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The onset of thermal convection in couple-stress fluid in hydromagnetics saturating a porous medium

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Abstract. In this paper, the effect of magnetic field on thermal convection in couple-stress fluid saturating a porous medium is considered. By applying linear stability theory and the normal mode analysis method, a mathematical theorem is derived which states that the viscoelastic thermal convection at marginal state, cannot manifest as stationary convection if the thermal Rayleigh number R, the medium permeability parameter P_l the couple-stress parameter F and the Chandrasekher number Q, satisfy the inequality

$$R \leq \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right),$$

the result clearly establishes the stabilizing character of couple-stress parameter and magnetic field whereas destabilizing character of medium permeability.

Key words: couple-stress fluid, magnetic field, porous medium, thermal convection.

Nomenclature

F – couple-Stress parameter,
d – depth of fluid layer (m),
P_l – dimensionless medium permeability,
v – filter velocity (m/s),
g – gravitational acceleration (m/s^2),
$\mathbf{g}$ – gravitational acceleration vector (m/s^2),
p – pressure (N/m^2),
T – temperature (K),
d – thickness of fluid layer (m),
t – time coordinate (s),
P_r – thermal Prandtl number,
$\boldsymbol{H}$ – magnetic field vector (G),
Q_r – magnetic Prandtl number,
h – perturbation in magnetic field,
l, m – wave numbers in x and y directions,
k – wave number of disturbance (1/m).

Greek symbols

β – adverse temperature gradient (K/m),
μ_c – couple-stress viscosity (kg/m/s),
η – electrical resistivity,
ρ – fluid density (kg/m^3),
μ – fluid viscosity (kg/m/s),
ν – kinematic viscosity (m^2/s),
μ_e – magnetic permeability (H/m),
ε – medium porosity,
δ – perturbation in respective physical quantity,
θ – perturbation in temperature,
η – electrical resistivity,
κ – thermal diffusivity (m^2/s),
α – thermal coefficient of expansion (1/K).

1. Introduction

The problem of thermal convection in porous media has attracted considerable interest during the last few decades, because it has various applications in geophysics, food processing, soil sciences, ground water hydrology and nuclear reactors etc. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3].

In all the above studies, the fluid is considered to be Newtonian. Although the problem of thermal convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. With the growing importance of non-Newtonian fluids with magnetic field in modern technology and industries, the investigations on such fluids are desirable. One such type of fluid is couple-stress fluid. The non-Newtonian behaviour of blood is mainly due to the suspension of red blood cells in the plasma. When neutrally buoyant corpuscles are contained in a fluid and there exists a velocity gradient due to shearing stress, corpuscles have rotatory motion. Furthermore, it is observed that corpuscles have spin an-

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gular momentum, in addition to orbital angular mo-mentum. As a result, the symmetry of stress tensor is lost in the fluid motion that is subjected to spin angular momentum. The fluid that has neutrally buoyant corpuscles, when observed macroscopically, exhibits non-Newtonian behaviour, and its constitutive equation is expressed by Stokes [4]. Couple-stress fluid theory developed by Stokes [4] is one among the polar fluid theories which considers couple stresses in addition to the classical Cauchy stress. It is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of couple stresses and body couples in the fluid medium. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid.

According to the theory of Stokes [4], couple-stresses are found to appear in noticeable magnitude in fluids. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [5] modeled synovial fluid as couple-stress fluid in human joints. Sharma and Thakur [6] have studied the couple-stress fluid heated from below in hydromagnetics. The investigation in porous media has been started with the simple Darcy model and gradually it was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [7], Ingham and Pop [8] and Nield and Bejan [9]. Sharma and Rana [10] have studied thermal instability of a incompressible Walters' (model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium whereas stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids in the presence of uniform horizontal magnetic field in porous medium studied by Rana et al. [11]. Recently, Kumar [12] has studied stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium whereas Rana and Sharma [13] studied the hydromagnetic thermosolutal instability of compressible Walters' (model B') rotating fluid permeated with suspended particles in a porous medium and found that magnetic field completely stabilizes the system.

Keeping in mind the importance in various applications aforementioned, our main aim in the present paper is to study the effect of magnetic field on thermal convection in couple-stress elastico-viscous fluid in a porous medium.

