Abstract. A comparison of the observed stellar distributions with a three-component model of the Galaxy is presented. The analysis is based on photometric data and absolute proper motions obtained in three fields of the program MEGA. Two of the fields are located near the North Galactic Pole and the third field is towards the galactic center. The model assumed considers the Galaxy as composed of the disk, thick disk and spheroid populations. To model the observed color distribution, we consider the main sequence stars and disk red giants as the disk subsystem, white dwarfs, subdwarfs and intermediate giants as the thick disk subsystem and extreme subdwarfs, spheroid giants and horizontal branch stars as the spheroid subsystem. To describe the observed distribution of absolute proper motions, a normal distribution of the spatial velocity components is considered.

The data were used to improve parameters of spatial and kinematical models of the Galaxy.

Key words: methods: observational, statistical – techniques: photometric – stars: HR diagram, luminosity function – ISM: extinction – the Galaxy: kinematics, stellar content, structure

1. INTRODUCTION

We present the results obtained within the program MEGA (Einasto et al. 1985) of studying the main meridian of the Galaxy.
The purpose of the program is to investigate the kinematics and spatial structure of stellar populations from observations of non-selected samples of stars and to specify the galactic models. The importance of investigations in the galactic meridian has been defined by the possibility to analyze the radial and vertical gradients of the galactic parameters. The spatial velocity component in the direction of the galactic rotation can be obtained directly from proper motions and distances. Moreover, these data are sufficient to derive the velocity component along the galactic radius in the polar areas and the velocity component along the galactic axis in the center–anticenter areas. In the galactic meridian and especially in the areas of the galactic poles, we are able to analyze the galactic subsystems from statistically significant stellar samples and, consequently, to study the Galaxy at different stages of its evolution.

The program includes 64 selected areas (Kharchenko 1983, Schilbach 1988), with 47 of them observed with the long-focus astrograph in Kiev and 17 with the Tautenburg Schmidt telescope. The location of the fields is fixed by the available astrometric plates which can serve as the first epochs to obtain stellar proper motions with respect to galaxies.

The aim of this study is to model the distributions of photometric data and absolute proper motions by the use of the fundamental equation of stellar statistics and to specify the galactic parameters from a comparison with the observational data.

2. OBSERVATIONAL DATA

2.1. Stellar data in 47 Kiev fields

Accurate coordinates, proper motions with respect to galaxies and \( B \) magnitudes were obtained for stars in 47 program fields using 65 pairs of plates taken with the long-focus astrograph (40/550 cm) in Kiev (Kharchenko 1984). Systematic errors in the proper motions (e.g., due to the magnitude equation) were analyzed and reduced by comparing the data for common stars from six other catalogs obtained within the Faint Stars Plan (Deutsch 1952).

Due to the lack of photometric standards, the new photoelectric observations were necessary to provide a photometric calibration. Photoelectric magnitudes and colors in the \( UBVR \) system were obtained for 943 stars in the program fields (Andruk et al. 1995). With
15–30 stars per field, this photoelectric subset (PES) provides a uniform photometric system over all program fields. The data were used to calibrate the plates and to obtain $B$ magnitudes for the program stars.

2.2. Stellar data from the Tautenburg Schmidt survey

Three Tautenburg fields, F14, F15 with the globular cluster M 3 (both towards the North Galactic Pole) and F17, were included in the analysis. The field F17 is located in the direction to the galactic center. Here, the line of sight crosses the space between the Local and Sagittarius-Carina spiral arms and then reaches the next one, nearer to the galactic center.

The Schmidt plates were scanned with two automatic measuring machines, MAMA in Paris (fields F14 and F17) and APM in Cambridge (field F15). Two pairs of plates in F14, five pairs of plates in F15 and three plates in F17 were used to determine proper motions and $B, V$ magnitudes.

In F14 and F15, a large number of background galaxies were used as reference points to determine the absolute proper motions directly. In the field F17, where no galaxies were identified, the relative proper motions were converted to the absolute ones with the data from the PPM-South catalog (Bastian & Röser 1993). The magnitude-dependent systematic errors in the proper motions were corrected by using the data for galaxies (F14), members of the globular cluster M3 (F15) and members of four open clusters (F17), with the assumption that proper motions of these objects are independent of stellar magnitudes (Scholz & Kharchenko 1994, Kharchenko & Schilbach 1995).

