart. The closer these particles come to each other, the stronger this force becomes. When two particles of different types are brought into close proximity to each other, a force grows between them that pulls them together. These two types of matter are designated as positive or negative and the property that differentiates between the two types of matter is called charge. When charges are static (i.e. not moving) these forces act radially from the centre of the charged particle. This radially acting force is caused by an electrical field.

When charge moves, a magnetic field comes into existence. When current (moving charges) flows through a wire, a magnetic field begins to circulate around the wire. If the wire is wound into a coil, the individual magnetic fields from each turn in the coil add to the magnetic field of its neighbour and so the magnetic field becomes stronger. This of course is an electro-magnetic coil. When the electrical current is turned off and the charges stop flowing, the magnetic field decays and disappears.

In a similar way, if a magnetic field exists in some volume of space and a charged particle travels into that volume, the charged particle experiences a phenomenon that forces it to follow a curved (or even circular) path through this space where as in the absence of any magnetic field it follows a straight path through space. Therefore it is apparent that electric fields and magnetic fields are linked. It also becomes apparent that torsion (or twisting action) is involved in these interactions between electric and magnetic fields.

The idea of a field is really an abstract thought that is used to better describe the effect of one object on another without being in contact with it. It is a little unclear, even in this modern era, what a field really is. If the field around a charged particle can be imagined like spokes on a wheel, it is clear that the spokes are closer together near the hub of the wheel and further apart from one another at the rim. In a similar way, field lines can be imagined to be closer together immediately near a charged particle and become further apart with distance from the particle. In a similar thought,
there is a curling effect of magnetic fields on moving charged particles and there is also a curling effect of moving charged particles on magnetic fields. There are two nice mathematical tools that can calculate how quickly field lines diverge from one another with distance from a charged particle and how much of a curling influence a field may have. It is no surprise that these are called the “divergence” and the “curl” operators. These operators must be applied to quantities called Vectors.

4.1 Electric and Magnetic Field Vectors

Because electric and magnetic fields exert forces on charged particles, they must be regarded as vector quantities. They have both a magnitude and direction of action. It is possible to perform various mathematical operations on vector quantities, including the application of calculus. Most of these mathematical operations will yield vector quantities; however some will yield non-vector quantities; which are called scalars. A scalar only has magnitude; there is no direction of operation associated with a scalar. Therefore any normal number is a scalar.

Changes in scalar quantities in space can be mapped using contours. For example, land elevation is a scalar quantity and is usually designated as some height above a datum. These values are designated by contours on a map. As another example, a synoptic weather map has contour lines to depict air pressure.

The steepness of a hill side can be determined by measuring how close together the contour lines are. This calculation is called the gradient; however the gradient has a particular direction, which is up the steepest part of the slope. Therefore the gradient of a scalar field is actually a vector quantity.

Gradient has a mathematical definition. The gradient of some scalar field \( f(x, y, z) \) has a defined value at every point \((x, y, z)\) is given by:

\[
\nabla f(x, y, z) = \hat{i} \frac{\partial f}{\partial x} + \hat{j} \frac{\partial f}{\partial y} + \hat{k} \frac{\partial f}{\partial z}
\]

In this case the unit (or basis) vectors in the x, y, and z directions are used to define the final direction of the vector of fastest change for the scalar quantity.

As mentioned earlier, electric and magnetic phenomenon depend on how quickly the electric and magnetic fields diverge from one another. This divergence value is simply a number, without any particular direction associated with it; therefore it is a scalar value.

Divergence also has a mathematical definition. The divergence of some vector field \( \mathbf{f}(x, y, z) \), which has a defined value at every point \((x, y, z)\) is given by:
So the divergence is a measure of the tendency for a field to converge or repel from within a given volume of space. Because there are no basis vectors involved in the calculation, the final result is a scalar quantity.

The other feature of vector fields is their rotational tendencies. This is particularly true of electromagnetic phenomenon. The degree of rotation in a given space can be determined by calculating the curl of the vector field.

Curl also has a mathematical definition. The curl of some vector field \( \mathbf{f}(x, y, z) \), which has a defined value at every point \((x, y, z)\) is given by:

\[
\nabla \times \mathbf{f}(x, y, z) = \hat{i} \left( \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \right) + \hat{j} \left( \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \right) + \hat{k} \left( \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \right)
\]  

(4.3)

Because there are basis vectors involved in the calculation, the final result is a vector quantity.

