Research Article

Lidia A. Egorova* and Valery V. Lokhin

Modeling of energy release at the final stage of the meteoroid movement

https://doi.org/10.1515/astro-2018-0035
Received Feb 06, 2018; accepted Aug 16, 2018

Abstract: The paper continues to build upon the author’s previous research on fireballs fragmentation. A model of the sudden explosive destruction of the cosmic body at the height of the maximum flash is used. After the fragmentation, the kinetic energy of the moving particles of a meteoroid passes into the thermal energy of the gas volume in which their motion takes place. The temperature of a gas cloud calculated analytically using energy conservation law and equations of physical theory of meteors. The mass distribution of fragments was taken from the literature. The high temperature of the gas in a cloud allows us to talk about the phenomenon of a “thermal explosion”.

Keywords: meteoroid fragmentation; fireball motion in atmosphere; physical theory of meteors

1 Introduction

Simulation of meteoroid interaction with the Earth atmosphere continues to be in a high interest since the Chelyabinsk event in 2013. The Chelyabinsk phenomena shows that even a body 10 m radius can cause catastrophic events. It occurred in a densely populated area so the numerous observations was made as mentioned by Popova et al. (2013). These data permit scientists to verify their models.

The phenomena of a similar size take place quite often and at the same time they are practically impossible to detect outside the atmosphere (see Popova & Nemchinov 2008; Shuvalov et al. 2013). Therefore, the study of this class of meteoroid motion and destruction are of particular interest to researchers.

Ground and space observations of meteoric bodies record their brightness along the trajectory. The results are usually represented as a graph - the luminosity curve. For meteoroids larger than 10 cm a sharp increase of emission is often recorded. This increase often interpreted as body fragmentation or even more as terminal explosion. The theory of terminal explosion based on the proposal that the maximum of deceleration should be achieved at the brightest point was proposed by Kruchynenko & Warmbein (2001).

Recent researches contain simulation of the entry and destruction of meteoric bodies in the atmosphere to evaluate the released energy. Shuvalov et al. (2013) introduced the concept of “air giant bolide (GB) or air “meteor explosion” (when the products of a completely disintegrated and evaporated meteoroid are decelerated in the atmosphere and do not reach the Earth’s surface but the shock wave and thermal radiation produce noticeable destruction and fires)” for the Chelyabinsk event. Fragmented body came under full vaporization according to model with air jet flow formation. Numerical simulations was in a good agreements with observation data.

The above works are very useful for practice, but have not clarified the source of released thermal energy after crushing. Thus we proposed a model of transferring the kinetic energy into thermal energy.

1.1 Catastrophic Fragmentation of the Meteoroid

There are plenty of models for meteoroid fragmentation. The review can be found in Popova (2004); Popova & Nemchinov (2008). We concern with a class of phenomena when the energy of a fragmented meteoroid reaches the Earth surface. We assumed that the body breaks up into separate fragments of different sizes moving independently. The explosive nature of the observed effects suggests that the aerodynamic drag is several times greater than the strength of the body or fragments of the pre-
destroyed body. The high velocity of the meteoroid and the exponential growth of the air density as the altitude decreases make this proposal reasonable. Under those conditions we assume the distribution of fragments in size similar to the distribution of fragments of a body suddenly destroyed by an explosion. There are a number of papers describing that distributions of fragments. For instance see Fujiwara (1986) and Wohletz & Brown (1995). We take into account the distribution from Nemchinov et al. (1999)

\[ \frac{dN_m}{dm} = C m^{1/2}, \quad k = 1.2. \quad (1) \]

Here \( m \) is a mass of fragment and \( N_m \) is a number of fragments having mass \( m \). \( C \) and \( k \) are constants.

In Egorova & Lokhin (2016) for this type of fragmentation we calculated the light curve analytically from the luminosity relation and the equations of the physical theory of meteors. The obtained analytical solution for luminosity along the trajectory has a close fit with the Marshall Islands fireball light curve. This fragmentation model is used in present paper.

