Research Article

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Dynamical investigations of the multiple stars

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Abstract: Two multiple stars — the quadruple star \( \kappa \) Bootis (ADS 9173) and the triple star TT Tauri were investigated. The visual double star \( \kappa \) Bootis was studied on the basis of the Pulkovo 26-inch refractor observations 1982-2013. An invisible satellite of the component A was discovered due to long-term uniform series of observations. Its orbital period is \( 20 \pm 2 \) years. The known invisible satellite of the component B with near 5 years period was confirmed due to high precision CCD observations. The astrometric orbits of the both components were calculated. The orbits of inner and outer pairs of the pre-main sequence binary TT Tauri were calculated on the basis of high precision observations by the VLT and on the Keck II Telescope. This weakly hierarchical triple system is stable with probability more than 70%.

Keywords: multiple stars: individual: \( \kappa \) Bootis, T Tauri

1 Introduction

The importance of the uniform observations produced under the same conditions on the same telescope is without of doubt. Regular photographic observations and later CCD observations of visual double stars have been executing with the 26-inch Pulkovo refractor since 1960 till today (Izmailov 2010; Kiselev 2014; Izmailov 2016, a). Now, this telescope is a robotic one, so the great amount of high precision observations allows us to analyze fine effects in detail. Two astrometric orbits for both visual components of the star ADS 9173 were obtained on the basis of deviations relative to the orbital motion of the visual outer AB pair. It became possible due to great precision of recent CCD observations.

In this work, we use also homogeneous observations which were performed by the best modern technique — VLT (Koehler 2016) and Keck II (Schaeffer 2014) — for determination of the orbits of inner and outer pairs in the system T Tauri. It should be noted that the orbit of outer pair was determined on the basis of a short arc of observations by the Apparent Motion Parameters (AMP) method (see Kiselev 1980). The investigation of stability of this weakly hierarchical triple star was initiated by V.V.Orlov (1956-2016), and we dedicate this report to his memory.

2 \( \kappa \) Bootis=ADS 9173=WDS 14135+5147=HIP 69483

This visual binary star was discovered by Struve in 1822. Component A, is a variable Star of \( \delta \) Sct type, component B is spectroscopic binary with period 4.9 years Bakos (1986). Observations with 26-inch Pulkovo refractor contain 48 photographic plates 1982-2004 with 10-20 images on each plate and 53 CCD series 2003-2012 with 50 images in each one. The motion of the component B relative to the component A is very slow. In the part of the orbit under consideration, it was represented by the formulas

\[
\begin{align*}
\rho &= (13.563 \pm 0.002)'' + (0.0045 \pm 0.0002)''(t - 2000.0) \\
\theta &= (235.589 \pm 0.010)^0 + (0.0002 \pm 0.0010)^0(t - 2000.0)
\end{align*}
\]

where \( \rho \) — distance between components, \( \theta \) — positional angle. We can see that now outer pair is moving in the direction \( \rho \), because the value of \( \dot{\theta} \) is less than its error according to the formula (2).

The residuals of observations relative to the orbital motion in right ascension (dx) and declination (dy) are initial data for this work. They are shown on Figure 1. We can see that the precision of CCD observations is much better than the precision of photographic ones. The influence of the invisible satellite of the component B on the behavior of the residuals \( dx \) of CCD observations is obvious here, and it seems unclear within the noise of photographic observations (Figure 1a). Previously, we had suggested that the A component has a satellite with a long period (Kiyaeva...
the orbits elements), which are used to determine the elements of the barycenter) and A, B, F, G (the Thiele–Innes elements) and other case. The orbitalelements P, T and e were taken from the spectroscopic orbit. To determinethe astrometric orbit, we used more precise CCD observations. Initial deviations were smoothed in the phase window, which is equal to 0.245P=4.9 year to exclude the influence of other satellite of the component B. Orbital elements e and T have been obtained by looking for the better fitting observations. There are missing observations for several phases, therefore it is impossible to determine the eccentricity. We present variants corresponding to e=0.5 and e=0. The comparison of the orbits with observations is presented on Figure 2.

