Xiang-Hui Lv*, Feng-Long Yin, Geng-Bo Zhang, Hao Hu and Ye Sun

The Ultrasonic Detection Terrain Data Processing Based on Adaptive Kalman Filtering

Abstract: In the ultrasonic terrain detection, individual data deviates from the actual data seriously. According to the continuity characteristics of terrain, the scale factor is added to the system process noise for perceiving the statistical characteristics of the noise and signal on the basis of traditional Kalman filtering. The method reached a certain degree matching between estimated value and observed value and achieved the adaptive filtering processing of the data. The algorithms were simulated by adding the noise single factitiously, and the results between traditional Kalman filtering were contrasted, it showed that the adaptive Kalman filtering has better estimation results. In order to test the terrain detection data recovery ability of the algorithms when the noise was added, the virtual data of hard wood terrain and gravelly soil terrain were managed, and the treatment effect of the two algorithms were analysed. The results show that the proposed algorithm can eliminate the adverse effect of noise on the system effectively, and can be used to deal with the real terrain data.

Keywords: Ultrasonic, terrain measurement, Kalman filtering, adaptive, simulation

1 Introduction

When using ultrasonic pulse echo method to detect terrain altitude, ultrasonic echo signal is actually combined signal[1,2] of reverse scattering echo and noise because of influence of the terrain being randomly ups and downs and rough, interference of environment noise, machine noise and noise of the system itself, making detected terrain altitude data abnormal, significantly deviating actual terrain.

Data filtering is a kind of data processing technique to get real data by removing noise; Kalman filtering can estimate the state of dynamic systems from a series of measurement noise in the case of measurement variance known. Because it is

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convenient for computer programming, and can real-time update and process data from collection locale, Kalman filtering is the most widely used in filtering method, communication, navigation, guidance and control in many fields has been well used. Kalman filtering estimates the current value of state variable using a previous state estimate value and the latest observation value without requiring reservation of all old data, it can realize effective estimation[3] of past and current state of estimated signal when specific form of the model is unclear. Using recurrence algorithm, realizing smooth processing of data based on minimum mean square error, Kalman filtering has been successfully used in such areas as automatic control, navigation, remote measurement and remote sensing, geophysics, etc.[4–6]. Yet it is hard to get higher accuracy to use traditional filtering algorithm due to diverse trajectory of target. So, in this paper we propose a kind of adaptive Kalman filtering algorithm that can achieve better data smoothing effect through comparison of simulation and terrain measured data.

2 Traditional Kalman Filtering Algorithm

Description of Kalman filtering consists of several mathematic recurrence formulas. It can realize minimize mean square error of estimated object by getting a kind of method to estimate process state from every recurrence formula. In real terrain measurement, observations are discrete value, so we mainly discuss the Kalman filtering algorithm of discrete system.

In the application of Kalman filter, estimated state value is expressed by $X \in \mathbb{R}^n$, system state model can be expressed as:

$$x[k] = Ax[k-1] + Bu[k-1] + w[k-1]$$

(1)

For observed variable use $Z \in \mathbb{R}^m$ to construct measuring equation:

$$z[k] = Hx[k] + v[k]$$

(2)

Where, $k$ is time $t_k$, $A$ is state transition matrix, $B$ is input matrix, $H$ is observation matrix and $u[k]$ is input signal.

$w[k]$ and $v[k]$ represent process noise and observation noise at time $k$ respectively, normally are defined as independent Gauss white noise, witch following natures:

$$P(w) \sim N(0, Q), P(v) \sim N(0, R)$$
Where, $Q$ and $R$ are square variance matrices of process noise and observation noise.

Kalman filtering adopts recurrence algorithm, first take state variable estimate at the time of $k - 1$ $\hat{x}[k - 1]$ as the prior estimate of state at No. $K$ step, the algorithm does not consider the influence of observation noise, formulas (1) and (2) transform into:

$$\hat{x}[k, k-1] = A\hat{x}[k-1] + Bu[k-1]$$  \hspace{1cm} (3)

And

$$\hat{z}[k] = H\hat{x}[k, k-1]$$  \hspace{1cm} (4)

Where, $\hat{x}[k, k-1]$ is the prior state estimate of state variable, $\hat{z}[k]$ is the initial estimate of state variable.