2. Mathematical model and perturbation equations

Here, we consider an infinite, horizontal, incompressible couple-stress viscoelastic fluid of the depth d, bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity and permeability k_1, which is acted upon by gravity $\mathbf{g}(0, 0, -g)$ and the uniform vertical magnetic field $H(0, 0, H)$ as shown below in the schematic sketch of a physical situation. This layer is heated from below in such a way that the steady adverse temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is maintained (Fig. 1). The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

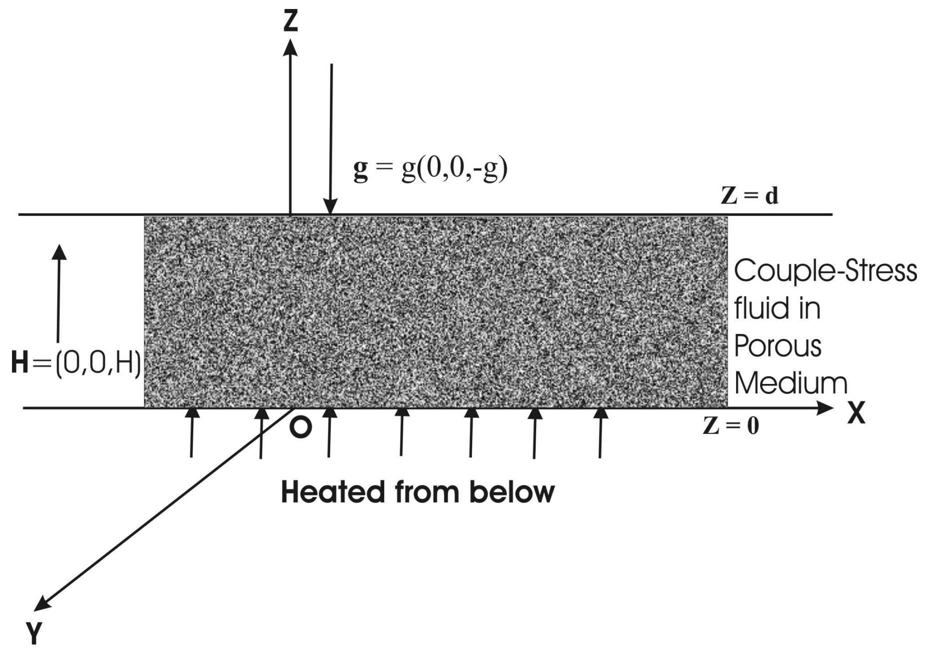

Fig. 1. Schematic sketch of physical situation

Let $v(u, v, w)$, ρ, ν, μ_c, μ_e, p, ε, T, α and $\boldsymbol{H}(0, 0, H)$ denote respectively, the velocity, density, kinematic viscosity, couple-stress viscosity, magnetic permeability, pressure, medium porosity, temperature, thermal coefficient of expansion, magnetic field...

The equations expressing the conservation of momentum, mass, temperature and Maxwell's equations for couple-stress fluid in a porous medium (Chandrasekhar [1], Sharma and Thakur [6], Kumar [12]) are

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\varepsilon}\left(\mathbf{v}.\nabla\right)\mathbf{v}\right]=-\frac{1}{\rho_0}\nabla p+\mathbf{g}\left(1+\frac{\delta\rho}{\rho_0}\right) -\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\mathbf{v}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times H\right)\times H, \tag{1}$$

$$\nabla.\mathbf{v}=0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+\left(\mathbf{v}.\nabla\right)T=\kappa\nabla^2 T, \tag{3}$$

$$\nabla.\mathbf{H}=0, \tag{4}$$

$$\varepsilon\frac{\partial \mathbf{H}}{\partial t}=\nabla\times\left(\mathbf{v}\times\mathbf{H}\right)+\varepsilon\eta\nabla^2\mathbf{H}, \tag{5}$$

where $E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$, which is constant, κ is the thermal diffusivity, η is the electrical resistivity, ρ_s, c_s; ρ_0, c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively.

The equation of state is

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \tag{6}$$

where the suffix zero refers to values at the reference level $z = 0$.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$\mathbf{v}=(0,0,0), \qquad T=-\beta z+T_0, \qquad \rho=\rho_0\left(1+\alpha\beta z\right). \tag{7}$$

is an exact solution to the governing equations.