For the B component’s photocenter orbit, we have other case. The orbital elements P, T and e were taken from the spectroscopic orbit. To determine the astrometric orbit, we used more precise CCD observations. Initial deviations...
Table 1. The star χ Bootis. Orbits of photocenters of the components A and B.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>A</th>
<th>A</th>
<th>B</th>
<th>B (Bakos)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{\text{ph}}), mas</td>
<td>35.1 ± 1.8</td>
<td>23.9</td>
<td>21.5 ± 4.3</td>
<td>-</td>
</tr>
<tr>
<td>(a_{\text{ph}} \cdot \sin i), AU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.0</td>
</tr>
<tr>
<td>(a), AU</td>
<td>9.8</td>
<td>9.5</td>
<td>3.4</td>
<td>-</td>
</tr>
<tr>
<td>(P), yr</td>
<td>20 ± 2</td>
<td>20</td>
<td>4.904</td>
<td>4.904 ± 0.009</td>
</tr>
<tr>
<td>(e)</td>
<td>0.5 ± 0.2</td>
<td>0</td>
<td>0.53</td>
<td>0.53 ± 0.09</td>
</tr>
<tr>
<td>(i), °</td>
<td>56 ± 4</td>
<td>59</td>
<td>77 ± 6</td>
<td>-</td>
</tr>
<tr>
<td>(\omega), °</td>
<td>15 ± 32</td>
<td>-</td>
<td>49 ± 15</td>
<td>96 ± 3</td>
</tr>
<tr>
<td>(\Omega), °</td>
<td>186 ± 16</td>
<td>187</td>
<td>7 ± 12</td>
<td>-</td>
</tr>
<tr>
<td>(T_p), yr</td>
<td>1994.9 ± 1.5</td>
<td>-</td>
<td>2006.05</td>
<td>1927.58 ± 0.02</td>
</tr>
<tr>
<td>(dx_0), mas</td>
<td>8.8 ± 5.3</td>
<td>1.8</td>
<td>10.0 ± 2.1</td>
<td>-</td>
</tr>
<tr>
<td>(dy_0), mas</td>
<td>43.5 ± 2.6</td>
<td>23.9</td>
<td>2.8 ± 1.6</td>
<td>-</td>
</tr>
<tr>
<td>(y_i), km/s</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-21.5 ± 0.4</td>
</tr>
<tr>
<td>(K_i), km/s</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.2 ± 0.3</td>
</tr>
</tbody>
</table>

\(m_i\) | 4.5        | 4.5        | 6.7        | -         |
| \(SP_i\) | A8IV       | A8IV       | F8V        | -         |
| \(M_1, M_\odot\) | 1.9        | 1.9        | 1.2        | -         |
| \(\pi_{\text{Hip}}\), mas | 19.9 ± 0.6 | 19.9 ± 0.6 | 21.5 ± 1.0 | -         |
| \(M_2, M_\odot\) | ≥ 0.42     | ≥ 0.27     | ≥ 0.49     | -         |

Fig. 3. ADS 9173. Comparison of the astrometric orbit Bab with CCD observations, which are presented as deviations \(dx\)(phase) (a) and \(dy\)(phase) (b) relative to the period \(P=4.904\) years; phase=0 corresponds to \(T_0 = 2008.0\). Notations are as follows: deviations are black circles, solid line is the ephemeris of the orbit.

Table 1. The star χ Bootis. Orbits of photocenters of the components A and B.

have been previously corrected taking into account the astrometric orbit of the component A (e=0.5). The comparison of this orbit with the observations is presented on Figure 3. Series \(dx_{\text{phase}}\) and \(dy_{\text{phase}}\) are not smoothed here.

It is necessary to take into account that there are different reasons for perturbations in the motion of outer pair. It can be seen on Figure 2a, that the influence of the orbital motion of the component B is not excluded completely. The aim of this investigation is to prove that this double star has two inner satellites and to determine their preliminary orbits which can be corrected in the future.

The orbital elements are presented in Table 1. Uncertainties were obtained by the Monte-Carlo method. Here, \(a_{\text{ph}}\) is the semi-major axis of the photocenter’s orbit, \(a\) is the semi-major axis of the relative orbit. Magnitudes (m), spectral classes (SP) and masses of visible components \(M_1\) have been taken from MSC catalog (Tokovinin 1997), parallaxes \(\pi_{\text{Hip}}\) have been taken from (Van Leeuwen 2007). Then we can calculate the minimal masses of the invisible satellites \(M_2\).

The spectroscopic orbit of the component B (Bakos 1986) is presented in the last column of Table 1 and on Figure 4 for comparison. There is a difference in the value of
Table 2. The star T Tauri. The Apparent Motion Parameters.

