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THE MULTIFRACTAL STRUCTURE OF SMALL-SCALE ARTIFICIAL IONOSPHERIC TURBULENCE

F. I. Vybornov and A. V. Rakhlin

Radiophysical Research Institute (RRI), B. Pecherskaya str., 25/12a, Nizhny Novgorod, 603950, Russia; vybornov@nirfi.sci-nnov.ru

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Abstract. We present the results of investigation of a multifractal structure of the artificial ionospheric turbulence when the midlatitude ionosphere is affected by high-power radio waves. The experimental studies were performed on the basis of the SURA heating facility with the help of radio sounding of the disturbed region of ionospheric plasma by signals from the Earth's orbital satellities. In the case of vertical radio sounding of the disturbed ionosphere region, the measured multipower and generalized multifractal spectra of turbulence coincide well with similar multifractal characteristics of the ionosperic turbulence under the natural conditions. In the case of oblique sounding of the disturbance region at small angles between the line of sight to the satellite and the direction of the Earth's magnetic field, a nonuniform structure of the small-scale turbulence with a relatively narrow multipower spectrum and small variations in the generalized multifractal spectrum of the electron density was detected.

Key words: techniques: radar astronomy – Earth: ionosphere: turbulence

1. INTRODUCTION

The investigation of inhomogeneous structure of electron density in the ionosphere is of great interest for fundamental science, expanding our understanding of the processes occurring in a magnetized plasma, as well as for for the applied tasks related to the problems of transionospheric communication, navigation, radio location and radio astronomy. After the discovery in the early 70's of the power spectrum of the ionospheric turbulence, much attention has been given to the study of its spectral characteristics in different geophysical conditions, including the impact on the ionosphere by a powerful short wave radiation (Gershman et al. 1984; Fremouw et al. 1978; Solodovnikov et al. 1988). Most of ionospheric studies used standard methods of spectral analysis of fluctuating signals, suitable for statistical analysis of quasi-stationary random processes. Earlier, the same methods were used in researches of atmospheric turbulence in probing it by electromagnetic and sound waves. At the same time, these studies used the method of the structure functions of the order 2 to determine the asymptotic behavior of fluctuations of the received signal. It was determined, that the random process of wave scattering in the Earth atmosphere of an isotropic turbulence has a quasi-stationary character. The spectral index is taken as a value $p_3 \approx 11/3$ almost in all cases and is uniquely associated with a single index of structure function $\varphi_2 = 2/3$ by a simple equation $p_3 = 3 + \varphi_2$ (Tatarsky 1967; Frisch 1995).

In studies of the inhomogeneous structure of the ionospheric turbulence, the structure function method has not been used. Moreover, there there is no study in which the structure function and spectral characteristics of the random process of scattering of high-frequency radio waves in the ionospheric plasma would be determined simultaneously. As a result of investigations of spectra of the ionospheric turbulence using the standard methods of spectral analysis of the fluctuating signals, large variations of the spectral index were registered depending on the state of the ionosphere and different geophysical conditions of observations. Since in these investigations the necessary control over stationarity of the random process was not carried out, the reliability of information on the spectral characteristics of small-scale ionospheric turbulence (SIT) is questionable. In addition, within the classical radio scintillation method, that was used in these studies, a uniform distribution of small-scale fluctuations of the electron density in the ionosphere has been always assumed. However, the results of our first experimental studies of intermittency of SIT suggest that it has a non-uniform fractal distribution in space (Alimov et al. 2008a,b). The first theoretical and experimental studies of the multifractal structure of the developed SIT were made in by Alimov et al. (2007, 2008c). It has been shown that in real nonstationary conditions of scattering of high frequency radio waves in the ionospheric plasma the classical method of spectral analysis of radio signals can lead to significant errors in determining spectral characteristics of SIT. The use of multifractal analysis of satellite signals, based on the methods of multidimensional structure functions and wavelet transform, appropriated in conditions of nonstationary stochastic processes, is more correct (Frisch 1995; Zosimov & Lyamshev 1995; Pavlov & Anishchenko 2007). These methods allow to obtain reliable information about the local structure of SIT (both in natural conditions and in impact on the ionosphere by powerful radio waves) in the studies of real irregular structure of ionospheric turbulence.