Then use estimation error of output signal to correct estimation of state variable, obtaining estimation result of posterior state $\hat{x}[k]$, see Formula (5).

$$\hat{x}[k] = \hat{x}[k, k-1] + K(z[k] - \hat{z}[k])$$  \hspace{1cm} (5)

Where, $z[k] - \hat{z}[k]$ is usually called innovation or remains, indicating difference between the estimation in Kalman filtering algorithm and actual value. If it is zero, it indicates estimation and actual value are consistent. $K$ in the formula is a $n \times m$ matrix, being the gain of innovation, also called Kalman gain matrix. It is typically expressed as:

$$K = P[k, k-1]H^T(V)\^{-1}$$  \hspace{1cm} (6)

Then

$$V = HP[k, k-1]H^T + R$$  \hspace{1cm} (7)

Where, $P[k, k-1]$ is the variance matrix of priori estimation error, see Formula (8).

$$P[k, k-1] = E[(x[k] - \hat{x}[k, k-1])(x[k] - \hat{x}[k, k-1])^T]$$  \hspace{1cm} (8)

The following relationship exists between $P[k]$ and $P[k, k-1]$:

$$P[k] = [I - KH]P[k, k-1]$$  \hspace{1cm} (9)

$P[k]$ is the error variance matrix at $k$ time, also known as posteriori estimation error.
Kalman filtering is essentially a kind of feedback control: i.e. while estimating state of a process at one point Kalman filter conducts closed loop feedback control over the estimation at that point using noise contained variable. Hence, Kalman filtering algorithm can be made up of time update and measurement equations. Like standard feedback control system, forward calculation of some process is completed by update equation through operations such as continuously resolving state variable and error estimation; meanwhile, the solution of last process provides reference for prior estimation of next time state. While forward calculation continuously proceeding, measure update equation also continuously instructs feedback mechanism, so posterior estimation is obtained as final calculation result by calculation prior estimate and measurement variable.

Update equation is as follows:

\[
\hat{x}[k, k-1] = \hat{x}[k-1] + Bu[k-1]
\]

\[
P[k, k-1] = AP[k-1]A^T + Q
\]

The update equation explains how to calculate state at time \( k \) through state at time \( k-1 \). When determining initial value \( \hat{x}[0] \) and \( P[0] \), current estimation can be obtained.

In traditional Kalman algorithm, when finishing a process of differential state iteration calculation, it does not focus on the relation between current value and some previous state values. With increase of observed data, errors between state estimation after filtering and actual state exceed its theoretical estimation, resulting in filter divergence that makes filter estimation deviate without achieving filtering effect [7].

### 3 Adaptive Kalman Filtering Algorithm

Based on continuity of terrain, dynamic ultrasonic terrain detection system can estimate statistic parameters according to observed data, estimating the system using estimated statistic parameters. Thinking of adaptive Kalman filter is: in order to sense statistic characteristics of noise and signal, add scale factor \( s \) to system process noise, when difference of process noise is updated, the difference in algorithm will change too. Main method is to present a threshold subtracting the variance of previous time from process noise variance and compare the differ-
ence with the threshold to determine whether the algorithm needs to reduce influence of probabilistic characteristics of relevant parameters.

Adding scale factor $s$ to system process noise, if input is not considered, then equation (1) will transform to:

$$x[k] = Ax[k-1] + sGw[k-1]$$  \hspace{1cm} (13)

Where, $G$ is known interference matrix, process noise and observation noise of the system is irrelated, initial state is $x[0]$.

Estimation part of Kalman filtering estimation model changes into:

$$\hat{x}[k] = A\hat{x}[k, k-1]$$  \hspace{1cm} (14)

And

$$P[k, k-1] = AP[k-1]A^T + s^2GG^T$$  \hspace{1cm} (15)

For calculation of scale factor $s$, mean estimation of the sum of diagonal of variance matrix can be conducted recurrence calculation with following formula:

$$\hat{b}_1 = v_1^Tv_1$$  \hspace{1cm} (16)

And

$$\hat{b}_k = (\hat{b}_{k-1} + \alpha v_k^Tv_k) / (1 + \alpha)$$  \hspace{1cm} (17)

Where, $\alpha$ is a positive constant. The expansion of formula (17) is:

$$\hat{b}_k = \frac{v_1^Tv_1}{(1 + \alpha)^{k-1}} + \frac{v_2^Tv_2}{(1 + \alpha)^{k-1}} + \frac{v_3^Tv_3}{(1 + \alpha)^{k-2}} + \ldots + \frac{v_k^Tv_k}{(1 + \alpha)}$$  \hspace{1cm} (18)

Hence, once estimation matches observed value somewhat, sum of $\hat{b}_k$ must be close to the sum of diagonal elements of covariance matrix $P[k]$. So equation (19) is valid:

$$\hat{b}_k \approx \text{tr}(P_k)$$  \hspace{1cm} (19)

Meanwhile, by using equation (7) and (19), the following is concluded:

$$\hat{b}_2 \approx \text{tr}(R_3) + \text{tr}(HAP[k-1]A^TH^T) + s^2\text{tr}(HGQ[k-1]G^TH^T)$$  \hspace{1cm} (20)
Where
\[
s^2 \approx \frac{\hat{b}_k - \text{tr}(\mathbf{R}_k) - \text{tr}(\mathbf{HAP}[k-1] \mathbf{A}^T \mathbf{H}^T)}{\text{tr}(\mathbf{HGQ}[k-1] \mathbf{G}^T \mathbf{H}^T)}
\]  

\[(21)\]

4 Simulation and Result

With distance of ultrasonic detection unchanged, i.e., in the case of front end sensor having no input data, manually adding Gauss noise, use traditional Kalman filtering algorithm and adaptive Kalman filtering algorithm to estimate the process under the state respectively. Because there is no real sensor data input, traditional Kalman filtering algorithm and adaptive Kalman filtering algorithm can be purely analyzed and compared without signal.