The photometric calibration of the MAMA density flux measurements in F14 and F17 was based on photoelectric standards taken from SIMBAD (Nicolet 1978) and photoelectric sequences by Walker (1961) and Sagar & Joshi (1979). In the field F15, the internal magnitude calibration was applied to the APM measurements, and the APM magnitudes were then converted to $B$ and $V$ magnitudes using the photoelectric photometry of Sandage (1970).

The data analyzed in this study are summarized in Table 1. In the following, we consider the Kiev stellar sample in six groups separated according to their location relative to the galactic meridian. The parts of the fields F15 and F17, occupied by cluster stars, were excluded from the analysis.
Table 1. Observational data

<table>
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<tr>
<th>Sky region</th>
<th>( \bar{l} ) [deg]</th>
<th>( \bar{b} ) [deg]</th>
<th>Area [sq.deg]</th>
<th>Number of stars</th>
<th>( B_{\text{lim}} ) [mag]</th>
<th>rms error ( B ) [mag]</th>
<th>rms error ( V ) [mag]</th>
<th>( \mu_x, \mu_y ) [mas/yr]</th>
<th>Number of galaxies</th>
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<td>1</td>
<td>188</td>
<td>+32</td>
<td>8.0</td>
<td>2475</td>
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<td>±0.3</td>
<td>±7.6</td>
<td>±0.3</td>
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<td>2658</td>
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<td>±0.3</td>
<td>±7.3</td>
<td>±0.3</td>
<td>24</td>
</tr>
<tr>
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<td>±2.5</td>
<td>±0.1</td>
<td>(V&lt;17)</td>
</tr>
<tr>
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<td>29125</td>
<td>17.9</td>
<td>±0.1</td>
<td>±2.7</td>
<td>±0.1</td>
<td>(V&lt;14.5) PPM stars</td>
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</tbody>
</table>

3. THE BASIC EQUATIONS AND RELATIONSHIPS

In this study we consider a three-component model of the Galaxy consisting of the thin disk (or disk), thick disk and spheroid populations. Further, we assume two groups of stars in the thin disk: the main sequence stars and the disk red giants. These two stellar groups have different scale heights and, contrary to the thick disk and spheroid populations, show a continuous set of the galactic parameters. Moreover, the correspondence between absolute magnitudes and scale heights for these groups is expressed by different relations.

To model the observed color index distribution, we consider eight types of stars: main sequence stars and disk red giants as the disk subsystem; subdwarfs and intermediate giants (or stars of the intermediate population before and after their main sequence turn-off point) and white dwarfs as the thick disk subsystem; extreme subdwarfs and spheroid giants (or stars of Population II before and after their main sequence turn-off point) and horizontal branch stars as the spheroid subsystem.
We used the first fundamental equation of stellar statistics for the $j$-th subsystem, or stellar group:

\[
A_j(V, l, b) = d\Omega \int_{V_{i1}}^{V_{i2}} dV \int_0^{R_{lim}} D_j(\vec{r}, M) F_j(M_V) R^2 dR,
\]

(1)

\[
A_j(B, l, b) = d\Omega \int_{B_{i1}}^{B_{i2}} dV \int_0^{R_{lim}} D_j(\vec{r}, M) F_j(M_B) R^2 dR
\]

(2)

to calculate the number $A_j$ of stars in a solid angle $d\Omega$ and in the $i$-th magnitude interval $\Delta V_i = V_{i2} - V_{i1}$ or $\Delta B_i = B_{i2} - B_{i1}$.

\[
A_j((B-V), l, b) = d\Omega \int_{(B-V)_{i1}}^{(B-V)_{i2}} d(B-V) \int_0^{R_{lim}} D_j(\vec{r}, M) F_j(M_V) R^2 dR
\]

(3)

to calculate the number of stars in the $i$-th color index interval $\Delta(B-V)_i = (B-V)_{i2} - (B-V)_{i1}$ and

\[
A_j(\mu, V, l, b) = 4.74 \cdot d\Omega \int_{\mu_{k1}}^{\mu_{k2}} d\mu \int_{V_{i1}}^{V_{i2}} dV \int_0^{R_{lim}} D_j(\vec{r}, M_V) F_j(M_V) P_j(V_t) R^2 dR
\]

