Third harmonic current injection into highly saturated multi-phase machines

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Abstract: One advantage of multi-phase machines is the possibility to use the third harmonic of the rotor flux for additional torque generation. This effect can be maximised for Permanent Magnet Synchronous Machines (PMSM) with a high third harmonic content in the magnet flux. This paper discusses the effects of third harmonic current injection (THCI) on a five-phase PMSM with a conventional magnet shape depending on saturation. The effects of THCI in five-phase machines are shown in a 2D FEM model in Ansys Maxwell verified by measurement results. The results of the FEM model are analytically analysed using the Park model. It is shown in simulation and measurement that the torque improvement by THCI increases significantly with the saturation level, as the amplitude of the third harmonic flux linkage increases with the saturation level but the phase shift of the rotor flux linkage has to be considered. This paper gives a detailed analysis of saturation mechanisms of PMSM, which can be used for optimizing the efficiency in operating points of high saturations, without using special magnet shapes.

Key words: dq-model, non-linear effects, PMSM, saturation, third harmonic current injection, torque improvement

1. Introduction

Multi-phase machines are rarely used in commercial applications due to high inverter costs. But compared to conventional three-phase machines they have the advantage of a higher power density due to higher winding factors and the possibility to use higher harmonics of the rotor flux for torque improvement by third harmonic current injection (THCI). In [1] five-phase machines with special magnet shapes are presented, which are optimised on third harmonic content in the magnet flux for maximizing the torque improvement by THCI. It has been shown in simulation, analytical results and measurement, that the optimal torque improvement can be achieved, if the third harmonic of the magnet flux is 1/6 of the first harmonic. The THCI with the presented magnet shape leads to a higher efficiency compared to motors with conventional magnet shapes without THCI [2] deals with the analytical determina-
tion of optimised THCI. In [3] control methods and systems for five-phase synchronous machines are presented to optimise the third harmonic current in phase and amplitude. With a table containing all operating points the optimal current configuration (direct current $i_d$, $q$-current $i_q$, direct current of the third harmonic $i_{d3}$, and the $q$-current of the third harmonic $i_{q3}$) concerning the motor losses is chosen. Another advantage of THCI is the higher phase control factor of the inverter and the peak voltage reduction.

Up to date no deliberations have been made on the influence of THCI taking into account the non-linear effects of the electromagnetic circuit of Permanent Magnet Synchronous Machines (PMSM). In this paper the THCI into a five-phase PMSM with conventional surface-mounted magnets and concentrated winding is described for different saturation levels concerning the effects on torque improvement, losses and peak voltage reduction by an analytical model, simulation and measurement.

The test drive is fed by an external decentralised inverter consisting of ten SST (smart stator teeth) modules, one for each coil. In the second prototype the SST-modules are integrated into the motor housing. The motor concept is described in [4]. The modeling and parameter identification of this drive is described in [5].

![Fig. 1. Cross-section of the test drive with phase numbers](image)

### 2. Motor concept and magnet shape

As shown in [1] a special magnet shape with a high third harmonic magnet flux is required for a considerable torque improvement by THCI. In this paper a segmented five-phase PMSM with 10 slots and 10 poles with loaf-shaped magnets is presented (Fig. 1). It also requires
a high overload and demagnetization withstand capability. This can only be achieved by conventional magnet shapes with evenly distributed magnet thickness over the pole circumference, for example arc-shaped and loaf-shaped magnets. But these magnets have a relatively low third harmonic component of the permanent magnet flux (about 5% of the first harmonic). Deliberations have been made, that arc-shaped magnets with a wide gap in the pole middle would have a higher percentage of the third harmonic permanent magnet flux. But according to FEM simulations the idea has been proven as ineffective as the first harmonic of the magnet flux has decreased to such a large extent that the motor efficiency decreased by about 4.5% at the same output torque. On the other hand it has been proven by simulation and measurements that the third harmonic of the permanent magnet flux linkage increases from 3 to more than 6% from no-load to double rated current due to saturation effects for the examined motor.

3. Simulation model and experimental setup

The experimental setup consists of a test drive with a rated power of 7.5 kW, a rated speed of 600 rpm and a rated torque of 120 Nm, and a geared load motor. As measurement devices a weighing cell for the torque, a Yokogawa WT 3000 power analyzer for current, voltage and power factor in one stator coil, and PT 100 temperature sensors on both end windings of each stator coil are used. For determination of the efficiency of the whole drive system voltage and current are also measured in the intermediate circuit. The control unit allows to separately provide \( d \)-current, \( q \)-current, \( d_3 \)-current and \( q_3 \)-current. An incremental encoder is used to detect the rotor position. The control unit considers the geometrical symmetry axis of the magnets as \( d \)-axis and \( d_3 \)-axis. All measurements are carried out at constant temperature (42°C).

For simulation the motor is modeled in the FEM-program Ansys Maxwell 2D taking into account the rotor position. The currents are given in the \( dq \)-system according to the analytical model described in section 5 and internally transformed into phase currents. Induced voltages, the total flux linkages and inductances of all phases as well as the copper losses, magnet losses and core losses and torque are calculated directly in Maxwell for every time step and transformed into the \( dq \)-system.

