

Erratum

Harmonic boundary value problems in a quarter ring domain

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In the article above the harmonic Neumann function for a quarter ring domain is studied. Due to a sign mistake in the calculations of the outward normal derivatives, formula (3.9) has to be changed. In the second line of (3.9), on the boundary part $\{z : |z| = r, \operatorname{Re} z > 0, \operatorname{Im} z > 0\}$ the constant 0 is to be replaced by $-\frac{8}{r}$. For $\zeta \in \partial R^*$ the boundary behavior of $\partial_{v_z} N_1(z, \zeta)$ on $|z| = r$ is modified to

$$\partial_{v_z} N_1(z, \zeta) = \begin{cases} -\frac{8}{r}, & \zeta \in \partial R^* \setminus \partial_2 R^*, \\ -\frac{8}{r} \operatorname{Re} \left(\frac{\xi^2}{\xi^2 - z^2} + \frac{\bar{\xi}^2}{\bar{\xi}^2 - \bar{z}^2} - 1 \right) - \frac{8}{r}, & \zeta \in \partial_2 R^*. \end{cases} \quad (*)$$

This fact entails some modifications of the considered Neumann problem. Theorem 3.4 is complemented with a solubility condition. It turns to be reformulated as

Theorem 3.4. *Let $f \in L_2(R^*; \mathbb{C}) \cap C(R^*; \mathbb{C})$, $\gamma \in C(\partial R^*; \mathbb{C})$ and $c \in \mathbb{C}$. Then the Neumann problem*

$$w_{z\bar{z}} = f \quad \text{in } R^*, \quad \partial_v w = \gamma \quad \text{on } \partial R^*, \quad -\frac{2}{\pi} \int_0^{\frac{\pi}{2}} w(re^{i\varphi}) d\varphi = c$$

is uniquely solvable if and only if

$$\frac{1}{4\pi} \int_{\partial R^*} \gamma(\xi) ds_\xi = \frac{1}{\pi} \int_{R^*} f(\xi) d\xi d\eta.$$

The solution is presented by

$$w(z) = \frac{1}{4\pi} \int_{\partial R^*} \gamma(\xi) N_1(z, \xi) ds_\xi + c - \frac{1}{\pi} \int_{R^*} f(\xi) N_1(z, \xi) d\xi d\eta.$$

The proof of Theorem 3.4 has to be modified with regard to the boundary behavior of the boundary integral on the boundary part $\partial_2 R^*$ due to (*).

For $|\zeta_0| = r$, $\operatorname{Re} \zeta_0 > 0$, $\operatorname{Im} \zeta_0 > 0$,

$$\begin{aligned}
 \partial_{\nu} w(z_0) &= - \lim_{z \rightarrow \zeta_0} \frac{1}{r} (z \partial_z + \bar{z} \partial_{\bar{z}}) w(z) \\
 &= - \lim_{z \rightarrow \zeta_0} \left\{ \frac{2}{\pi i r} \int_{\substack{|\xi|=1, \\ 0 < \operatorname{Im} \xi, \\ 0 < \operatorname{Re} \xi}} \gamma(\xi) \frac{d\xi}{\xi} \right. \\
 &\quad + \frac{1}{2\pi i r} \int_{\substack{|\xi|=r, \\ 0 < \operatorname{Im} \xi, \\ 0 < \operatorname{Re} \xi}} \gamma(\xi) \left[\left(\frac{\xi}{\xi - z} + \frac{\bar{\xi}}{\bar{\xi} - \bar{z}} - 1 \right) \right. \\
 &\quad + \left(\frac{\xi}{\xi + z} + \frac{\bar{\xi}}{\bar{\xi} + \bar{z}} - 1 \right) + \left(\frac{\bar{\xi}}{\bar{\xi} - z} + \frac{\xi}{\xi - \bar{z}} - 1 \right) \\
 &\quad \left. \left. + \left(\frac{\bar{\xi}}{\bar{\xi} + z} + \frac{\xi}{\xi + \bar{z}} - 1 \right) \right] \frac{d\xi}{\xi} \right. \\
 &\quad + \frac{2}{\pi i r} \int_{\substack{|\xi|=r, \\ 0 < \operatorname{Im} \xi, \\ 0 < \operatorname{Re} \xi}} \gamma(\xi) \frac{d\xi}{\xi} + \frac{2}{\pi r} \int_r^1 \gamma(t) dt \\
 &\quad \left. - \frac{2}{\pi r} \int_r^1 \gamma(it) dt - \frac{8}{\pi r} \int_{R^*} f(\xi) d\xi d\eta \right\} \\
 &= - \lim_{z \rightarrow \zeta_0} \left\{ \frac{1}{2\pi i} \int_{|\xi|=r} \Gamma_1(\xi) \left(\frac{\xi}{\xi - z} + \frac{\bar{\xi}}{\bar{\xi} - \bar{z}} - 1 \right) \frac{d\xi}{\xi} \right. \\
 &\quad \left. - \frac{2}{\pi r} \int_{\partial R^*} \gamma(\xi) ds_{\xi} + \frac{8}{\pi r} \int_{R^*} f(\xi) d\xi d\eta \right\} \\
 &= \gamma(\zeta_0) - \frac{2}{\pi r} \int_{\partial R^*} \gamma(\xi) ds_{\xi} + \frac{8}{\pi r} \int_{R^*} f(\xi) d\xi d\eta.
 \end{aligned}$$

For the boundary behavior of the boundary integrals on the real and imaginary axes the kernels $\Gamma_2(t)$ in (2.10) and $\Gamma_2(it)$ in (2.11) are modified to

$$\Gamma_2(t) = \begin{cases} \gamma(t), & r \leq t \leq 1, \\ \frac{1}{t^2} \gamma\left(\frac{1}{t}\right), & 1 \leq t \leq \frac{1}{r}, \\ 0, & |t| \leq r, |t| \geq \frac{1}{r}, \\ \gamma(-t), & -1 \leq t \leq -r, \\ \frac{1}{t^2} \gamma\left(-\frac{1}{t}\right), & -\frac{1}{r} \leq t \leq -1, \end{cases}$$

and

$$\Gamma_2(it) = \begin{cases} \gamma(it), & r \leq t \leq 1, \\ \frac{1}{t^2} \gamma(-\frac{1}{it}), & 1 \leq t \leq \frac{1}{r}, \\ 0, & |t| \leq r, |t| \geq \frac{1}{r}, \\ \gamma(-it), & -1 < t < -r, \\ \frac{1}{t^2} \gamma(\frac{1}{it}), & -\frac{1}{r} < t < -1. \end{cases}$$

These last corrections do not impact the results.

The author apologizes for the mistakes made in that article.

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