Simulation result is shown in Fig.1, green full line in it indicates actual distance between sensor and measured terrain, namely 800mm, “+” indicates actual distance measured by the detection system between sensor and measured terrain, its value is obtained by adding actual distance to Gauss noise, blue full line indicates estimation of Kalman filtering.

(a) Traditional Kalman filtering
From Fig.1 it can be found that in this state, actual value is a straight line with amplitude being zero, but measurement value distribute like the form of red “+” in the two figures due to interference of various errors and noise. Through Kalman filtering, measurement value rapidly approached intrinsic distance value between sensor and measured terrain. But comparing Fig. 1(a) and Fig.1(b), effect of traditional Kalman filtering is less good than adaptive Kalman filtering, so in this case adaptive Kalman filtering has better estimation effect.

5 Actual Terrain Data Processing

Measure elevation of hardwood terrain and gravels terrain in laboratory environment with ultrasonic terrain detection system, with sampling interval being 10mm, both are arc terrains with radius of 2,000 mm, as shown in Fig. 2.
Keeping same condition with simulation experiment, use data of hardwood, gravel terrain as data set to conduct filtering processing with traditional and adaptive methods respectively, with result shown in Fig.3 and Fig.4.

(a) Traditional Kalman filtering

(b) Adaptive Kalman filtering

**Fig. 3:** Filtering effect of hardwood terrain
Fig. 4: Filtering effect of gravels terrain.
In Fig. 3, 4, the green full line indicates data of actual terrain elevation which was obtained by manually measuring at the highest point of gravels terrain. Red full line indicates measurement data of ultrasonic detection system; blue full line indicates estimation of Kalman filter. Traditional Kalman filter can correct outliers somewhat, but it has weaker correcting effect than adaptive Kalman filter; meanwhile, where there is no outliers, estimation of adaptive Kalman filter is more close to actual distance value. Calculate errors respectively by comparing data before filtering and actual terrain elevation data of both terrains, result shown in Fig. 5, 6.
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Fig. 5: Filtering error of hardwood terrain

(a) Traditional Kalman filtering

(b) Adaptive Kalman filtering
In order to check reliability of filtering model, compare two algorithms more clearly, this paper evaluated two Kalman filtering algorithm by adopting Error rate [8]. For calculating formula of error rate see equation (22).

\[
\text{error rate} = \left| \frac{\text{Filtering estimation} - \text{actual value}}{\text{actual value}} \right| \times 100\% \quad (22)
\]

According to equation (22), the higher the error rate, the larger error between result of a Kalman filtering algorithm and actual value, the worse the filtering effect, likewise, if a Kalman filtering algorithm has a result with lower error rate, it shows the algorithm has better filtering effect. Filtering result of two filtering algorithm is shown in Table 1.

**Tab. 1: Filtering result of two filtering algorithm**

<table>
<thead>
<tr>
<th>type</th>
<th>traditional</th>
<th></th>
<th>adaptive</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>error rate</td>
<td>Max- error</td>
<td>error rate</td>
<td>Max- error</td>
</tr>
<tr>
<td>Hardwood terrain</td>
<td>7.45%</td>
<td>0.91</td>
<td>6.04%</td>
<td>0.52</td>
</tr>
<tr>
<td>Gravels terrain</td>
<td>8.27%</td>
<td>9.19</td>
<td>6.39%</td>
<td>5.89</td>
</tr>
</tbody>
</table>
According to Table 1, error rate of using adaptive Kalman filtering algorithm is lower than that of using traditional Kalman filtering algorithm, result after filtering is also more close to known terrain, comparing processing effect of local outliers, adaptive Kalman filtering algorithm can reduce error to about 60% that of traditional Kalman algorithm, with more significant smoothing effects.

6 Conclusion

Based on traditional Kalman filtering algorithm this paper proposes adaptive Kalman filtering algorithm, then simulate filtering effect of two algorithms through Matlab, and use them in ultrasonic detection result processing of hardwood and gravels terrain. By comparison, both algorithms can remove abnormal value in measurement, effect of adaptive Kalman filtering algorithm is better than that of traditional Kalman filtering algorithm, realizing optimization of traditional Kalman filtering algorithm, with better performance in estimation of terrain elevation. In the future, we will make further efforts to optimize Kalman filtering algorithm to further improve the filtering accuracy.

Reference