(4)

to calculate the number of stars in the $k$-th interval of proper motions $\Delta \mu_k = \mu_{k2} - \mu_{k1}$ and in the $i$-th magnitude interval $\Delta V_i = V_{i2} - V_{i1}$. Here $D_j(\vec{r}, M)$ is the space density as a function of the absolute magnitude $M$ and of the position $\vec{r}$ relative to the galactic center; $F_j(M)$ is the luminosity function in the solar neighborhood; $P_j(V_t)$ is the distribution function of the tangential velocity component $V_t$ with respect to the Sun; $R$ is the distance from the Sun; and $R_{lim}$ is the maximum distance from the Sun which a star of a given absolute magnitude can reach within a given apparent magnitude interval.

The expressions for the density function $D(\vec{r}, M)$ of each subsystem are taken from Bahcall (1986):
\[
D_{\text{disk}}(\vec{r}, M_{V \text{or } B}) = \exp \left[ -\frac{Z}{h_Z}(M_{V \text{or } B}) - \frac{(X - R_0)}{h_X} \right],
\]
\[
D_{\text{thick disk}}(\vec{r}, M) = \exp \left[ -\frac{Z}{h_Z} - \frac{(X - R_0)}{h_X} \right],
\]
\[
D_{\text{spheroid}}(\vec{r}, M) = (\frac{R_s}{R_0})^{-7/8} \exp \left[ -10.093 (\frac{R_s}{R_0})^{0.25} + 10.093 \right] \left[ 1 - 0.08669/\left( \frac{R_s}{R_0} \right)^{0.25} \right],
\]

where \( X = (R_0^2 + R^2 \cos b - 2RR_0 \cos b \cos l)^{0.5} \) is the distance in the galactic plane from the galactic center to a star; \( Z = R \sin b \) is its distance vertically to the galactic plane; \( r = (X^2 + Z^2)^{0.5} \) and \( R_0 \) are distances from the galactic center to a star and to the Sun, respectively; \( R_s = (X^2 + Z^2/k^2)^{0.5} \), \( k \) is the axial ratio of the spatial distribution of the spheroid; \( h_X \) is the disk scale length; \( h_Z \) is the disk scale height. As an indicator of the age, \( h_Z \) is related to the stellar absolute magnitude differently for main sequence stars and for red giants in the thin disk.

The luminosity functions \( F_j(M_V) \) and \( F_j(M_B) \) are shown in Fig. 1. The disk luminosity functions are computed by the use of the equations given in Bahcall & Soneira (1980) but excluding disk red giants and taking into account the dip \( d[\log F_{\text{disk}}(M)] \), i.e. the lack of stars within \( M_V \) interval from 6 to 10 mag (Starikova 1960, Wielen 1974):

\[
d[\log F_{\text{disk}}(M_{V \text{or } B})] = a_W \exp \left[ -0.5((M_{V \text{or } B} - m_1)/0.7)^2 \right],
\]
\[
M_V \leq 7, \ M_B \leq 8.1,
\]
\[
d[\log F_{\text{disk}}(M_{V \text{or } B})] = a_W \left( m_2 - M_{V \text{or } B} \right)/3),
\]
\[
7 < M_V \leq 10, \ 8.1 < M_B \leq 11.1,
\]

where \( m_1 \) is 7 and 8.1 mag, \( m_2 \) is 10 and 11.1 mag for \( V \) and \( B \), respectively. The value \( a_W \) of the dip is obtained by a fit of the model to the observed stellar distribution (see subsections 4.2 and 4.3).

The fraction of the main sequence stars in the solar neighborhood is

\[
1 - 0.477 \exp \left[ -0.5((M_{V,B} + 0.5)/0.5)^2 \right], \quad M_{V,B} < -0.5;
\]
\[
0.44 \exp \left[ 0.00015 (M_{V,B} + 8)^{3.5} \right], \quad -0.5 \leq M_V < 3.7, \quad -0.5 \leq M_B < 4.1;
\]
\[
1, \quad M_V \geq 3.7, \ M_B \geq 4.1.
\]
The luminosity functions $F_j(M_V)$ and $F_j(M_B)$ in stars/mag/pc$^3$ assumed in the present analysis. The luminosity function of the disk main sequence (thick lines) and disk red giants (thin lines) are based on the relationships given in Bahcall & Soneira (1980, 1981) and our results. The luminosity functions of the thick disk (short-dashed lines) and spheroid (long-dashed lines) as well as the normalization are taken from Gilmore (1984), whereas the horizontal branch was modeled with the data of Da Costa (1982). The dots, crosses and triangles correspond to $F(M_V)$ obtained by Starikova (1960), Wielen (1974) and McCuskey (1966), respectively. The open circles are the data of Allen (1973) for the sum of giants, subgiants and supergiants. The diamonds mark the data of Da Costa (1982) shifted along the vertical axis. The squares and asterisks correspond to $F(M_B)$ of Luyten (1968) and Allen (1973), respectively.