### 4.2 Maxwell’s Equations for Electro-magnetism

James Clerk Maxwell combined the fundamental properties of electrical and magnetic behaviour into a mathematical model of electromagnetism, which he based on earlier work carried out by Faraday, Ampere and Gauss. Maxwell published this work in “A Treatise on Electricity and Magnetism”, 1873. Others have refined Maxwell’s original equations, using the ideas of divergence and curl. The final result is four fundamental equations that when used in their proper combinations can describe every electrical or magnetic phenomenon.

Maxwell’s equations can be defined as:

\[
\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}
\]  

(4.4)

\[
\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}
\]  

(4.5)

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}
\]  

(4.6)
where $\varepsilon$ is the electrical permittivity of the space in which the electrical field $E$ exists; $\mu$ is the magnetic permeability of the space in which the magnetic field $H$ exists; $\sigma$ is the conductivity of the space in which the electrical field exists; $J_s$ accounts for any source current densities in the space of interest; and $\rho$ accounts for any stationary charges in the space of interest.

In anisotropic media, where the orientation of the electric or magnetic fields is critical to the electrical behaviour of the material: the dielectric permittivity; magnetic permeability; and electrical conductivity are tensors (having different values depending on the orientation of the field vectors) rather than single values. In vacuum, which is isotropic, the dielectric permittivity and the magnetic permeability are constants and the conductivity is zero. In this case $\varepsilon_o = 8.854187817 \times 10^{-12} \text{ (F/m)}$ and $\mu_o = 4\pi \times 10^{-7} \text{ (H/m)}$.

One of the most important contributions made by Maxwell was to clarify that light was actually electromagnetic waves.

Although it takes a while to go through the mathematical derivation, Maxwell’s equations can be combined and manipulated to ultimately yield the following relationship:

$$
\nabla \cdot \vec{H} = 0 \quad (4.7)
$$

This is called a wave equation, because the solution to this type of equation behaves like a wave. A similar wave equation for the magnetic field can also be derived from Maxwell’s equations.

In particular, equation (4.8) is called a “forced, damped wave equation”. It is made up of three components:

1. $$\nabla^2 \vec{E} = \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \frac{\partial J_s}{\partial t} + \nabla \left( \frac{\rho}{\varepsilon} \right) \quad (4.8)$$

2. $$\mu \frac{\partial J_s}{\partial t}$$

which is a time varying current that “forces” (or creates) the damped wave; and
3.

\[ \nabla \left( \rho \frac{\mu}{\varepsilon} \right), \]

which is a static component to the electrical field, associated with any stationary charges.

Waves of various kinds transfer energy through space and time. Waves in water and sound waves in air are two examples of mechanical waves. Mechanical waves are caused by a disturbance or vibration in matter, whether solid, gas, liquid, or plasma. Electromagnetic waves differ from mechanical waves in that they do not require a medium through which to propagate. This means that electromagnetic waves can travel not only through gasses, liquids or solids, but they can also travel through a vacuum. This implies that space-time itself has inherent electromagnetic properties. As mentioned in chapter 1, it has even been suggested that electromagnetic phenomena may be a space-time phenomenon, with electromagnetic behaviour being the result of space-time torsion (Evans 2005).

Maxwell was the first to formalise the concept of electromagnetic waves when he combined the fundamental properties of electrical and magnetic behaviour into a mathematical model of electromagnetism. Heinrich Hertz, a German physicist, applied Maxwell’s theories to the production and reception of radio waves (Seitz 1996, Ramsay 2013). His experiment with radio waves solved two problems. Firstly, he had demonstrated in practice, what Maxwell had only theorised, that the velocity of radio waves was equal to the velocity of light. This proved that radio waves were a form of light. Secondly, Hertz demonstrated that: electric and magnetic fields can generated from electrical currents in wires (antennae); propagated as electromagnetic waves through open space; and be received by other wires (antennae) and turned back into electrical currents.