Let $\mathbf{v}(u, v, w)$, $\boldsymbol{h}(h_x, h_y, h_z)\theta$, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $\mathbf{v}(0, 0, 0)$, magnetic field $\boldsymbol{H}(0, 0, H)$, temperature T, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ in temperature is given by

$$\delta\rho=-\alpha\rho_0\theta.. \tag{8}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{v}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\mathbf{g}\alpha\theta-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\mathbf{v} +\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\mathbf{h}\right)\times\mathbf{H}, \tag{9}$$

$$\nabla.\mathbf{v}=0, \tag{10}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \tag{11}$$

$$\nabla.\mathbf{h}=0, \tag{12}$$

$$\varepsilon\frac{\partial\mathbf{h}}{\partial t}=\nabla\times\left(\mathbf{v}\times\mathbf{H}\right)+\varepsilon\eta\nabla^2\mathbf{h}. \tag{13}$$

3. Normal mode analysis

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,\theta,\gamma]=[W(z),\Theta(z),K(z)]\exp\left(ilx+imy+nt\right), \tag{14}$$

where l and m are the wave numbers in the x and y directions, $k=\left(l^2+m^2\right)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (14) in Eqs. (9), (11) and (13) the following is obtained

$$\frac{n}{\varepsilon}\left(\frac{d^2}{dz^2}-k^2\right)W=-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\left(\frac{d^2}{dz^2}-k^2\right)W -gk^2\alpha\theta+\frac{\mu_e H}{4\pi\rho_0}\frac{d}{dz}\left(\frac{d^2}{dz^2}-k^2\right)K, \tag{15}$$

$$\varepsilon nK=H\frac{dW}{dz}+\varepsilon\eta\left(\frac{d^2}{dz^2}-k^2\right)K, \tag{16}$$

$$E\frac{\partial\Theta}{\partial t}=\beta W+\kappa\left(\frac{d^2}{dz^2}-k^2\right)\Theta. \tag{17}$$

Equations (15) and (17) in non-dimensional form, become

$$\left[\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W =-\frac{g\alpha a^2 d^2\Theta}{\nu}+\frac{\mu_e H d}{4\pi\nu\rho_0}\left(D^2-a^2\right)DK, \tag{18}$$

$$\left(D^2-a^2-Q_r\sigma\right)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW, \tag{19}$$

$$\left(D^2-a^2-EP_r\sigma\right)\Theta=-\frac{\beta d^2}{\kappa'}W, \tag{20}$$

where we have put $a = kd$, $\sigma=\frac{nd^2}{\nu}$ and $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $P_r=\frac{\nu}{\kappa}$, is the thermal Prandtl number, $Q_r=\frac{\nu}{\eta}$, is the magnetic Prandtl number, $F=\frac{\mu_c}{\mu d^2}$, is the couple-stress parameter and $D'=d\frac{d}{dz}=dD$ and dropping 'dash' for convenience.

Substituting $W = W'$, $K=\left(\frac{Hd}{\varepsilon\eta}\right)K'$ and $\Theta=\frac{\beta d^2}{\kappa}\Theta'$ in Eqs. (18)–(20) and dropping 'dash' for convenience, we obtain

$$\left[\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W =-Ra^2\Theta+\frac{Q}{\varepsilon}\left(D^2-a^2\right)DK, \tag{21}$$

$$\left(D^2-a^2-Q_r\sigma\right)K=-DW, \tag{22}$$

$$\left(D^2-a^2-EP_r\sigma\right)\Theta=-W, \tag{23}$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$, is the thermal Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\nu\rho_0\eta}$, is the Chandrasekhar number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, [1])

$$W = D^2W = \Theta = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad 1 \tag{24}$$

and the components of $\boldsymbol{h}$ are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$$DK = 0, \tag{25}$$

on the boundaries.

Then, we prove the following theorem:

Theorem. If $R > 0$, $F > 0$, $Q > 0$ and $\sigma = 0$, then the necessary condition for the existence of non-trivial solution (W, K, Θ) of Eqs. (21)–(23) together with the boundary conditions (24) and (25) is that

$$R \leq \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right).$$

Proof. If the instability sets in stationary convection and 'principle of exchange of stability' is valid, the neutral or marginal state will be characterized by $\sigma = 0$. Thus the relevant governing Eqs. (21)–(23) reduces to

$$\left[\frac{1 - F\left(D^2 - a^2\right)}{P_l}\right]\left(D^2 - a^2\right)W = -Ra^2\Theta + \frac{Q}{\varepsilon}\left(D^2 - a^2\right)DK, \tag{26}$$

$$\left(D^2 - a^2\right)K = -DW, \tag{27}$$

$$\left(D^2 - a^2\right)\Theta = -W, \tag{28}$$

together with the boundary conditions (24) and (25).