2. FRACTAL CORRELATION OF RADIO SCINTILLATIONS OF A SATELLITE SIGNAL IN THE IONOSPHERIC PLASMA

The question about the development of local structures of the ionospheric turbulence is closely related to the question about the fractal properties of small-scale ionospheric irregularities, considered by Alimov et al (2007, 2008c, 2009). In this case, the problem of the developed SIT local structure is considered as the study of a multifractal structure of the amplitude fluctuations of sounding signals using the method of multidimensional structure functions.

The structure function of the amplitude fluctuations *A(t)* of the received signal with a little time delay τ can be discribed by the well-known relation (Rabinovich & Sushchik 1990):

$$\langle|\Delta A(t)|^q\rangle = \frac{1}{T}\int_0^T |A(t+\tau) - A(t)|^q dt \propto \tau^{\varphi_A(q)} \propto \tau^{\alpha_q q + 1 - D_A(\alpha_q)}, \qquad (1)$$

where T is the time interval of the amplitude signal recording, $\varphi_A(q)$ is the scaling index for the experimentally measured structure function, the qth-order the amplitude fluctuations of the received signal, and $D_A(\alpha_q)$ is the fractal dimension of amplitude fluctuations, which is determined on a set q of structural functions from the obvious parametric dependence

$$D_A(\alpha_q) = \alpha_q \cdot q + 1 - \varphi_A(q), \alpha_q = d\varphi_A(q)/dq. \quad (2)$$

Finally, the processing of experimental records of the received signals in accordance with Equation (2) gives information about the multifractal spectrum of the amplitude fluctuations $D_A(\alpha_q)$ of the received signal.

It can be recalled that at radio sounding of the ionosphere one-dimensional integral (along the line of sight to the satellite) of the electron density fluctuations is measured (Alimov et al. 2007). The local structure function $\overline{[\Delta A(\tau)]_q^2}$ for the weak amplitude fluctuations of the received signal will be proportional to the corresponding integral of structure function of the order 2 (see Alimov et al. 2007):

$$\overline{[\Delta A(\tau)]_q^2} \propto \overline{\Delta N_{lq}^2(r_x = vt)} \propto \int_0^L \int_0^L [\overline{[\Delta N(\bar{r})]_q^2} - \overline{[\Delta N(0,0,l_2 - l_1)]_q^2}] dl_1 dl_2. \quad (3)$$

Here $r = \sqrt{r_x^2 + (l_2 - l_1)^2}$ is the spatial diversity of the observation points along the direction of the difraction pattern velocity v in the projection on the Earth, L is the total thickness of the inhomogeneous layer along the line of sight to the satellite, l_1 and l_2 are the current values of the coordinate along the line of sight.

In the case of natural SIT at the middle latitudes the local structure can be characterized by a quasi-isotropic three-dimensional spectrum of electron density fluctuations in the following form (see Alimov et al. 2007):

$$\Phi_{N_q}(\overrightarrow{k}) \propto [k_\perp^2 + k_l^2]^{-p_{3q}/2} \equiv k^{-p_{3_q}}, \quad (4)$$

where $k_\perp$,k_l are transverse and longitudinal (relative to the magnetic field of the Earth) wave number of irregularities, respectively, p_{3_q} is the spectral index of the irregularities for the qth component, appropriate to the local structural function $\overline{[\Delta N(\bar{r})]_q^2}$ according to Equation (3) (see (Alimov 2007; Tatarsky 1967).

In the case of small-scale artificial ionospheric turbulence (SAIT) at mid-latitudes, caused by impact on the ionosphere of powerful radio waves, its local structure can be characterized by three-dimensional anisotropic spectrum of fluctuations of the electron density (see Alimov et al. 2007):

$$\Phi_{N_q}(\overrightarrow{k}) \propto k_\perp^{-p_{2_q}} \cdot \Phi_{N_q}(k_l), \quad (5)$$

where p_{2_q} is the index qth component of multidegree spectrum SAIT in the transverse direction of the magnetic field of the Earth.