If rectangular coordinates are used and propagation is assumed to be in the \( \hat{z} \) direction with the electric field vector is polarized in the \( \hat{y} \) direction, then equation (4.9) is a useful solution (Sadiku 2001a, Sadiku 2001b) to equation (4.8). This solution applies a Fourier series to represent any complex wave patterns induced by the source currents.

\[
\vec{E} = \sum_{n=1}^{\infty} \Re \left\{ E_n \cdot e^{j k_n x} \cdot e^{j k_n y} \cdot e^{-j \left( \omega t - k_n z \right)} \right\} \cdot e^{-j \alpha_n \cdot \hat{z}} \cdot \hat{y} + \Phi \quad (4.9)
\]

where

\[
\alpha_n = \omega_n \sqrt{\varepsilon_o \mu_o} \sqrt{\frac{k_n^2}{2} \left( 1 + \left( \frac{k_n}{k} \right)^2 \right) - 1} \quad (m^{-1}) \quad (4.10)
\]

and

\[
\beta_n = \omega_n \sqrt{\varepsilon_o \mu_o} \sqrt{\frac{k_n^2}{2} \left( 1 + \left( \frac{k_n}{k} \right)^2 \right) + 1} \quad (m^{-1}) \quad (4.11)
\]
Where \( k' \) is the real part of the relative dielectric constant \( \left[ \varepsilon' = \frac{\varepsilon}{\varepsilon_0} \right] \) of the space through which the wave travels and \( k'' \) is the imaginary part of the relative dielectric constant \( \left[ \varepsilon'' = \frac{\varepsilon}{\varepsilon_0} \right] \).

The constants \( \varsigma \) and \( \xi \) are determined by the boundary conditions imposed onto the wave by any structures the wave may encounter. The magnitude \( (E_n) \) and frequency \( (\omega_n) \) of each wavelet are related to the source current density \( \mathbf{j}_s \) and the distance from the source where the wave is detected, while \( \Phi \) represents a static component of the electric field associated with any stationary charge density \( \rho \).

In the case where \( \varsigma = \xi = k'' = \alpha = \rho = 0 \), equation (4.9) reduces to the equation for a plane wave:

\[
\mathbf{E} = \sum_{n=1}^{\infty} E_n \cdot \cos (\omega_n t - \beta_n z) \cdot \mathbf{\hat{y}}
\]

4.3 Magnetic Vector Potential

Vector potential, which is often perceived as a somewhat abstract idea, has a relatively simple interpretation, something that makes it appear quite intuitive. The path of deriving the vector potential from Maxwell’s equations is not simple; however the result is very useful. The vector potential is constantly proportional to the field momentum contained within a system.

The complete derivation of the magnetic vector potential is a complex process; however it essentially depends on the mathematical properties that \( \nabla \cdot \mathbf{H} = 0 \), \( \nabla \cdot \mathbf{\times} \mathbf{A} = 0 \), \( \nabla \mathbf{\times} \mathbf{A} = \mu \mathbf{H} \), and \( \nabla \times \nabla \Phi = 0 \). When these are substituted into Maxwell’s equations and manipulated, they ultimately lead to three important equations:

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi
\]

\[
\nabla \cdot \mathbf{A} = -\mu \varepsilon \frac{\partial \Phi}{\partial t}
\]
and

\[ A = \sum_{n=0}^{\infty} M_n \int \frac{\mu \cdot d \vec{J}_s}{4\pi} \cdot \frac{e^{-j\omega_n \sqrt{\mu \omega_n r}}}{r} \cdot dv \]  

(4.15)

where \( M_n \) is the magnitude of the \( n^{th} \) harmonic term, \( r \) is the distance from the current source that created the magnetic potential, and \( w_n \) is the angular frequency of the \( n^{th} \) harmonic term. Note that the angular frequency is related to standard frequency \( (f) \) by:

\[ \omega = 2\pi f \]

(4.16)

These equations allow the electrical field to be calculated from any known distribution of source currents and charges.

### 4.4 Continuity

It is useful to know the relationship between currents and charges. This implies that net charge is conserved throughout any electrical phenomenon. More specifically, it implies that electrical currents crossing a closed surface of a fixed volume of space must be equal to the rate of change of the charge within the volume.