1.2 Solution of Equations of the Physical Theory of Meteors

To find the parameters it is necessary to solve the equations of the physical theory of meteors for particle

\[ M \frac{dV}{dt} = -\frac{1}{2} S C_D \rho V^2 \quad (2) \]
\[ Q \frac{dm}{dr} = -\frac{1}{2} S C_H \rho V^3 \quad (3) \]

where \( V, m, S, Q \) – velocity, mass, midsection area of particle and enthalpy of mass loss due to aerodynamic forces and heat transfer, \( C_D \) and \( C_H \) are the drag and heat transfer coefficients (constants), \( \rho \) is atmosphere density.

Let us divide equation (3) by (2) and transfer terms to body velocity before the fragmentation \( V_0 \) after fragmentation. Let us introduce the initial relative velocity \( V_0 \) (8) and relative velocity \( V = \frac{V}{V_0} \). Then we integrate kinetic energy of particle over all radii and subtract from the initial kinetic energy

\[ E = M_0 \frac{V_0^2}{2} \]
\[ - M_0 V_0^2 \int_0^1 \left[ N(\tau_0) \frac{d}{d\tau_0} \left( \frac{\bar{V}(\tau_0, z)}{2} \right) \right] d\tau_0 \]
\[ = M_0 V_0^2 \frac{1}{2} \left\{ 1 - \frac{2}{3} \left( \tau_0^{-9/5} - 1 \right) \frac{d}{d\tau_0} \left( \frac{3}{8} \frac{2^{4/5} + \frac{5}{2} \frac{z}{\delta}}{2^{4/5}} \right) \right\} \]

Denoting \( B = 2A \left( 1 + kV_0 \right) \frac{z}{R} \)
we calculate

\[ \frac{6}{5} \int_0^1 \tau_0^{-1/5} e^{-B \tau_0} d\tau_0 = e^{-B} - B^{6/5} \int_B^\infty y^{-6/5} e^{-y} dy \]

The solution of equation (5) could be found in Stulov et al. (1995) and is rather complicated. So the simplifications are following relying on the nature of its constituent constants.

We propose \( C_H = 0.01 \) as we consider rather small particles of the meteoroid, \( C_D = 1 \) for the spherical particles at altitude less than 50 km, \( Q = 1.6747 \cdot 10^4 \text{ m}^2/\text{c}^2 \) as suggested in Stulov et al. (1995). Thus we estimate the parameter of ablation \( \kappa = \frac{C_d}{C_H} \ll 1 \).

1.3 Temperature of a Gas Cloud

We assumed the destruction of the meteoroid into many fragments. The part of kinetic energy of the moving particles passes into the thermal energy of the gas volume in which their motions take place. Let us estimate the loss of the kinetic energy \( E \) of the whole ensemble of particles with a size distribution (1), which has converted to thermal energy during deceleration (during the traversed path \( z \) ) after fragmentation. Let us introduce the initial relative radius \( \tau_0 = r_0/R \), mass \( m(\tau_0) = m_0/M \) and relative velocity \( V(\tau_0, z) = V/V_0 \). Then we integrate kinetic energy of particle over all radii and subtract from the initial kinetic energy

\[ E = M_0 \frac{V_0^2}{2} \]
\[ - M_0 V_0^2 \int_0^1 \left[ N(\tau_0) \frac{d}{d\tau_0} \left( \frac{\bar{V}(\tau_0, z)}{2} \right) \right] d\tau_0 \]
\[ = M_0 V_0^2 \frac{1}{2} \left\{ 1 - \frac{2}{3} \left( \tau_0^{-9/5} - 1 \right) \frac{d}{d\tau_0} \left( \frac{3}{8} \frac{2^{4/5} + \frac{5}{2} \frac{z}{\delta}}{2^{4/5}} \right) \right\} \]

