Corrigendum

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Remarks on L^2 boundedness of Littlewood–Paley operators

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Corrigendum to: K. Yabuta, Remarks on L^2 boundedness of Littlewood–Paley operators, Analysis 33 (2003), 209–218

Abstract: In our paper in this journal, entitled "Remarks on L^2 boundedness of Littlewood–Paley operators", there are two incomplete statements and incompleteness in the proof of the main theorem. In this short note we will correct them.

Keywords: Littlewood–Paley operators, Littlewood–Paley g-function, L^2 boundedness

MSC 2010: Primary 42B25; secondary 42B20

In our paper [2], entitled "Remarks on L^2 boundedness of Littlewood–Paley operators", there are two incomplete statements and incompleteness in the proof of the main theorem.

1. From line 11 to line 12 in the Introduction, the statement "g_ψ is bounded on $L^2(\mathbb{R}^n)$ if and only if

$$\sup_{\xi' \in S^{n-1}} \bigg| \iint_{\mathbb{R}^n \times \mathbb{R}^n} \psi(x)\overline{\psi(y)} \log|\xi' \cdot (x-y)|\, dx\, dy \bigg| < \infty."$$

should be replaced by "under the assumption

$$\iint_{\mathbb{R}^n \times \mathbb{R}^n} \big|\psi(x)\overline{\psi(y)} \log|\xi' \cdot (x-y)|\big|\, dx\, dy < \infty \quad \text{for a.e. } \xi' \in \mathcal{S}^{n-1}, \tag{1.0}$$

g_ψ is bounded on $L^2(\mathbb{R}^n)$ if and only if

$$\sup_{\xi' \in S^{n-1}} \bigg| \iint_{\mathbb{R}^n \times \mathbb{R}^n} \psi(x)\overline{\psi(y)} \log|\xi' \cdot (x-y)|\, dx\, dy \bigg| < \infty."$$

2. In Remark 1.1, the statement

$$\text{“}\Omega \in \mathcal{F}_1(S^{n-1}) := \bigg\{ \Omega \in L^1(S^{n-1}) : \sup_{\xi' \in S^{n-1}} \int_{S^{n-1}} |\Omega(y')| \log \frac{1}{|\xi' \cdot y'|}\, d\sigma(y') < \infty \bigg\}.\text{”}$$

should be replaced by

$$\text{“}\Omega \in \mathcal{F}_1(S^{n-1}) := \bigg\{ \Omega \in L^1(S^{n-1}) : \sup_{\xi' \in S^{n-1}} \bigg| \int_{S^{n-1}} \Omega(y') \log \frac{1}{|\xi' \cdot y'|} \bigg|\, d\sigma(y') < \infty \bigg\}.\text{”}$$

3. In line 9 on page 216, the statement "Since $\psi \in L^1(\mathbb{R}^n)$, this shows the desired assertion." should be replaced by "Next we check (1.0). In the case $n = 1$, $\xi' = 1$ or $= -1$ for $\xi' \in S^0$, and so we trivially have

$$\iint_{\mathbb{R}^1 \times \mathbb{R}^1} |\psi(x)\overline{\psi(y)}| \log \frac{2}{\sqrt{(\xi' \cdot x')^2 + (\xi' \cdot y')^2}}\, dx\, dy = \log \sqrt{2} \|\psi\|^2_{L^1(\mathbb{R})} \quad \text{for } \xi' \in S^0.$$

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In the case $n \geq 2$, we have

$$\begin{aligned}\int_{S^{n-1}} \iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \log \frac{1}{|\xi'\cdot x'|}\,dx\,dy\,d\sigma(\xi') &= \iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \int_{S^{n-1}} \log\frac{1}{|\xi'\cdot x'|}\,d\sigma(\xi')\,dx\,dy\\ &= \iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \int_{S^{n-1}} \log\frac{1}{|\xi'_1|}\,d\sigma(\xi')\,dx\,dy\\ &= \omega_{n-2}\iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \int_{-1}^{1}\Big(\log\frac{1}{|s|}\Big)(1-s^2)^{\frac{n-3}{2}}\,ds\,dx\,dy\\ &= C_n\|\psi\|^2_{L^1(\mathbb{R}^n)},\end{aligned}$$

where ω_{n-2} is the surface area of the unit sphere in $\mathbb{R}^{n-1}$ (see [1, Section 5.2.2]). Hence we get

$$\iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \log\frac{1}{|\xi'\cdot x'|}\,dx\,dy < \infty \quad \text{for a.e. } \xi' \in S^{n-1}.$$

Thus we have

$$\iint_{\mathbb{R}^n\times\mathbb{R}^n} |\psi(x)\overline{\psi(y)}| \log\frac{2}{\sqrt{(\xi'\cdot x')^2+(\xi'\cdot y')^2}}\,dx\,dy < \infty \quad \text{for a.e. } \xi' \in S^{n-1}.$$

Using the above estimate, and observing the proof of estimates (2.4)–(2.11), we see that

$$\begin{aligned}&\iint_{\mathbb{R}^n\times\mathbb{R}^n} \big|\psi(x)\overline{\psi(y)}\log|\xi'\cdot(x-y)|\big|\,dx\,dy\\ &\leq \iint_{\mathbb{R}^n\times\mathbb{R}^n} \Big|\psi(x)\overline{\psi(y)}\log\sqrt{(\xi'\cdot x')^2+(\xi'\cdot y')^2}\Big|\,dx\,dy\\ &\qquad + \int_{\mathbb{R}^n\times\mathbb{R}^n} \Big|\psi(x)\overline{\psi(y)}\log\sqrt{|x|^2+|y|^2}\Big|\,dx\,dy\\ &\qquad + \iint_{S^{n-1}\times S^{n-1}} \bigg(\int_0^\infty\bigg[\int_0^{\frac{\pi}{2}}\Big|\psi(r\cos\theta\, x')\overline{\psi(r\sin\theta\, y')}\Big|(\cos\theta\sin\theta)^{n-1}\\ &\qquad\qquad \times\bigg|\log\bigg|\cos\bigg(\theta+\tan^{-1}\frac{\xi'\cdot y'}{\xi'\cdot x'}\bigg)\bigg|\bigg|\,d\theta\bigg]r^{2n-1}\,dr\bigg)\,d\sigma(x')\,d\sigma(y') < \infty\end{aligned} \tag{2.13}$$

for a.e. $\xi' \in S^{n-1}$. Thus, by (2.12) we obtain the desired assertion.

References

[1] L. Grafakos, *Classical Fourier Analysis*, 2nd ed., Grad. Texts in Math. 249, Springer, New York, 2008.
[2] K. Yabuta, Remarks on L^2 boundedness of Littlewood–Paley operators, *Analysis* **33** (2003), 209–218.