Taking the divergence of equation (4.5) gives:

\[ \nabla \cdot \left( \nabla \times \vec{H} \right) = \varepsilon \frac{\partial \left( \nabla \cdot \vec{E} \right)}{\partial t} + \sigma \left( \nabla \cdot \vec{E} \right) + \left( \nabla \cdot \vec{J}_s \right) \]  

(4.17)

Mathematically, \( \nabla \cdot \left( \nabla \times \vec{H} \right) = 0 \), and \( \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \). Therefore:

\[ \frac{\partial \rho}{\partial t} = -\sigma \left( \nabla \cdot \vec{E} \right) - \left( \nabla \cdot \vec{J}_s \right) \]  

(4.18)

Therefore the divergence of all currents (including conductivity currents caused by an electrical field) is balanced by a change in charge with respect to time. More simply, currents are moving charges and total charge is conserved.
4.5 Conservation of Electromagnetic Energy

A natural extension of the continuity equation is Poynting’s Theorem. Poynting’s theorem is a powerful statement of energy conservation. It can be used to relate the power absorption in an object to the electromagnetic fields that are incident on the surface of that object.

After another lengthy derivation from Maxwell’s equations, it can be shown that:

\[
-\oint (\mathbf{E} \times \mathbf{H}) \cdot ds = \frac{1}{2} \frac{\partial}{\partial t} \int (\varepsilon \mathbf{E}^2 + \mu \mathbf{H}^2) \cdot dv + \int \sigma \mathbf{E}^2 \cdot dv + \int \mathbf{E} \cdot \mathbf{J}_S \cdot dv \tag{4.19}
\]

Each term in equation (4.19) represents a component of the total electromagnetic energy in a fixed volume of space:

1. \( E \times H \) is the instantaneous Poynting Vector. This term represents the instantaneous power flow across a closed surface \( s \) into the enclosing volume of interest \( v \);
2. \( \frac{1}{2} \varepsilon \mathbf{E}^2 \) is the instantaneous stored electrical energy density inside the enclosed volume;
3. \( \frac{1}{2} \mu \mathbf{H}^2 \) is the instantaneous stored magnetic energy density inside the enclosed volume;
4. \( \sigma \mathbf{E}^2 \) is the instantaneous power dissipated inside the enclosed volume by electrical currents generated in the material by resistive losses (this component of the equation equals the heat that can be generated inside objects by electromagnetic energy); and
5. \( \mathbf{E} \cdot \mathbf{J}_S \) is the instantaneous electromagnetic power generated inside the volume by any current sources.

By way of explanation, a closed surface is any surface that completely encloses a fixed volume (perhaps the surface of a 3-dimensional object). The integral over the volume inside the closed surface corresponds to a sum of the terms in the integrand over all the points inside the volume.

Effectively, Poynting’s theorem is a power balance equation. It implies that the electromagnetic power that goes across the surface of an object must either be stored as electromagnetic energy or be converted to heat. When real objects are being considered, the amount of electromagnetic power that crosses the surface into the contained volume of the object will depend on what happens at the surface of the object.
4.6 Boundary Conditions

Consider an electromagnetic field at the boundary between two materials with different properties. The tangent and the normal component of the fields must be examined separately, in order to understand the effects of the boundary on the behaviour of the electromagnetic wave.

![Figure 4.1: Magnetic fields at the boundary of two different materials (Source: Georgieva 2001).](image)

Consider the situation shown in Figure 4.1, where the magnetic fields, both inside and outside the boundary, are being studied. From equation (4.5) it follows that:

\[
\vec{\nabla} \times \vec{H} = \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{z} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}_\varepsilon
\]

(4.20)

The curl part of this equation can be expressed in a difference form as:

\[
\left\{ \frac{H_{1\alpha} - H_{1\delta}}{b} - \frac{H_{1\alpha} - H_{1\delta}}{a} \right\} \hat{z} = \varepsilon \frac{\partial \vec{E}_z}{\partial t} + \vec{J}_\varepsilon
\]

(4.21)

where the components of the magnetic field are shown in Figure 4.1.

If the boundary region shrinks, with dimension “a” going to zero faster than dimension “b”, then:

\[
H_{1\alpha} - H_{1\delta} = \lim_{a \to 0} \left\{ a \frac{\partial \vec{E}_z}{\partial t} + a \vec{J}_z - \frac{a(H_{1\alpha} - H_{1\delta})}{b} \right\} = 0
\]

(4.22)
If there is a surface current at the interface so that no matter how small the dimension “a” becomes, this current is still evident, then equation (4.22) needs to be modified to become:

\[
H_{i1} - H_{i2} = \lim_{a \to 0} \left\{ a \varepsilon \frac{\partial E_i}{\partial t} + a J_i + J_s - \frac{a (H_{s1} - H_{s2})}{b} \right\} = J_s
\]  

(4.23)

where \( J_s \) is the surface current immediately on the interface itself.

