14 Simultaneous Heat and Moisture Movement

In experiments using wool fibres, described by Cassie, King and Baxter (1940 in Crank 1979), the isothermal moisture diffusion coefficient predicted that moisture equilibrium should be reached within seconds of a sudden change in external humidity. However their experiments demonstrated that equilibrium was only reached after an hour or more of exposure to the new external conditions (Crank 1979).

Any realistic analysis of microwave heating in moist organic materials must account for simultaneous heat and moisture diffusion through the porous material. The coupling between heat and moisture transfer is well known but not very well understood (Chu and Lee 1993). Henry (1948) was one of the first to propose a theory for simultaneous diffusion of heat and moisture into a texture package. Crank (1979) presented a more thorough development of Henry’s work. The theory of simultaneous heat and moisture diffusion through porous materials, based on Henry’s original work, has been rewritten and used by many authors (Chu and Lee 1993, Vos, et al. 1994, Fan, et al. 2000, Casada 2002, Fan 2004, Barba 2005, Fan 2005, Frydrych and Ralek 2005).

For a porous material, the amount of water vapour moving into a small section of material is the sum of any net increase in moisture content in the air space and the net increase in moisture content of the material’s fibres. Therefore:

\[
a_{a} \tau D_{a} \cdot \nabla^{2} M_{a} = a_{a} \frac{\partial M_{a}}{\partial t} + (1 - a_{a}) \rho_{a} \frac{\partial M_{a}}{\partial t} \tag{14.1}
\]

Heat is evolved when moisture is absorbed by a material, therefore the thermal diffusion equation, allowing for a volumetric heat source is:

\[
C_{p} \frac{\partial T}{\partial t} = k \nabla^{2} T + L \rho \frac{\partial M_{s}}{\partial t} + q(x) \tag{14.2}
\]

If it is assumed that some linear relationship exists between the moisture content of a material, the moisture vapour concentration in the air spaces in the material and the temperature, then:

\[
\frac{\partial M_{s}}{\partial t} = \sigma \frac{\partial M_{v}}{\partial t} - \omega \frac{\partial T}{\partial t} \tag{14.3}
\]

Substituting into equations (14.1) and (14.2) and combining the two equations as described by Henry (1948) yields:
Simultaneous Heat and Moisture Movement

\[ \nabla^2 (pM_v + nT) - \frac{\partial}{\partial t} \left( \frac{1}{\tau_r D_a} \left( 1 + \frac{(1-a_r) \sigma \rho_v}{\alpha_v} \right) \frac{n \rho_v L}{pk} pM_v + \frac{C_p}{k} \left( 1 + \frac{\omega L}{C} \right) p(1-a_r) \sigma \rho_v nT \right) + \frac{n q(x)}{k} = 0 \]  \hspace{2cm} (14.4)

This can be expressed in a simpler form if \( \Omega = pM_v + nT \) and \( p \) and \( n \) are chosen such that:

\[ \frac{1}{\gamma} \left[ \frac{1}{\tau_r D_a} \left( 1 + \frac{(1-a_r) \sigma \rho_v}{\alpha_v} \right) \frac{n \rho_v L}{pk} \right] = \frac{C_p}{k} \left( 1 + \frac{\omega L}{C} \right) \frac{p(1-a_r) \sigma \rho_v}{n \tau_r D_a \alpha_v} \]  \hspace{2cm} (14.5)

Equation (14.4) can be rewritten as:

\[ \nabla^2 \Omega - \frac{1}{\gamma} \frac{\partial \Omega}{\partial t} + \frac{n q(x)}{k} = 0 \]  \hspace{2cm} (14.6)

Equation (14.6) is now a diffusion equation in terms of \( \Omega \) and equation (14.5) is a quadratic polynomial in \( \gamma \) and yields:

\[ \gamma = \frac{2 \psi \xi}{\psi + \xi \pm \sqrt{(\psi - \xi)^2 + 4 \psi \xi \chi}} \]  \hspace{2cm} (14.7)

where \( \psi = \frac{k}{C_p \left( 1 + \frac{L}{C} \omega \right)} \), \( \xi = \frac{\tau_r D_a}{1 + \frac{(1-a_r) \rho_v \sigma}{\alpha_v}} \) and \( \chi = \frac{\sigma \rho_v \omega L}{a_r \omega L \left( 1 + \frac{L}{C} \omega \right)^{1/2}} \left( 1 + \frac{(1-a_r) \rho_v \sigma}{\alpha_v} \right) \).

Equation (14.7) implies that the combined heat and moisture diffusion coefficient \( (\gamma) \) has two independent values. This is consistent with Henry’s (1948) equation for simultaneous heat and moisture diffusion in textiles. The combined processes of heat and moisture diffusion are equivalent to the independent diffusion of two quantities, each of which is a linear function of moisture vapour concentration and temperature. The diffusion coefficients of these two quantities are always such that one is much less and the other much greater than would be observed, were moisture and heat diffusion not coupled together. Therefore the independent solution of the heat and moisture diffusion equations is inadequate to describe their combined influence (Henry 1948). The diffusion coefficient for the slower quantity of the coupled system is always less than either the isothermal moisture diffusion constant or the constant vapour...
concentration coefficient for heat diffusion, which ever is less, but never by more than one half (Henry 1948). The faster diffusion coefficient may be many times greater than either of the independent diffusion constants.