<table>
<thead>
<tr>
<th></th>
<th>T₁ – T₂</th>
<th>ρ</th>
<th>θ</th>
<th>μ</th>
<th>ψ</th>
<th>ρc</th>
<th>σρc, mas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sa–Sb</td>
<td>2008.0–2014.0</td>
<td>126.69</td>
<td>321.93</td>
<td>10.63</td>
<td>60.0</td>
<td>64</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>2010.5</td>
<td>±0.12</td>
<td>±0.13</td>
<td>±0.12</td>
<td>±0.3</td>
<td>±2</td>
<td>5.57</td>
</tr>
<tr>
<td>N–S</td>
<td>2002.8–2014.9</td>
<td>684.11</td>
<td>187.939</td>
<td>8.45</td>
<td>289.4</td>
<td>840</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>2009.0</td>
<td>±0.17</td>
<td>±0.015</td>
<td>±0.06</td>
<td>±0.4</td>
<td>±10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3. The star T Tauri. The orbits of the inner pair Sₐ – Sₜ.

<table>
<thead>
<tr>
<th>Author</th>
<th>a</th>
<th>P</th>
<th>e</th>
<th>i</th>
<th>ω</th>
<th>Q</th>
<th>Tₚ</th>
<th>ΣM</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>88.0 ± 1.5</td>
<td>27.4 ± 0.7</td>
<td>0.55±0.02</td>
<td>25.8 ± 2.8</td>
<td>237.5 ± 5.1</td>
<td>264.7 ± 5.1</td>
<td>1996.3 ± 0.4</td>
<td>2.9 ± 0.1</td>
</tr>
<tr>
<td>Schaefer et al., 2014</td>
<td>89 ± 7</td>
<td>29 ± 4</td>
<td>0.49±0.12</td>
<td>29 ± 12</td>
<td>218 ± 14</td>
<td>287 ± 19</td>
<td>1995.8 ± 0.9</td>
<td>2.7 ± 0.1</td>
</tr>
<tr>
<td>Koehler et al., 2016</td>
<td>85 ± 6</td>
<td>27 ± 2</td>
<td>0.56±0.09</td>
<td>20 ± 10</td>
<td>228 ± 34</td>
<td>272 ± 36</td>
<td>1991.1 ± 0.8</td>
<td>2.65 ± 0.10</td>
</tr>
</tbody>
</table>

Note: ΣM of each orbit fits the parallax 6.8 mas.

Fig. 4. ADS 9173. Comparison of the astrometric orbit Bab (solid line) with the spectroscopic orbit (dash line) and radial velocity observations (black circles).

ω, but our orbit (obtained on the basis of deviations) fits observations of radial velocity rather acceptably.

3 T Tauri=WDS 04220+1932=HIP 20390

This star was discovered as a double star (T Tau N and T Tau S) in 1982 (Dyck 1982). In 1983 T Tau S was resolved into two objects (Sₐ and Sₜ), but more reliable observations are being executed since 1997 (see Koresko 2000). We have already tried to investigate the stability of this system (Zhuchkov 2010), but it was not possible to determine an outer orbit rather exact. Now we can do it due to the uni-
form series of recent observations, which were obtained by the best modern technique: VLT (2001-2015) and Keck II (2002-2014). We used published data from the articles (Schaeffer 2014 and Koehler 2016). The elements of orbits for the inner pair (Sₐ and Sₜ) were presented in both articles, but the orbit determination of the outer pair is presented only in the second one.

The orbit of the inner pair can be obtained by any method, because the arc covered by observations since 1997 is rather long. We have obtained it by the AMP method. We have used the value of parallax, which was obtained with VLBA (see Loinard 2007), it is equal to 6.8 ± 0.1 mas. The Apparent Motion Parameters — an apparent separation between components (ρ), a position angle (θ), an apparent relative motion (μ), a position angle of the direction of the relative motion (ψ) and a radius of curvature (ρc) — were calculated at the middle instant t₀ = 2010.5, using the short arc of Keck II observations 2008-2014. They are presented in Table 2. The second line gives the uncertainties, the last column contains the rms scatter in ρ and τ, in mas (σr = ρση/180). The remaining observations were used for estimation of sum of masses (Mₐ = 2.9 ± 0.1 Mₒ) and relative radial velocity (∆Vₙ = 1.5±0.3 km/s). It has been done according to the algorithm, which was described many times (Kiyaeva 1983, 2015, 2017). The orbital elements are presented in Table 3. The published orbits, which were obtained by the authors of the observations, are presented here as well. A comparison of all orbits with observations is presented on Figure 5. All orbits are in agreement with observations and fit each other.

