SIMULATING THE MECHANICS OF MYOCARDIAL TISSUE USING STRONGLY SCALABLE PARALLEL ALGORITHMS

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Abstract: Due to preferential orientations of fibers, such as collagen or myocytes, the modeling of the mechanics of myocardial tissue leads to anisotropic and highly nonlinear material models. For micro-anatomically realistic geometries the computational effort to handle these sophisticated models is very challenging and demands the usage of strongly scalable parallel algorithms. In this context we mention two possible approaches and show numerical examples.

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Modeling

For the numerical simulation of the elastic behavior of biological tissue we consider the stationary equilibrium equations

\[- \text{Div} \ F S(u, x) = 0, \quad \text{for } x \in \Omega \]

(1)

to find a deformation \(u\) in a domain \(\Omega\), in our case the myocardium. In (1) we introduce the deformation gradient \(F = I + \text{Grad}(u)\) and the second Piola-Kirchhoff stress tensor \(S\). In addition, we incorporate Dirichlet and Neumann boundary conditions to describe fixed displacements and surface tractions on \(\Gamma = \partial \Omega\). The total stress tensor \(S\) is obtained by an addition of a passive and an active stress part

\[ S = S_p + S_a, \]

(2)

which will be described in the following.

Passive Stress

For the derivation of a constitutive equation for the passive stress tensor \(S_p\), we introduce the strain-energy function \(\Psi(\mathbf{C})\). Using this, we can state the constitutive equation

\[ S_p = 2 \frac{\partial \Psi(\mathbf{C})}{\partial \mathbf{C}}, \]

(3)

where \(\mathbf{C} = F^\top F\) is the right Cauchy-Green tensor. The specific form of the strain-energy function varies from material to material, but in the case of myocardial tissue we use the common structure

\[ \Psi = \Psi_{\text{vol}} + \Psi_{\text{iso}} + \Psi_{\text{aniso}}. \]

(4)

Here, the volumetric contribution \(\Psi_{\text{vol}}\) is used to handle the nearly incompressibility condition which is typical for most biological tissues.

Active Stress

The active stress component is generated by the electrical activation in the myocardial tissue. In this context we make use of the bidomain equations which describe the spread of electrical activation in the heart, e.g., see [2], to compute the transmembrane voltage \(V_m\). A scalar-valued active stress term \(S_a\) is then retrieved by

\[ S_a = h(V_m, \eta, u), \]

(6)

where the function \(h\) and the state variables \(\eta\) depend on the respective cell model, see CellML (www.cellml.org).
Hence, the active contribution to the second Piola-Kirchhoff stress tensor is obtained by
\[ S_a = S_a I_f^{-1}(f_f \otimes f_f), \]  
(7)
where \( f_f \) is the main orientation of the myocyte network and \( I_f \) was already introduced in (5). For more details concerning the modeling of cardiac materials see [1, 3].

**Numerical Methods and Results**

In order to obtain a numerical solution of Eqs. 1-7 we use variational and finite element techniques. To solve the non-linear system we perform a linearization to apply Newton’s method:
\[ K'(u^k) \Delta u = F(u^k) - K(u^k), \quad u^{k+1} = u^k + \Delta u. \]  
(8)

In this equation \( K'(u^k) \) is the tangent stiffness matrix generated by using finite element techniques, \( u^k \) is the displacement vector at Newton step \( k \) and \( F(u^k) \) captures the loads acting on the boundary of \( \Omega \). \( \Delta u \) is the update to be calculated from the linearized system (8). For a fine grid, resulting from detailed geometries, this linearized system of equations involves a very large number of degrees of freedom, which can not be handled by direct solving methods or single core algorithms in a reasonable time.

Hence, we have to apply parallel solving methods such as the finite element tearing and interconnecting (FETI) method, see [4], or algebraic multigrid (AMG) methods, see [5], which both share the same principle: a decomposition of the domain \( \Omega \) into subdomains (Fig. 3).

The strong scalability of the implementation up to 256 cores is illustrated by means of Fig. 2 on the example of the FETI method. We obtain similar results using the AMG approach.

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**Bibliography**