As in the control unit of the realised drive system the geometrical symmetry axis of the magnets is assumed to be the \( d \)-axis and the \( d_3 \)-axis. As the maximum current which can be carried by the SST modules does not affect demagnetization, the BH-curve of the permanent magnet material can be assumed as linear, but the non-linear BH-curve of the laminated core is considered. The segmentation of the magnets in axial direction as well as the segment air gap between the single stator tooth modules are neglected.

4. Analytical model

The electromagnetic torque \( T \) for \( m \)-phase machines is defined according to the Park model by:
Here $i$ is the current vector, $J$ the coefficient matrix from the derivation of the rotation matrix, $M$ the inductance matrix, $p$ the number of pole-pairs and $\Psi_{PM}$ the permanent magnet flux linkage. For five-phase systems the components of the vectors and matrices from Equation (1) are defined as:

$$i = \begin{bmatrix} i_d \\ i_q \\ i_{d3} \\ i_{q3} \\ i_0 \end{bmatrix} \quad ; \quad M = \begin{bmatrix} L_{dd} & L_{dq} & L_{dd3} & L_{dq3} & L_{d0} \\ L_{dq} & L_{qq} & L_{q3d} & L_{q3q} & L_{q0} \\ L_{dd3} & L_{d3q} & L_{dd3} & L_{dq3} & L_{d30} \\ L_{q3d} & L_{q3q} & L_{q3d} & L_{q3q} & L_{q30} \\ L_{d0} & L_{q0} & L_{d03} & L_{q03} & L_{q00} \end{bmatrix} .$$

As only static operation points are considered $i_d, i_q, i_{d3}, i_{q3}$ and the zero-current $i_0$ are constant. The inductances are the absolute inductances:

$$L_{ik} = \frac{\Psi_i}{i_k} .$$

$L_{ik}$ is the absolute inductance between phase $i$ and $k$, $\Psi_i$ the flux linkage generated by phase $k$ and crossing phase $i$ and $i_k$ is the current of phase $k$. When the absolute inductances are transformed into the $dq$-system, the minor elements can be neglected, but for non-linear operating points they have to be considered.

The components of the permanent magnet flux linkage are defined analogously to the current, whereas for ideal linear conditions the $q$-component and $q^3$-component of the permanent magnet flux linkage is 0. But for higher saturation levels, when the $d$-axis and $d3$-axis are defined as the geometrical symmetry axis of the magnets, the phases of the harmonics of the magnet flux linkage shift so that:

$$\Psi_{PM,q} \neq 0 \quad \text{and} \quad \Psi_{PM,q^3} \neq 0 ,$$

with $\Psi_{PM,q} = 0$ and $\Psi_{PM,q^3} = 0$ only $i_q$ and $i_{q3}$ affect the output torque considerably. But when Equation (4) is fulfilled, an additional $d3$-current can be injected to achieve further torque improvement as the phase of the third harmonic current is shifted to fit the phase of the third harmonic of the magnet flux linkage. The permanent magnet flux linkage is calculated out of the total flux linkage $\Psi$ calculated in Maxwell:

$$\Psi_{PM} = \Psi - M_i .$$

In the following the different torque generating mechanisms from Equation (1) are distinguished. Firstly, the torque $T_{PM}$ generated by the first and third harmonic of the permanent magnet flux is defined as:
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\[ T_{PM} = \frac{m}{2} p \left( \Psi_{PM,d} i_q^d - \Psi_{PM,q} i_q^d + 3 \Psi_{PM,d3} i_{q3}^d - 3 \Psi_{PM,q3} i_{q3}^d \right). \]  

Furthermore the reluctance torque \( T_{rel} \) considering the cross-saturation effects is:

\[ T_{rel} = \frac{m}{2} p i^d JMi. \]

The total current for a single phase is defined as:

\[ I_t = \sqrt{I_d^2 + I_q^2 + I_{d3}^2 + I_{q3}^2}. \]

5. Results

5.1 Saturation behavior

To show the saturation behavior of the machine the phase voltage and the torque are calculated in the FEM model for \( q \)-current \( I_q \) in steps of 20\% up to 300\% of the rated current (\( I_r = 16.54 \) A). In the range of the total phase current given in Equation (8), which can be carried by the SST modules (between 200\% and 250\% of rated current) measurements and simulation results are in good agreement with each other. According to the simulation results the electromagnetic circuit is fully saturated at total phase currents beginning with 300\% of the rated current, where the phase voltages and the torque start to increase linearly with the phase current. That means full saturation cannot be reached with the given SST modules, but nonlinear effects already occur beginning with 140\% of the rated current. Simulation results are presented up to 300\% of the rated current only to show the behavior at full saturation. According to analytic thermal estimations and measurements the winding insulation can withstand total phase currents of about 180\% of the rated current in permanent operation. Concerning the measured thermal capacity of the machine the winding can withstand the highest overload current of 250\% which can be carried by the SST modules for about 5 minutes.