To determine the orbit of outer pair by the AMP method, one must estimate the masses of all components. It was carried out by the astrometric method. We suggested
where the barycenter of the component $N$ coincides with its photocenter. The position of the center of masses of this triple system can be calculated for each observation

$$x_c = \frac{(x_1 - x_2)q_1 + x_2}{1 + q_3}, \quad y_c = \frac{(y_1 - y_2)q_1 + y_2}{1 + q_3} \quad (5)$$

where

$$q_1 = \frac{M_{S_a}}{M_{S_a} + M_{S_b}}, \quad q_3 = \frac{M_N}{M_{S_a} + M_{S_b}}$$

$(x_1, y_1)$ and $(x_2, y_2)$ are coordinates of the components $S_a$ and $S_b$ relative to the component $N$; $M_{S_a}$, $M_{S_b}$, $M_N$ — masses of the components.

If to combine observations for two nearby instants, then it is possible to look for values $q_1$ and $q_3$ corresponding to the best fitting the velocities of components $S_a$ and $S_b$ relative to the center of masses and their velocity relative to each other. We adopt the following limits for $q_1$ and $q_3$: 

$$0.10 \pm 0.09 \quad 293 \pm 50 \quad 137.6 \pm 9.2 \quad 1842 \pm 74$$

$$0.39 \pm 0.06 \quad 48.2 \pm 6.2 \quad 255.1 \pm 3.0 \quad 1601 \pm 140$$

### Table 4. The star T Tauri. The orbits of the outer pair $N - S$.

<table>
<thead>
<tr>
<th>$M_\Sigma$</th>
<th>$\Delta V_r$</th>
<th>$a$</th>
<th>$P$</th>
<th>$e$</th>
<th>$i$</th>
<th>$\omega$</th>
<th>$\Omega$</th>
<th>$T_\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\odot$</td>
<td>km/s</td>
<td>mas</td>
<td>yr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td>2.0 $\pm$ 1.0</td>
<td>800 $\pm$ 125</td>
<td>516.9 $\pm$ 126.1</td>
<td>0.10 $\pm$ 0.09</td>
<td>43.0 $\pm$ 3.6</td>
<td>293 $\pm$ 50</td>
<td>137.6 $\pm$ 9.2</td>
<td>1842 $\pm$ 74</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>1.6 $\pm$ 1.0</td>
<td>763 $\pm$ 91</td>
<td>526.2 $\pm$ 96.3</td>
<td>0.30 $\pm$ 0.07</td>
<td>31.3 $\pm$ 3.1</td>
<td>37.5 $\pm$ 5.3</td>
<td>263.0 $\pm$ 16.2</td>
<td>1601 $\pm$ 113</td>
</tr>
</tbody>
</table>
Fig. 6. T Tauri outer (NS) orbits. Comparing with observations: ρ(t) (a), y(t) (b), y(x) (c). The N-Sa observations are denoted by open circles, the N-Sb observations — by open squares, the photocenter observations — by crosses, the excluded observations are selected in the square; the observations of the SaSb center mass relative to the N component are denoted by black squares (KeckII) and by open circles, the N-Sa observations — by open squares, the photocenter (dash line), the circle orbit (dot line). On Figs. 6a) and 6b) all lines coincident.

$q_3: 0.5 \leq q_3 \leq 1.0, 0.5 \leq q_3 \leq 2.0$. The value of top limit of $q_3$ is much more than the expected one.

We have combined 27 pairs of uniform observations (separately for VLT and KeckII) under condition $1 \leq \Delta t \leq 2$ years and calculated the following mean values: $q_1 = 0.84 \pm 0.03$, $q_3 = 1.1 \pm 0.1$. We have already obtained $M_{S\text{a}} + M_{S\text{b}} = 2.9M_\odot$, then $M_{S\text{a}} = 2.44 \pm 0.07M_\odot$, $M_{S\text{b}} = 0.46 \pm 0.07M_\odot$, $M_N = 3.2 \pm 0.3M_\odot$, $M_{S\text{a}}/M_{S\text{b}} = 0.19 \pm 0.04$.