Denoting \( B = 2A \left( 1 + kV_0 \right) \frac{z}{R} \)
we calculate

\[ \frac{6}{5} \int_0^1 \tau_0^{-1/5} e^{-B \tau_0} d\tau_0 = e^{-B} - B^{6/5} \int_B^\infty y^{-6/5} e^{-y} dy \]
\[ e^{-B} - 5Be^{-B} + 5B^{6/5} \left( \Gamma \left( \frac{4}{5} \right) - \int_0^B y^{-1/5} e^{-y} \, dy \right) . \]

Here \( \Gamma \) is gamma function. Estimations shows that \( B \ll 1 \), then the integrand can be expanded in a series and then it is easy to calculate

\[ \int_0^B y^{-1/5} e^{-y} \, dy = \frac{5}{4} B^{4/5} + O \left( B^{4/5} \right) , \tag{9} \]

where

\[ E = M_0 V_0^2 \left\{ 1 - e^{-B} + (B)^{5/6} \int_B^\infty y^{-1/5} e^{-y} \, dy \right\} \tag{10} \]

\[ \frac{E}{M_0 V_0^2} = 1 - 6B + 5B^{6/5} \Gamma \left( \frac{4}{5} \right) + O(B) \tag{11} \]

The volume of the gas heated by the particles in first approximation is a cylinder of length \( L \) (for a typical case of a fireball flight we put it equal to 1 km) and radius \( R^* \). The radius of the cylinder is equal to the speed of propagation of the perturbations (we set it equal to the speed of sound) multiplied by the time \( t \). Having differentiated \( E \) and divided by heat capacity \( C_V \), volume of a gas cloud and gas density we obtain the temperature of a gas cloud.

\[ T = \frac{dE}{dV} \frac{dz}{dV} = \frac{4\pi R^3 \delta V_0^2}{C_V \rho \pi R^2} \frac{dB}{dz} \left[ 1 - B^{1/5} \Gamma \left( \frac{4}{5} \right) \right] \tag{12} \]

Substituting \( B \), we obtain

\[ T = \frac{3 R^2 V_0^2 C_D}{C_V R^2} \left( 1 + \kappa V_0^2 \right) \tag{13} \]

\[ \cdot \left[ 1 - \left( \frac{3 \rho C_D}{4 \delta} \right)^{1/5} \left( 1 + \kappa V_0^2 \right)^{1/5} \Gamma \left( \frac{4}{5} \right) \left( \frac{z}{R} \right)^{1/5} \right] \]

For the case when ablation is neglected \( \kappa = 0 \). These calculations was done in Egorova & Lokhin (2017).

2 Results

Figure 1 show the temperature in a gas cloud calculated for body of radius (a) 9 m and (b) 6 m entering with velocity of 20 km/s and 15 km/s. Solid line referred to model taking to account ablation and dashed ones to model without ablation of particles. The ablation of particles in the cloud raises the temperature. The decrease in particle size leads to more intensive deceleration. The lower altitude of fragmentation gives a more rapid fall of the temperature in a cloud. A direct relationship between cloud temperature and body size is evident. The larger the size of the parent body the higher the temperature in the cloud.

3 Conclusion

Assuming a known distribution of meteoroid fragments by mass after the fragmentation of the explosive mechanism, a change in the temperature in the cloud of gas is obtained.
The high temperature of the gas in a cloud allows us to talk about the phenomenon of a "thermal explosion" suggested by Shuvalov et al. (2013). The calculation of temperature of a gas cloud is a first step for estimation of energy release by fragmenting meteoroid into the Earth atmosphere. Further investigations will be needed to find the pressure of a cloud and to solve the gas-dynamic problem of disturbances propagation in atmosphere.

Acknowledgment: The authors are cordially thankful to Dr. Irina Brykina for the fruitful discussion about the article. This work was done under the state contract and was partially supported by RFBR according to the research project No 18-01-00740.

References