Fig. 2. Amplitude and phase of the first and third harmonic of the permanent magnet flux linkage depending on the \( q \)-current from 0 to 300\% of rated current in steps of 20\%
Another special non-linear effect in five-phase PMSM, which has been mentioned in the previous section, is the phase shift of the permanent magnet flux linkage in the first and third harmonic and the increase of the amplitude of the third harmonic flux linkage depending on saturation, which can be seen in simulation results in Fig. 2.

In Fig. 2 it can be seen, that the amplitude of the first harmonic almost remains constant between 0 and 200% of phase current. For further increase of the phase current the first harmonic decreases linearly. The phase shift of the first harmonic starts to increase until the machine is fully saturated. Much more significant are the changes of the third harmonic of the permanent magnet flux linkage. Here the phase shift increases to a much larger extent, than for the first harmonic. The maximum phase shift is reached at about the same current. The amplitude for the third harmonic permanent magnet flux linkage is negligible for the unsaturated motor, but it starts to increase significantly at $I = 120\%$. The maximum gradient of the third harmonic flux is reached at full saturation.

5.2. Torque improvement

Firstly the torque depending on the $d$- and $q$-current for the first harmonic is examined. In Fig. 3 it can be seen, that for constant $q$-current ($I_q = 150\%$ of rated current) a significant dependence on $d$-current can be observed. Closer examinations with the analytical model show, that the permanent magnet torque $T_{PM}$ almost remains constant, while the reluctance torque $T_{rel}$ considerably rises with negative $d$-current. The torque improvement by $d$-current reaches its maximum at about $i_d = 50\%$ of rated current. Simulation and measurement agree very well with each other.

In the following the torque output is simulated for $i_q$ from 20 to 240% of rated current in steps of 20%. Furthermore for each value of $i_q$ the effect of third harmonic current is examined and the optimal relation of $i_{d3}$ and $i_{q3}$ concerning torque and the efficiency are determined.

The TCHI is examined for different values of $i_{d3}$ where low values turn out to be more effective because the third harmonic of the permanent magnet flux is rather small. For all
$q$-current values the efficiency and the torque slightly improve with THCI without $d3$-current. For each value of $i_q$ and $i_{d3}$ the $d3$-current is increased until the efficiency starts to decrease and the torque improvement $\Delta T$ compared to the same $q$-current without THCI is detected. These values of $i_{d3}$ and $\Delta T$ are given in Fig. 4 exemplary for a constant $q3$-current (5%). In Fig. 4 it can be seen, that the THCI only has a considerable effect on torque improvement for higher saturation levels (about 1.3% related to the torque value with $i_{d3} = i_{q3} = 0$). But the required value for $i_{d3}$ increases with the saturation level until $i_q = 220\%$ as the phase shift of the third harmonic of the magnet flux linkage increases. At $i_q = 200\%$ and higher, $q$-component of the flux component even outweighs the $d$-component. That means $d3$-current injection becomes more effective than $q3$-current injection. The measurements in Fig. 4 agree well with the simulations.

![Fig. 4](image_url)

Fig. 4. Relative torque improvement $\Delta T$ (to $i_{d3} = i_{q3} = 0$) with $i_{q3} = 5\%$ (of rated current) and optimal value for $i_{d3}$ (constant efficiency)

The dependence of efficiency and torque improvement on $d3$-current is shown more clearly in Fig. 5 and 6 for $I_q = 140\%$ and $I_q = 200\%$ with $I_{q3} = 5\%$. For both operation points the efficiency is increased by THCI. The efficiency for the operation without TCHI is given here as reference (blue line). Here it can be seen that for $I_q = 140\%$ the efficiency decreases at lower $d3$-currents below the reference efficiency than for higher currents ($I_q = 200\%$) and torque improvement is much lower.

![Fig. 5](image_url)

Fig. 5. Relative torque improvement $\Delta T$ (to $i_{d3} = i_{q3} = 0$) with $i_q = 140\%$ and $i_{q3} = 5\%$ (of rated current) and efficiency depending on $i_{d3}$
The torque generating effects are more closely observed in Fig. 7, where different from the $d$-current injection for the first harmonic, the permanent magnet torque has a much greater effect compared to the reluctance torque. This torque improvement is only weakened by reduction of the first harmonic of the permanent magnet flux linkage by increasing $d3$-current.

The agreements with the measurement results are acceptable concerning the relatively low torque differences to be measured, the measurement tolerances and high interferences of the technical equipment despite good shielding. The measurements in Fig. 7 show that the motor efficiency and also the total efficiency of the drive system including the rectifier losses almost remain constant with third harmonic current injection in the investigated range of $i_{d3}$.

6. Conclusions

In PMSM with conventional magnet shapes with low third harmonic content in the permanent magnet flux linkage, the effects of THC1 with unsaturated electromagnetic circuit are negligible. But for higher saturation levels third harmonic linkage significantly rises and THC1 can be very beneficial with consideration of the $q3$-component of the permanent magnet flux linkage. For high overload a higher efficiency in a motor and inverter can be reached.
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References