Equation (4.22) reveals that for many materials, such as insulators, the tangential components of the magnetic fields are continuous across the boundary between two materials; however in the case of conducting materials, where a surface current may be possible, Equation (4.23) reveals surface currents modify the magnetic field as it crosses the material boundary.

Now considering the electric fields:

![Electric fields at the boundary of two different materials](Source: Georgieva 2001).

Consider the situation shown in Figure 4.2, where the electric fields, both inside and outside the boundary, are being studied. From equation (4.4) it follows that:

\[
E_{i1} - E_{i2} = \lim_{a \to 0} \left\{ a \varepsilon \frac{\partial H_i}{\partial t} - \frac{a (E_{s1} - E_{s2})}{b} \right\} = 0
\]  

(4.24)

Equation (4.24) implies that tangential electrical fields are continuous across the boundary of any object.
Now consider a small box that encloses an area with surface charge \( \rho_s \):

![Figure 4.3: Fields at the boundary of two different materials (Source: Georgieva 2001).](image)

From equations (4.6) and (4.7) it follows that:

\[
\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s 
\]

\[
\mu_1 H_{1n} - \mu_2 H_{2n} = 0
\]

These equations imply that magnetic fields that are perpendicular to the surface are modified by any change in permeability as they cross the material’s surface, while electric fields that are perpendicular to the surface are modified by changes in the permittivity of the two spaces on either side of the surface and by any surface charges that may be on the interface.

Because the electric and magnetic fields change in response to a material, it is useful to define a single parameter that can describe the behaviour of the space through which the electromagnetic wave may propagate.

### 4.7 Wave Impedance

If the simplest solution of the electrical field wave, equation (4.12) with

\[
\beta = \omega \sqrt{\mu \varepsilon},
\]

is directly substituted into equation (4.4), then the only non-zero term in \( \nabla \times \vec{E} \) is:

\[
\frac{\partial E_z}{\partial z},
\]

therefore:
This implies that:

$$H = \frac{\beta E_o \cdot \sin(\omega t - \beta z)}{\mu} \cdot \hat{x}$$  \hspace{1cm} (4.28)

This yields:

$$\vec{H} = \frac{\beta E_o \cdot \cos(\omega t - \beta z)}{\mu \omega} \cdot \hat{x}$$  \hspace{1cm} (4.29)

Therefore

$$\vec{H} = \sqrt{\frac{\kappa \varepsilon_o}{\mu}} \cdot E \cdot \cos(\omega t - \beta z) \cdot \hat{x}$$  \hspace{1cm} (4.30)

Equation (4.22) implied that the magnetic field is at right angles to the electric field.

Equation (4.30) can be simplified to:

$$\vec{H} = \frac{\vec{E}}{\eta}$$  \hspace{1cm} (4.31)

This can be rearranged to yield:

$$\frac{\vec{E}}{\eta} = \sqrt{\frac{\mu}{\varepsilon}}$$  \hspace{1cm} (4.32)

Equation (4.32) defines the wave impedance of the space through which the electromagnetic wave is propagating. The wave impedance determines how difficult it is for the wave to propagate. When there is a sudden change in the wave impedance, such as at the surface of a material, the wave behaviour abruptly changes.
4.8 Reflection and Transmission at an Interface

When a plane wave propagating in a homogenous medium encounters an interface with a different medium, a portion of the wave is reflected from the interface while the remainder of the wave is transmitted. The reflected and transmitted waves can be determined by enforcing the electromagnetic field boundary conditions described earlier.

When the wave is propagating perpendicular to the interface between the two materials, the total tangential electric field on the incident side of the interface is the sum of the tangential components of the incident and reflected waves. The electric field on the transmission side of the interface is simply the tangential component of the transmitted wave. If the reflection coefficient for the interface is defined as:

\[ \Gamma = \frac{E_r}{E_i} \]  

(4.33)

Where: \( \Gamma \) is the reflection coefficient of the surface; \( E_r \) is the reflected wave; and \( E_i \) is the incident wave. The transmission coefficient is defined as:

\[ \tau = \frac{E_t}{E_i} \]  

(4.34)

These can be applied to the boundary condition for the tangential electric fields yields:

\[ E_i + E_r = E_i \quad or \quad 1 + \Gamma = \tau \]  

(4.35)

When the magnetic fields are being considered, the total perpendicular magnetic field on the incident side of the interface is the difference between the components of the incident and reflected waves.