The remaining stars are disk red giants. The expressions for $M_{V,B} > -0.5$ mag were taken from Bahcall & Soneira (1981), whereas for $M_{V,B} < -0.5$ mag we derived the fraction of the main sequence stars by assuming the absence of very luminous red giants in the disk.

The luminosity functions of the thick disk and spheroid and the local density ratio were taken from Gilmore (1984): thin disk : thick disk : spheroid as 1 : 0.02 : 0.00125. Additionally, the results obtained by Da Costa (1982) for globular clusters were used for modeling the influence of the horizontal branch in the spheroid luminosity function of the spheroid.
Fig. 2. $M_V, B-V$ diagram with mean sequences for main sequence stars and red giants of the disk population (solid lines); white dwarfs, subdwarfs and giants of the thick disk population (short-dashed lines); subdwarfs, giants and horizontal branch stars of the spheroid population (long-dashed lines).

The absolute magnitudes of stars were obtained from color-magnitude sequences of different populations shown in Fig. 2. For the disk stars, we used the data of Chiu (1980) for the lower main sequence and those of Straizys (1977) for stars earlier than G0. For the disk red giants, $M_V$ were taken from Morgan & Eggleton (1978), corrected by $\Delta M_V^{\text{red giants}}$ (see subsection 4.3). For subdwarfs and giants of the disk, the color-magnitude diagram of the globular cluster 47 Tuc (Lee 1977) was used. For extreme subdwarfs and giants of the spheroid, the sequence of the old globular cluster M92 (Sandage 1982) was used. For white dwarfs and horizontal branch stars, the data were taken from Chiu (1980).

The absolute stellar magnitudes in Eqs. (1), (2) and (4) are

$$M_{V_{\text{or}} B} = (V_{\text{or}} B) - 5 \log R + 5 - A_{V_{\text{or}} B}(R, b),$$

where $A_{V_{\text{or}} B}$ is the total interstellar extinction at a distance $R$ from the Sun. The corresponding reddening is $E_{B-V} = A_V/3$.

Three different expressions are considered to specify the interstellar extinction $A_V(R, b)$ (see subsection 4.1): the Parenago (1940), or cosecant, law

$$A_V(R, b) = a_V h_{Za} |\csc b| \cdot [1 - \exp (-R \sin b |h_{Za}^{-1}|)], \quad (5)$$
the modified Sandage (1972) law

\[
A_V(R, b) = 0.165 \left(1.192 - |\tan b|\right) \cdot |\csc b| \cdot \left[1 - \exp \left(-R |\sin b| h_Z^{-1}\right)\right],
\]

for \(|b| \leq 50^\circ\) (6),

\[
A_V(R, b) = 0, \quad |b| > 50^\circ,
\]

and

\[
A_V(R, b) = 0.
\]

Here \(a_V\) is the extinction for the unit distance and \(h_Z\) is the scale height of the extinction layer.

We assume in this work a normal distribution of the spatial velocity components (see §4.4):

\[
P_j(V_t) = \sigma_{V_{ij}}^{-1} \cdot \exp \left[-\frac{(V_{tj} - \bar{V}_{tj})^2}{2\sigma_{V_{ij}}^2}\right],
\]

where \(\bar{V}_{tj}\) is the mean value and \(\sigma_{V_{ij}}\) is the dispersion of the velocity component \(V_{tj}\) of the \(j\)-th subsystem in a given direction. There are three contributors to the velocity component \(V_{tj}\): the spatial velocity component of the \(j\)-th subsystem relative to the LSR, the solar velocity relative to the LSR and the differential galactic rotation. Here, the influence of the proper motion error on the velocity dispersion was taken into account.