The index of multidegree spectrum of local small-scale structure of the ionospheric turbulence at mid-latitudes in natural conditions can be easily obtained as (Alimov et al. 2009):

$$p_{3_q} = 2 + 2\alpha_q, \quad (6)$$

The index of multidegree spectrum of the local small-scale artificial ionospheric turbulence across the Earth's magnetic field can be described by the expression:

$$p_{2_q} = 1 + 2\alpha_q. \quad (7)$$

Thus, the local structure of small-scale ionospheric turbulence, described by multidegree spectrum $\Phi_{N_q}(\overrightarrow{k})$, is uniquely defined by the corresponding Hölder exponent α_q (see Equation (2)) from a multidegree spectrum of the amplitude fluctuations taken from a satellite signals after sounding of the midlatitude ionosphere. In turn, these exponents are defined in the standard multi-fractal signal processing (see (1), (2) and Alimov et al. 2007, 2008a).

Note that the turbulence value is measured in the experiment spectrum index p_A of amplitude fluctuations, does not permit to distinguish the origin (natural or artificial) of the SIT. Unlike the method of spectral analysis of fluctuating signals, multifractal processing of the received signals makes it possible to obtain information for a set of scaling indexes of different order q for relevant structural functions of the amplitude fluctuations, see Equation (1). In this case, the width Δp of the local spectrum of SIT is the difference relevant parameters p_{2_q} and p_{3_q} taken for two fixed values q ($q_{\min} \le q \le q_{\max}$). According to (6) and (7) we find that the value Δp is determined by a uniform way in the satellite radio sounding signals of SIT (natural and artificial origin) for all situations, mentioned above:

$$\Delta p = 2(\alpha_{q_{\min}} - \alpha_{q_{\max}}). \tag{8}$$

It is important to note that the width Δp of the desired local spectrum of SIT (Eq. 8) is the measured value and it does not depend on the model of the inhomogeneous ionospheric layer and the method of radio sounding. Therefore, in principle, it can be used as an indicator of the real state of the SIT study, and in particular, to determine the kind of turbulence (natural or artificial).

The unequal spatial distribution of isotropic small-scale ionospheric irregularities in general is characterized by a set of fractal dimensions that are related to the fractal dimension of the amplitude fluctuations of the received signal by the simple expression (Alimov et al. 2007, 2008d; Rabinovich & Sushchik 1990):

$$D_N(\alpha_q) = 2 + D_A(\alpha_q). \tag{9}$$

In the case of an anisotropic spectrum (5) for the local structure of small-scale artificial ionospheric turbulence, following to Alimov et al. (2007), it is easy to obtain a simple relation between the fractal dimension $D_N(\alpha_q)$ of the space occupied by the irregularities of SAIT and the fractal dimension $D_N(\alpha_q)$ of the amplitude fluctuations (compare with Alimov et al. 2007),

$$D_N(\alpha_q) = \frac{3}{2}[1 + D_A(\alpha_q)]. \tag{10}$$

For the parameters α_q, as well as for indexes of multidegree spectrum p_{2_q} and p_{3_q} (7) and (8), it follows that the measured in the experiment multifractal spectrum of amplitude fluctuations actually characterizes unequal spatial distribution of small-scale ionospheric irregularities for different turbulent structures with different indexes of multidegree spectrum.

3. FRACTAL STRUCTURE OF THE DEVELOPED ARTIFICIAL SMALL-SCALE TURBULENCE

Experiments to receive the navigation satellite signals (satellites of the system "Sail" (Russia), transmit a continuous signal at frequencies 150 and 400 MHz) in 2002–2010 years conducted in Arya and Vasilsursk, Nizhny Novgorod region mainly in quiet the geophysical conditions. Selected satellite trajectory is close to the plane of the magnetic meridian. The scheme of experiments and their technical implementation are described in detail by Alimov et al. (2006). Heating of the upper ionosphere carried out O-wave polarization at the frequency 4.6 or 5.75 MHz near the critical frequency F2 layer of the ionosphere using three transmitters of SURA (each transmitter has a power of 250 kW). Application of the regime to an inclined position of orientation diagram of antenna (12° from the vertical to the south in the plane of the magnetic meridian) provided the increase in the intensity of artificial ionospheric irregularities due to the effect of the magnetic zenith. Heating facility turned on for 15 minutes before the satellite passes over the disturbed region of the ionosphere.