The magnetic field on the transmission side of the interface is simply the perpendicular component of the transmitted wave.

\[ H_i - H_r = H_i \quad or \quad H_i - \Gamma H_i = H_i \quad or \quad \frac{E_i}{\eta_1} - \Gamma \frac{E_i}{\eta_1} = \tau \frac{E_i}{\eta_2} \]  

(4.36)
This yields:

\[ \frac{1 - \Gamma}{\eta_1} = \frac{\tau}{\eta_2} \]  \hspace{1cm} (4.37)

If equation (4.37), multiplied by \( \eta_2 \), and equation (4.35) are combined and simplified, then:

\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \]  \hspace{1cm} (4.38)

From equation (4.35) it immediately follows that:

\[ \tau = \frac{2\eta_2}{\eta_2 + \eta_1} \]  \hspace{1cm} (4.39)

From equations (4.38) and (4.39) it is clear that the wave impedances of the spaces on either side of an interface determine how much of the wave is transmitted and how much is reflected. This analysis has so far only considered when a wave is propagating perpendicular to the interface. In the case where the incident wave is not perpendicular to the interfacial surface, two extreme cases exist: the electrical field may be oriented perpendicular to the plane of incidence to the surface, as shown in Figure 4.4; or the electric field may be oriented to be parallel to the plane of incidence to the surface as shown in Figure 4.5.

**Figure 4.4:** Analysis of electromagnetic waves at medium interface for perpendicular polymerization (Source: Georgieva 2001.)
In analysing this problem, continuity of the magnetic components must be modified to account for the angle of incidence:

\[
H_i \cos \theta_i - H_r \cos \theta_r = H_t \cos \theta_t
\]

\[
\Rightarrow \frac{E_i}{\eta_1} \cos \theta_i - \Gamma \frac{E_i}{\eta_1} \cos \theta_r = \tau \frac{E_i}{\eta_2} \cos \theta_t
\]

\[
\Rightarrow \frac{\cos \theta_i}{\eta_1} - \Gamma \frac{\cos \theta_r}{\eta_1} = \tau \frac{\cos \theta_t}{\eta_2}
\]  \hspace{1cm} (4.40)

Where \( \theta_i \) is the incident angle of the wave, \( \theta_r \) is the reflection angle of the wave and \( \theta_t \) is the transmission angle of the wave. Equation (4.40) can be manipulated to eventually yield:

\[
\Gamma = \frac{\eta_2 \cos \theta_r - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_r + \eta_1 \cos \theta_i}
\]  \hspace{1cm} (4.41)

It follows that:

\[
\tau = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_r + \eta_1 \cos \theta_i}
\]  \hspace{1cm} (4.42)

**Figure 4.5:** Analysis of electromagnetic waves at medium interface for parallel polymerization (Source: Georgieva 2001).
Applying a similar analysis to the parallel polarization case yields:

\[ \Gamma_{\parallel} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i} \]  
\hspace{1cm} (4.43)

It follows that:

\[ \tau_{\parallel} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_i} \]  
\hspace{1cm} (4.44)

4.9 Electromagnetic Behaviour of Materials

Material can be categorized as transparent, absorbing or reflective, according to their electromagnetic behaviour at any given frequency. If the attenuation factor (\(\alpha\)) defined in equation (4.10) is negligible, then the material can be regarded as transparent at the electromagnetic frequency of interest. If the attenuation factor is not negligible, then the material can be regarded as absorbing at the electromagnetic frequency of interest. Finally, if the reflection coefficient for the surface of the material (\(\Gamma\)) is very high, the material can be regarded as reflective.

4.10 Conclusions

In summary, Maxwell’s equations can be used to describe the wave behaviour of electromagnetic phenomenon. These waves propagate through space; transport energy from one part of space to another; consist of orthogonal electric and magnetic vector fields; are reflected from and refracted through interfacial boundaries of materials; and are attenuated in materials that have non-zero electrical conductivity. There must be an energy balance associated with these waves, so as electromagnetic waves are attenuated, heat energy is generated in a material.

This chapter has introduced many of the key concepts associated with radio frequency and microwave propagation through space. Many of these concepts are complex and therefore required more than a cursory presentation of the facts. The remainder of this book will explore how these propagating waves can be used to do useful things in an agricultural context.
References