4. MODELING THE OBSERVED STELLAR DISTRIBUTIONS

To model the distributions given by Eqs. (1)–(4), we assume the following parameters: the distance of the Sun from the galactic center is \(R_0 = 8.5\) kpc; the height of the Sun above the galactic plane is 20 pc; the axial ratio of the spatial distribution for the spheroid is \(k = 0.85\); the scale length of the disk subsystems is \(h_X = 4\) kpc and the scale height of the thick disk is \(h_Z = 1.3\) kpc. For main sequence stars of the thin disk, we assume the scale heights from 90 to 350 pc corresponding to increasing \(M_V\) from 2.3 to 5.4 mag; for the disk red giants, the scale heights change from 250 to 400 pc for \(M_V\) from -0.75 to +2.6 mag (Schmidt 1963, Mihalas & Binney 1981). The parameters used in Eqs. (5)–(6) are taken from Sharov (1963): \(a_V = 1.6\) mag/kpc and \(h_Z = 114\) pc. The dependence of \(M_V\) on metallicity (or on the distance from the galactic plane, \(Z\)) for the main sequence stars can be expressed by the equation:
AM metallicity = (0.6 + q) \cdot [1 - \exp(-Z/150)],

with \( q = 0 \) for \( B-V > 0.5 \); \( q = (B-V) - 0.5 \) for \( B-V \leq 0.5 \).

The velocity components of the Sun relative to the LSR are assumed as +10, +10, +7 km/s in \( X, Y, Z \) directions, respectively. The dependence of the rotation constant \( A \) on the age of subsystems is given in Kharchenko (1992). In converting the ages to absolute magnitudes, we used the results of Tinsley (1980). The values of the dispersion gradient \( \Delta \sigma_v \) along the galactic radius and vertically to the galactic plane (for the main sequence and red giants of the thin disk only) are \(-10 \text{ km/s/kpc} \) and \( 10 \cdot [1 - \exp(-Z/150 \text{ pc})] \text{ km/s} \) (Kharchenko 1993), respectively. The values of the proper motion errors were taken from Table 1.

Considering a total number of stars \( A_j(V, l, b) \) in a disk-like subsystem \( j \) as

\[
A_j(V, l, b) = \frac{4}{3} \pi R_{\text{lim}}^3 D_j(\tau, M) F_j(M_V)
\]

and setting its partial logarithmic derivative with respect to \( V \) to zero, i.e. \( \frac{\partial \log}{\partial V}(A_j) = 0 \), we obtain the relation between the apparent magnitude \( V_m \) and the absolute magnitude \( M_V \) of stars with the maximum contribution to the star counts for the subsystem \( j \)

\[
V_m = -2.614 + M_V + A_V(R, b) - 5 \log \left( \frac{\sin b}{h_Z} + \frac{\cos b \cos l}{h_X} \right). \tag{8}
\]

4.1. The interstellar extinction law

The data for 943 stars of the photoelectric subset (PES) was used to specify the expression for interstellar extinction \( A_V \) in Eqs. (5)-(7). The PES stars are located in the program fields outside the galactic plane (\( |b| \geq 25^\circ \)) and they are selected randomly relative to their colors. As the number of stars fainter than \( V = 13 \text{ mag} \) decreases considerably in the PES, we can assume that only a few spheroid stars are in the sample. The majority of the PES stars belong to the disk populations, i.e. disk main sequence stars, disk red giants and stars of the thick disk before and after their main
Stellar counts and galactic models

Fig. 3. Observed (crosses) and modeled (solid lines) numbers of stars versus color index $B-V$. Solid lines connecting open circles, triangles, squares and crosses correspond to the modeled distributions of main sequence stars, disk red giants, intermediate subdwarfs and giants, respectively. The interstellar extinction in (a), (b) and (c) is taken into account by Eqs. (5), (6) and (7), respectively.

According to (3), we computed the distribution of stars over the color index $B-V$ using Eqs. (5), (6) and (7) to take into account the interstellar extinction. The distribution of observed and modeled numbers of the PES stars over the color index $B-V$ is shown in Fig. 3. We repeated the calculations in three intervals of galactic latitudes.
Fig. 4. Observed (crosses) and modeled (lines) numbers of stars versus color index $B-V$ in three latitude intervals. Solid, dashed and dotted lines correspond to the interstellar extinction taken into account by Eqs. (5), (6) and (7), respectively.