For analysis of experimental data, in accordance with the above concepts, about 10 recorded signals from satellites were selected with clearly expressed chaotic structure of fast amplitude fluctuations. Length of individual fragments of the amplitude records usually was from 20 to 80 s. At fractal processing, according to (1), empirical correlations $\lg\langle|\Delta A(\tau)|^q\rangle = f(\lg\tau)$ were calculated for different values of the parameter q with time delay $\tau \approx (0.1 - -0.3)$ s.

Figure 1 (curves a and b) display the plots of the generalized multifractal spectrum $D_N(p_{2_q})$ and $D_N(p_{3_q})$ of small-scale ionospheric turbulence determined from the results of the experiment 2007-08-22 for the Vasilsursk receiver center (for intervals $T \approx 20$ s). Figure 2 shows the plots of the multifractal spectra for the experiment of 2002-08-20 for the Arya receiving point (about 100 km north of Vasilsursk). Here, the first recording interval (a) corresponds to observation of the satellite when it is to the north near the the main beam of the heating facility antenna. The second interval (b) matches a received signal at small angles between the line of sight to the satellite and the magnetic field of the Earth (5–7°) and it is almost within the main beam of the heating facility antenna south of the vertical. It should be noted that the generalized multifractal spectrum of isotropic SIT, defined by the results of the experiment on 2006-04-29 in a naturally disturbed ionosphere, is a little different from those shown in Figure 1.

4. DISCUSSION OF EXPERIMENTAL RESULTS

For the first (Figure 1, curve a) and the second (Figure 1, curve b) observation interval in the experiment of 2007-08-22 the width of the multidegree spectrum was $\Delta p_{2_q} \approx \Delta p_{3_q} \approx 0.3$ i.e. it was almost identical to the corresponding value of the width of the multifractal spectra of small-scale ionospheric turbulence in natural conditions. In this case, variations in the dimensions of the space D_N occupied by small-scale irregularities in both cases (see Fig. 1 and Fig. 2, curve a) are equal to $\Delta D_N \approx 0.6$. During the experiment on 2007-08-22 the fluctuation amplitude of the received signals was observed far beyond the field of artificial ionospheric disturbances over Vasilsursk. In the field of artificial disturbances they were superimposed on the fluctuations caused by the anisotropic turbulence and small-scale ionospheric turbulence caused by decay phenomena in artificial traveling ionospheric disturbances of electron density.

During the experiment on 2002-08-20 the background values of amplitude fluctuations of signals taken at 150 MHz were negligible. The multifractal characteristics of ionospheric turbulence near the area of maximum artificial disturbances inside the main beam of the heating facility antenna (Figure 2, curve a) were

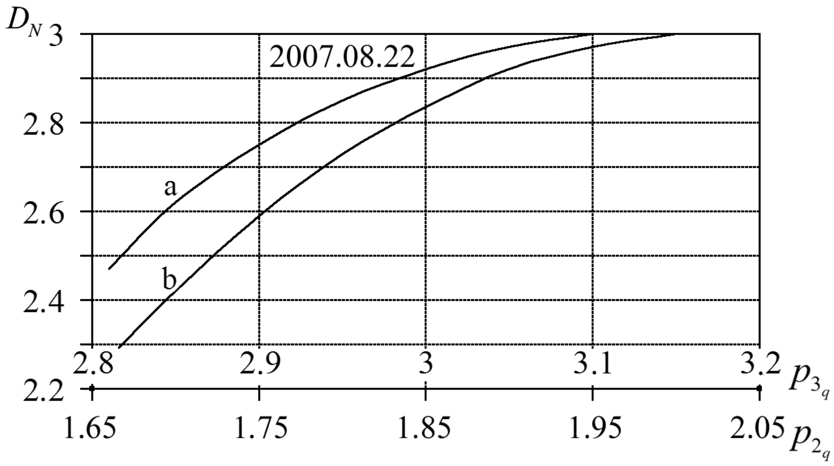

Fig. 1. Plots of the generalized multifractal spectrum $D_N(p_{2_q})$ (a) and $D_N(p_{3_q})$ (b) of small-scale ionospheric turbulence.