Henry (1948) presents a nomogram, shown in Figure 14.1, relating the fast diffusion coefficient to the default diffusion coefficient for the material at 20 °C and 65 % relative humidity. This nomogram can be used to forecast the thermal diffusivity of the system under different conditions. For example, if the relative humidity remains constant and the temperature of some part of the system rapidly rises to 55 °C, Henry’s nomogram suggests that the diffusion coefficient for the faster wave will be about 7.5 times higher than the standard thermal diffusivity of the material.

![Figure 14.1: Nomogram of variation in g with temperature and relative humidity (Modified from: Henry 1948).](image)

It must be remembered that this data was for cotton packaging in the absence of internal heat sources; however Henry (1948) states that a similarly coupled response should be expected when either moisture or heat are released inside the material in a way that is independent of the diffusion processes. Dielectric heating, from microwave interactions inside the material, is independent of diffusion processes; therefore a rapidly propagating wave of heat and moisture should be expected during microwave heating in any moist materials.
Having now identified some key principles of heat diffusion while heat is being generated inside an object, the specific problem of determining the temperature distribution inside objects during microwave heating can be addressed provided the spatial distribution $q(x)$ can be determined for each microwave heating system configuration. The spatial distribution of heat generated inside the object depends on how microwave energy interacts with the material itself.

Considerable evidence exists in literature for rapid heating and drying during microwave processing (Rozsa 1995, Zielonka, et al. 1997, Zielonka and Dolowy 1998); therefore it is reasonable to assume that the faster diffusion wave should dominate microwave heating in moist materials. A slow heat and moisture diffusion wave should also exist; however observing this slow wave during microwave heating may be difficult and no evidence of its influence has been seen in literature so far.

This theory was successfully applied to microwave heating of wood samples (Figure 14.2).

![Figure 14.2](image)

**Figure 14.2:** Comparison of measured temperature, published by Zielonka and Dolowy (1998), with theoretical temperature, estimated using equation (13.24) - (a) using the published constant vapour concentration thermal diffusivity of wood and (b) using a thermal diffusivity that is 7.8 times larger than the published values, as suggested by Henry’s nomogram (Source: Brodie 2007).

In his discussion of simultaneous heat and moisture diffusion, Henry (1948) also stated that thermal diffusion would approach the constant vapour concentration thermal diffusivity of the material as the combined heat and moisture diffusion processes became decoupled. The decline in the multiplying factor, implied by the data shown in Figure 14.3, may be linked to the loss of moisture and subsequent decoupling of the combined heat and moisture transfer mechanisms. This requires further investigation.
14.1 Temperature Sensing in Electromagnetic Fields

Temperature measurement with electromagnetic fields is challenging. Conventional temperature sensors based on thermo-electric effects, such as thermocouples and thermistors, distort the electromagnetic fields in the vicinity of the probes, giving erroneous readings and in some cases causing localised plasma (Tang 2006).

Fibre-optic temperature sensors provide comparable accuracy to thermocouples in a normal heating medium. The probe sizes of fibre-optic sensors are generally small. Fibre-optic temperature sensors are developed based on one of three methods: fluorescence decay time, Fabry–Pe´rot interferometry, and transmission spectrum shift in semiconductor crystals (Tang 2006). These systems can be safely placed into electromagnetic fields because the fibre-optic is transparent at microwave frequencies.

Fluorescence technology uses a fibre-optic cable to connect a phosphor element, which is attached to the tip of a silica fibre and encapsulated in Teflon tube, to an electronic instrument. The phosphor material is excited with a light source, which excites the phosphorescent sensor causing it to emit light. The emitted light returns to the instrument through the same fibre-optic cable. The rate of after-glow decay in this system depends on the temperature of the phosphor material, which is placed in contact with the heated material (Tang 2006).

The Fabry–Pe´rot interferometer consists of two parallel reflective surfaces (mirrors) that form a cavity resonator. These sensors often use a thermally expandable piece of glass between the two reflected surfaces, which cause a perturbation in the resonance of the reflective cavity. These sensors can also respond to pressure changes as well and thermal expansion (Tang 2006).
Semiconductor sensors use the temperature dependent light absorption/transmission characteristics of semiconductor crystals such as gallium arsenide (or GaAs). A unique feature of this crystal is that when temperature increases, the crystal's transmission spectrum shifts to a higher wavelength (Tang 2006). Measuring the position of the absorption shift, using a grating-based near infrared spectrometer, provides information on the temperature of the sensing element.

Another technique for measuring temperature in electromagnetic fields is to use remote infra-red detectors (Chemat, et al. 2005, van Dam 2005, Arrieta, et al. 2007). These systems respond to increases in the infra-red radiation from materials as the temperature increases; however because the wavelength of thermal infra-red energy is very small, the resulting measurements are limited to surface temperature only.

References