(see Fig. 4). According to Fig. 3, Fig. 4 and the $\chi^2$-test, the modified Sandage law (6) provides the best fit to the observational data.
Fig. 5. Observed (crosses) and modeled (solid lines) numbers of stars (up to a completing magnitude) versus $B$ magnitude in 6 groups of the Kiev MEGA fields. Solid, short-dashed, long-dashed, dotted and dashed-dotted lines correspond to the modeled number of all stars, to the disk main sequence stars, disk giants, thick disk and spheroid, respectively.

4.2. Distributions of $B$ and $V$ magnitudes

For modeling the magnitude distribution, we used Eqs. (1) and (2) and the values of the galactic parameters given above. Figs. 5, 6 and upper panels of Fig. 7 show the observed and modeled star counts in the Kiev and Tautenburg fields (see Table 1). According to Figs. 5 and 6, the disk red giants make the largest contribution to the star counts within $V = 6 - 9$ mag at high galactic latitudes. The results obtained from the PES data confirm this conclusion. The thick disk and spheroid populations become most numerous at $V > 17$ mag. From Eq. (8), the dip of the luminosity function at $M_V \approx 7$ mag can be observed for the disk subsystem most clearly towards the galactic pole with stars of $V \approx 17$ mag in our sample. We modeled the observed star count data in two Tautenburg fields using three different values of the dip $a_W = 0.0, 0.3, 0.6$ dex. The best fit was obtained with the value $a_W = 0.6$ dex.
Fig. 6. Observed (crosses) and modeled (solid lines) numbers of stars versus $B$ and $V$ magnitudes in the Tautenburg MEGA fields F14 and F15. The thick lines represent the disk luminosity function modeled with a value of the dip $a_W = 0.6$ dex. The thin lines show the modeled distributions with a dip size of 0.0 and 0.3 dex. The lines mean the same as in Fig. 5.

4.3. The color index $B-V$ distribution

The data in the Tautenburg fields were used to compare the observed and modeled distributions of the color index $B-V$ (Fig. 8). The magnitude intervals were chosen in such a way that the disk red giants ($V < 13$) and the disk main sequence stars with $M_V = 7$ mag ($15 < V < 17$) were mixed as little as possible.

We computed the numbers of stars assuming three different corrections $\Delta M_V^{\text{red giants}}$, $+0.5$, $+0.25$ and $0$ mag, to the absolute magnitude of disk red giants in the color-magnitude diagram of Morgan and Eggleton (1978). According to the $\chi^2$-test and Fig. 8 (the left panels), the best fit of the observed color distribution by model (3) is obtained for $V < 13$ mag, if the correction $\Delta M_V^{\text{red giants}} = +0.5$ mag is assumed.
Fig. 7. Observed and modeled numbers of stars versus $V$ magnitude (upper panels) and color $B-V$ (lower panels) in four areas of F17. The crosses, open circles and triangles correspond to the observed number of all stars, main sequence stars and red giants, respectively. The lines mean the same as in Fig. 5.

The color distribution computed with three values of the dip in the luminosity function is demonstrated in the right panels ($15 < V < 17$) of Fig. 8. Again, the best agreement between the observed and modeled color distributions was obtained with $a_W = 0.6$ dex.

In contrast to the stars towards the galactic pole (fields F14 and F15), the stars of the same color in the field F17 show a non-regular distribution over the field. This effect may be explained by two main reasons: first, a non-regular distribution of interstellar matter towards the galactic center and second, a non-regular spatial distribution of the bluest and youngest stars in the Sagittarius-Carina spiral arm. We divided the field F17 to four areas assuming for each area its own law of interstellar extinction. The size and location of the interstellar clouds was specified by comparing the modeled and observed numbers of stars as a function of magnitude or color index (see Fig. 7). We computed a total interstellar extinction $A_V(R)$ as
Fig. 8. Observed (crosses) and modeled (solid lines) numbers of stars versus the color index $B-V$ in the F14 and F15 fields and in three $V$ magnitude intervals. In the left panel ($V < 13$ mag): thick lines present the model computed with the correction $\Delta M_V^{\text{red giants}} = +0.5$ mag to absolute magnitudes of the disk red giants given in Morgan & Eggleton (1978). Thin lines correspond to $\Delta M_V^{\text{red giants}}$ of $+0.25$ and $0.0$ mag. In the right panel ($15 \leq V < 17$): thick lines mark the disk luminosity function modeled with a dip value $\alpha_W = 0.6$ dex. Thin lines correspond to $\alpha_W$ of $0.3$ and $0.0$ dex. The lines mean the same as in Fig. 5.

\[
A_V(R) = \int_{0}^{R} a_V(r) \, dr
\]

in each area separately. The best agreement between the observed and modeled distributions of magnitudes and colors in the field F17 was obtained with the values of interstellar extinction $a_V$ given in Table 2.
Table 2. Characteristics of the areas in the field F17.