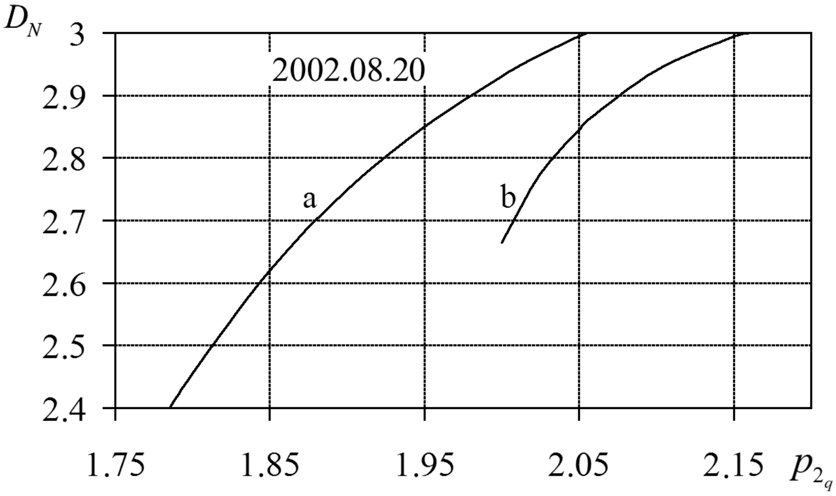

Fig. 2. Plots of the multifractal spectra of small-scale ionospheric turbulence (receiver at the Arya center).

virtually identical to the characteristics of the experiment on 2007-08-22. The multidegree spectrum width was $\Delta p_{2_q} \approx 0.3$, and the variations of the generalized multifractal spectrum of the electron density was about $\Delta D_N \approx 0.6$. In this case, the detected parameters of the convective decay irregularities in a thick layer (about 100 km) are over the area of heating, the characteristics of which are identical with the characteristics of natural irregularities.

However, the width of the multidegree spectrum and the variation of the generalized multifractal spectrum of the electron density $\Delta D_N(p_{2_q})$ in the ionospheric disturbances inside the main beam of the antenna in the experiment on 2002-08-20 were significantly lower ($\Delta p_{2_q} \approx 0.15$ and $\Delta D_N \approx 0.3$). This indicates that the strongly anisotropic small-scale artificial ionospheric turbulence in the region of maximum ionospheric disturbances inside a thin nonuniform layer of the thickness 10–20 km is an irregular structure.

5. CONCLUSIONS

The measured width Δp of the local spectrum of small-scale ionospheric turbulence does not depend on the model of the nonuniform ionospheric layer and the method of its radio sounding. It can be used as an indicator of the real state of SIT and, in particular, to determine the kind of turbulence (natural or artificial origin).

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REFERENCES

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2006, Radiophysics and Quantum Electronics, 49, 506

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2007, Radiophysics and Quantum Electronics, 50, 274

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2008a, Radiophysics and Quantum Electronics, 51, 20

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2008b, Radiophysics and Quantum Electronics, 51, 170

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2008c, Radiophysics and Quantum Electronics, 51, 259

Alimov V. A., Vybornov F. I., Myasnikov E. N., Rakhlin A. V. 2008d, Radiophysics and Quantum Electronics, 51, 874

Alimov V. A., Vybornov F. I., Rakhlin A. V. 2009, Radiophysics and Quantum Electronics, 52, 13

Gershman B. N., Erukhimov L. M., Yashin U. Y. 1984, *Wave Phenomena in the Ionosphere and Space Plasma*, Nauka Publishers, Moscow (in Russian)

Fremouw E. J., Leadabrand R. L., Livingston R. C. et al. 1978, Radio Sci., 13, 167

Frisch U. 1995, *Turbulence: the Legacy of A. N. Kolmogorov*, Cambridge University Press

Pavlov A. N., Anishchenko V. S. 2007, Physics Uspekhi (Advances in Physical Sciences), 50, 819

Rabinovich M. I., Sushchik M. M. 1990, Physics Uspekhi (Advances in Physical Sciences), 33, 1

Solodovnikov G. K. Sinelnikov V. M., Krohmalnikov E. B. 1988, *Remote Sensing of the Earth's Ionosphere Using a Beacon Satellite*, Nauka Publishers, Moscow (in Russian)

Tatarsky V. I. 1967, *Propagation of Radio Waves in the Turbulent Atmosphere*, Nauka Publishers, Moscow (in Russian)

Zosimov V. V. , Lyamshev L. M. 1995, Physics Uspekhi (Advances in Physical Sciences), 38, 347