Multiplying Eq. (26) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\begin{aligned}\frac{1}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)W dz - \frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz \\ = -Ra^2\int_0^1 W^*\Theta dz + \frac{Q}{\varepsilon}\int_0^1 W^*\left(D^2 - a^2\right)DK dz.\end{aligned} \tag{29}$$

Taking complex conjugate on both sides of Eq. (28), we get

$$\left(D^2 - a^2\right)\Theta^* = -W^*. \tag{30}$$

Therefore,

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2 - a^2\right)\Theta^* dz. \tag{31}$$

Now, taking complex conjugate on both sides of Eq. (27), we get

$$\left(D^2 - a^2\right)K^* = -DW^*. \tag{32}$$

Therefore,

$$\int_0^1 W^*\left(D^2 - a^2\right)DK dz = -\int_0^1 DW^*\left(D^2 - a^2\right)K dz. \tag{33}$$

Using Eqs. (31) and (33) in the right hand side of Eq. (29), we obtain

$$\begin{aligned}\frac{1}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)W dz - \frac{F}{P_l}\int_0^1 W^*\left(D^2 - a^2\right)^2 W dz \\ = Ra^2\int_0^1 \Theta^*\left(D^2 - a^2\right)\Theta dz - \frac{Q}{\varepsilon}\int_0^1 DW^*\left(D^2 - a^2\right)K dz.\end{aligned} \tag{34}$$

Integrating term by term on both sides of Eq. (34) for an appropriate number of times by making use of boundary conditions (24) and (25), we obtain

$$\begin{aligned}\frac{1}{P_l}\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz + \frac{F}{P_l}\int_0^1 \left(\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz \\ = \frac{Ra^2}{B}\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz + \frac{Q}{\varepsilon}\int_0^1 \left(|DW|^2\right)dz.\end{aligned} \tag{35}$$

Since W, K and θ satisfy $W(0) = 0 = W(1)$, $\theta(0) = 0 = \theta(1)$, $K(0) = 0 = K(1)$, we have by Rayleigh-Ritz inequalities

$$\int_0^1 |DW|^2 dz \geq \pi^2\int_0^1 |W|^2 dz, \tag{36}$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2\int_0^1 |\Theta|^2 dz \tag{37}$$

and

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^4 \int_0^1 |W|^2 dz. \tag{38}$$

Further, multiplying Eq. (28) by θ^* (the complex conjugate of θ), integrating by parts each term of resulting equation on the right hand side for an appropriate boundary condition, namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\begin{aligned} &\int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2\right) dz \\ &= \text{Real part of } \left(\int_0^1 \Theta^* W dz\right) \\ &\leq \left|\int_0^1 \Theta^* W dz\right|, \\ &\leq \int_0^1 |\Theta^* W|\, dz, \\ &\leq \int_0^1 |\Theta^*|\,|W| dz, \\ &\leq \int_0^1 |\Theta|\,|W| dz, \\ &\leq \left(\int_0^1 |\Theta|^2 dz\right)^{1/2} \left(\int_0^1 |W|^2 dz\right)^{1/2} \\ &\text{(by using Cauchy-Schwartz inequality).} \end{aligned} \tag{39}$$

Thus, inequalities (39) can be written as

$$\left(\pi^2 + a^2\right) \left(\int_0^1 |\Theta|^2 dz\right)^{1/2} \leq \left(\int_0^1 |W|^2 dz\right)^{1/2}. \tag{40}$$

Combining inequalities (36) and (37), we obtain

$$\int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2\right) dz \leq \frac{B}{\pi^2 + a^2} \int_0^1 |W|^2 dz. \tag{41}$$

Thus, if $R > 0$, $F > 0$, $Q > 0$, using the inequalities (36), (38) and (41), Eq. (35) becomes

$$\left[\frac{1}{P_l}\left(\pi^2 + a^2\right) + \frac{F}{P_l}\left(\pi^2 + a^2\right)^2 - \frac{Ra^2}{(\pi^2 + a^2)} + \frac{Q}{\varepsilon}\pi^2\right] \cdot \int_0^1 |W|^2 dz < 0. \tag{42}$$