Loinard (2007) estimated the following limits for $M_N$: $1.7 \leq M_N \leq 2.2M_\odot$, which is essentially less than the value we have obtained, but now we adopted our result as a first approximation. Thus, the total mass of the system is equal $6.1 \pm 0.3M_\odot$. It should be noted, that Koehler (2016) used estimations of Loinard and obtained the total mass of the triple system $4.6M_\odot$. Their estimation of the mass relation is $M_{S\text{a}}/M_{S\text{b}} = 0.25 \pm 0.03$, but they had obtained other value of this relation (0.18) earlier, in 2008 (see Koehler 2008), which fits our result.

All observations of outer pair relative to the component N (according to WDS catalog (Mason 2016) and considered articles) are presented on Figure 6 (a and b). The apparent motion parameters have been calculated on the basis of the uniform Keck II series of observations (2002-2014) and the last observation VLT (2015), which fits this series. They are presented in Table 4. The observations were previously corrected to the center of masses of the star $S$.

Now it is necessary to estimate only the relative radial velocity to determine the AMP-orbit. We used the archive observations of the photocenter of the star S from WDS catalog since 1990. The radio observations with VLA (since 1982) were excluded, because they have essential systematic errors in comparison with other observations. They are selected in the square on Figure 6. The search value of $ΔV_r$ fits minimum of a function $S(ΔV_r)$ (see Figure 7), it is equal to $ΔV_r = 2 \pm 1$kms$^{-1}$. As a result, we obtained two AMP-orbits, which are presented in Table 4. The accuracy of the radial velocity is poor, therefore we have calculated also a circle orbit. The determination of radial velocity isn’t demanded in this case. This orbit is presented in Table 4 and on Figure 6c too. We can calculate also a dynamic parallax $\pi_d$ on the basis of the circle orbit, using apparent motion parameters and total mass of the system. If the total mass is equal to $6.1M_\odot$, then $\pi_d = 6.4''$. It fits the observational value, because an error of this approximation (ε=0) cannot be more than 1.2 mas (20%) for ε=0.4 (see Kiyaeva 1982). However, there is also the orbital solution with ε=0.1, therefore we can follow conversely and ignore the error of the approximation. If $\pi_d = 6.8''$, then the total mass is equal to $5.1M_\odot$. We have repeated the calculation with a new value of the total mass. The orbital elements
are presented also in Table 4. As we can see, they do not change essentially.

Our orbital solution differs markedly from the orbit presented in the article Koehler (2016). The authors obtained the family of orbits, and according the criterion $\chi^2$, the orbital period of the best solution is equal to 4000 years.

The decay probability of this triple system was calculated by numerical simulations (see Orlov 2005; Zhuchkov 2005), by the criteria of Aarseth (Aarseth 2003) and of Valtonen et al. (Valtonen 2008). For $e=0.1$ the decay probability is less than 2%; for $e=0.39$ maximal values of the decay probabilities are following: 15% (simulations), 26% (Aarseth), 29% (Valtonen). A difference between decay probabilities obtained by different methods is usual. All criteria present only an approximation of the stability area. In that case numerical modelling gives the most definite result. To determine decay probability we used Monte-Carlo method and simulations at 1 MYr into the past and into the future. Thus, this system is stable with probability more than 70%.

4 Conclusion

The aim of this article is to show the possibilities of the methods of double and multiple stars investigations which were developed at the Pulkovo observatory in recent decades.

The increased capabilities of the Pulkovo 26-inch refractor are shown on the example of the visual double star $\kappa$ Bootis. The accuracy of automated CCD observations is much better than the accuracy of photographic ones. It has allowed us to discover the the invisible satellite of the component A with the 20 years period and to confirm the invisible satellite of the component B with the 4.9 years period.

On the example of the famous star T Tauri, we have shown that the combination of new technical capabilities of modern telescopes with the AMP method allows to obtain the reliable orbits on the basis of short arc of the uniform high precision observations. We have determined the masses of the components of this triple star by the astronomical method. The new orbit of outer pair is obtained in this work. It is proved the stability of this triple system.

Acknowledgment: The authors are thankful to I.S. Izmailov for the organization and maintenance of automatic CCD observations at the 26-inch refractor. We are thankful to all observers of the VLT and Keck telescopes for possibility to use results of their work.

In this paper we use CDS and WDS data. We wish also to thank the editors of these catalogs.

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References