<table>
<thead>
<tr>
<th>No. of area</th>
<th>Area size [sq. degree]</th>
<th>No. of stars [star/sq. degree]</th>
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<tr>
<td>2</td>
<td>1.9214</td>
<td>4275</td>
<td>$1.018+a1$</td>
</tr>
<tr>
<td>3</td>
<td>1.7741</td>
<td>4796</td>
<td>$0.916+a1$</td>
</tr>
<tr>
<td>4</td>
<td>1.6387</td>
<td>3785</td>
<td>$1.018+a1$</td>
</tr>
</tbody>
</table>

Note: $a1 = 0.6 \cdot \exp[-(2.5 - r)/0.3]$ if $r < 2.5$ kpc, $a1 = 0.6$ if $r \geq 2.5$ kpc; $r$ is given in kpc.

4.4. Distribution of absolute proper motions

The proper motion data and Eq. (4) were used to derive the spatial velocity components in the direction of the galactic rotation ($V$), along the galactic radius ($U$) and perpendicularly to the galactic plane ($W$) and the corresponding velocity dispersions $\sigma_V$, $\sigma_U$ and $\sigma_W$. Eq. (4) includes a number of parameters which can be changed within reasonable limits to fit the observed proper motion distributions and to derive numerical values of kinematical parameters. For the disk subsystem, the spatial velocities $U, V, W$ relative to the LSR and their dispersions have been estimated more or less accurately. We used the corresponding kinematical parameters for the disk subsystem given in Mihalas & Binney (1981), Allen (1973) and Casertano et al. (1990) assuming a linear change of the numerical values within the considered absolute magnitude intervals for the main sequence stars and red giants of the thin disk (see Table 3). Since the results for the thick disk and spheroid obtained by different authors show a rather large scattering, we fitted the spatial velocity component $V$, the dispersion $\sigma_V$ and the dispersion ratio for these subsystems to obtain the best agreement between the model and observations. The results of calculations are given in Table 3. Figs. 9 and 10 show the modeled and observed distributions of proper motion components in the direction of the galactic rotation.
Table 3. The spatial velocity parameters of the disk main sequence stars (MS), disk red giants (RG), thick disk and spheroid.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MS, ( M_V &lt; )</th>
<th>MS, ( M_V &gt; )</th>
<th>RG, ( M_V &lt; )</th>
<th>RG, ( M_V &gt; )</th>
<th>Thick Disk</th>
<th>Spheroid</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{U} ), [km/s]</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \overline{V} ), [km/s]</td>
<td>+0.8</td>
<td>-14.0</td>
<td>-8.3</td>
<td>-16.7</td>
<td>-40.0</td>
<td>-200.0</td>
</tr>
<tr>
<td>( \overline{W} ), [km/s]</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \sigma_V ), [km/s]</td>
<td>14.1</td>
<td>18.4</td>
<td>16.7</td>
<td>19.2</td>
<td>50.0</td>
<td>130.0</td>
</tr>
<tr>
<td>( \sigma_U : \sigma_V : \sigma_W )</td>
<td>100:62:50</td>
<td>100:62:50</td>
<td>100:67:54</td>
<td>100:67:54</td>
<td>100:80:70</td>
<td>100:88:88</td>
</tr>
<tr>
<td>( \mathcal{A} ), [km/s/kpc]</td>
<td>20.0</td>
<td>8.1</td>
<td>12.6</td>
<td>5.8</td>
<td>8.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Fig. 9. Observed (crosses) and modeled (solid lines) numbers of stars versus the proper motion component in the direction of the galactic rotation for six groups of the Kiev fields. The lines mean the same as in Fig. 5.
Fig. 10. Same as Fig. 9 but for the Tautenburg fields F14 and F15 in three intervals of the $V$ magnitudes. The lines mean the same as in Fig. 5.

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