Therefore, we must have

$$R > \frac{\left(\pi^2 + a^2\right)^2}{P_l a^2}\left[1 + F\left(\pi^2 + a^2\right) + \frac{Q P_l \pi^2}{\varepsilon\left(\pi^2 + a^2\right)}\right] \cdot \int_0^1 |W|^2 dz < 0.$$

Since the minimum value of $\dfrac{\pi^2\left(\pi^2 + a^2\right)}{a^2}$ is $4\pi^4$ at $a^2 = \pi^2 > 0$, hence, we necessarily have

$$R > \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right), \tag{43}$$

which completes the proof of the theorem.

From physical point of view, the above theorem states that the onset of instability at marginal state in a couple-stress fluid heated from below in hydromagnetics saturating a porous medium cannot manifest as stationary convection, if the thermal Rayleigh number R, the couple-stress parameter F, medium permeability and the Chandrasekhar number Q, satisfy the inequality

$$R \leq \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right). \tag{44}$$

4. Conclusions

The effect of magnetic field on thermal convection in couple-stress fluid in a porous medium has been investigated. From the above theorem, the main conclusions are as follows:

(i) The necessary condition for the onset of instability as stationary convection for couple-stress elastico-viscous fluid is

$$R > \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right).$$

(ii) The sufficient condition for non-existence of stationary convection at a marginal state is

$$R \leq \frac{4\pi^2}{P_l}\left(1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon}\right).$$

(iii) In the inequality (43), the thermal Rayleigh number $R > 0$, is directly proportional to the couple-stress parameter F. Thus, couple-stress parameter has stabilizing effect on the system which is identical with the results derived by Sharma and Thakur [6] and Kumar [12].

(iv) In the inequality (43), the thermal Rayleigh number $R > 0$, is directly proportional to the Chandrasekhar number Q, which mathematically established the stabilizing effect of magnetic field on the system as derived by Sharma and Thakur [6], Kumar [12] and Rana and Sharma [13].

(v) The medium permeability has a destabilizing effect on the system as can be seen from inequality (43), which is an agreement with the earlier work of Sharma and Thakur [6], Rana et al. [11], Kumar [12] and Rana and Sharma [13].

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REFERENCES

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability,* Dover Publication, New York, 1981.
[2] E.R. Lapwood, "Convection of a fluid in porous medium", *Proc. Camb. Phil. Soc.* 44, 508–519 (1948).
[3] R.A. Wooding, "Rayleigh instability of a thermal boundary layer in flow through a porous medium", *J. Fluid Mech*. 9, 183–192 (1960).
[4] V.K. Stokes, "Couple-stress in fluids", *Phys. Fluids* 9, 1709–1715 (1966).
[5] E. Walicki and A. Walicka, "Inertial effect in the squeeze film of couple-stress fluids in biological bearings", *Int. J. Appl. Mech. Engg.* 4, 363–373 (1999).
[6] R.C. Sharma and K.D. Thakur, "Couple-stress fluid heated from below in hydromagnetics", *Czech. J. Phys*. 50, 753–758 (2000).
[7] D. Ingham and L. Pop, *Transport Phenomena in Porous Media*, Elsevier, New York, 1981.
[8] D.A. Nield and A. Bejan, *Convection in Porous Medium*, Springer, New York, 2006.
[9] K. Vafai, and H.A. Hadim, *Hand Book of Porous Media*, M. Decker, New York, 2000.
[10] V. Sharma and G.C. Rana, "Thermal instability of a Walters' (model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium", *J. Non-Equilib. Thermodyn.* 26, 31–40 (2001).
[11] G.C. Rana, V. Sharma, and S. Kumar, "Stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids in the presence of Uniform horizontal magnetic field in porous medium", *J. Appl. Math. and Fluid Mech.* 2, 41–47 (2011).
[12] V. Kumar, "Stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium", *Appl. And Appl. Math*. 6, 510–521 (2011).
[13] G.C. Rana and V. Sharma, "Hydromagnetic thermosolutal instability of compressible Walters' (model) rotating fluid permeated with suspended particles in porous medium", *Int. J. Multiphysics* 5, 325–338 (2011).