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DATA SCATTERING IN STRENGTH MEASUREMENT OF STEELS AND GLASS/EPOXY COMPOSITE

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Abstract. *The strength of materials is a complex function which involve two main components, material nature and the presence of defects. Usually glasses exhibit a fragile behavior due to a numerous flaws and the effect is a large range of data scattering in tensile strength measurement. The Weibull probability density function was applied to describe the scatter of experimental data in tensile test, which emphasize a difference between variance in case of tensile strength of three stainless steel grades and glass epoxy composite. The main goal is mathematical modeling of those distributions and finding of equations which predict the probability of failure for a sample subjected to a specific stress.*

Keywords: *Weibull distribution function, tensile strength data scattering, glass/epoxy composite, analysis of variance*

1. INTRODUCTION

Prediction of failure probability under a specific stress level could be an interesting tool in analysis of material behavior. The variability of a material characteristic it can be described by using Weibull distribution function [1] which cover a large scale of application, such as: fatigue performance [2, 3, 4], scatter of fracture toughness [5], probabilistic characterization in different static tests up to failure (four-point bending [6] or tensile test [7]).

In this paper, in order to obtain a relationship between stress level and failure probability was used Weibull distribution. For comparison, analysis of experimental data scattering in tensile test was performed also by using normal distribution.

The standard Weibull distribution function, which is shown in equation (1) has three parameters.

$$f(x;\beta,\lambda,\delta)=\frac{\beta}{\delta}\cdot\left(\frac{x-\lambda}{\delta}\right)^{\beta-1}\cdot e^{-\left(\frac{x-\lambda}{\delta}\right)^{\beta}} \tag{1}$$

where
β - is the shape parameter,
δ- is the scale parameter
λ is the location parameter of the distribution.

Each parameter has a specific significance as function of analyzed application. In case of tensile strength analysis, the current variable x is σ – ultimate tensile strength, λ- location parameter is the level of stress below which no fracture occurs, δ - scale parameter is usually the average of entire data set and shape parameter β (Weibull modulus) is a measure of experimental data scattering.

If the general function (1) is customized for distribution of fracture strength, the failure probability of a specimen subjected to stress level σ is given by:

$$F(\sigma;\beta,\sigma_{lim},\sigma_0)=1-e^{-\left(\frac{\sigma-\sigma_{lim}}{\sigma_0}\right)^{\beta}} \tag{2}$$

Usually, for brittle materials is difficult to estimate a minimum value of tensile strength σ_{lim} under the material does not break certainly and in this case, the location parameter will be set to zero.

The same presumption can be also done for other type of materials in case of lack of safety data.
The strength of materials is a complex function which involve two main components, material nature and the presence of defects.

Usually ceramics and plastic materials exhibit a fragile behavior due to a numerous flaws and the effect is a large range of data scattering in tensile strength measurement.

The number of defects is dependent on sample size (large samples could have more defects as smaller ones, short glass fibers have higher strength then longer ones, etc.).

For this reason, in case of large size fragile materials samples there is a higher scattering of experimental data in tensile strength measurement than in case of smaller samples [6, 8, 9]. This phenomena (sample volume contribution in data scattering) is shown also in case of high isotropic materials such as stainless steel (Fig.1).

Specimen size is taken into account by changing the exponent of function which describes the failure probability. If is considered the breaking probability of a specimen of the volume V_0 as described by expression (2), the probability of fracture for a specimen of the same material and the volume V, under the same stress level σ is given by the equation (3) [9]:

$$F(\sigma; V) = 1 - e^{-\left(\frac{V_0}{V} \cdot \frac{\sigma}{\sigma_0}\right)^{\beta}} \qquad (3)$$

From here, it can be formulated a practical conclusion, for specimens of material with the same resistance, more voluminous specimens show a higher probability of failure. In a similar manner, in case of the same probability of failure, specimens which exhibit higher volume have a lower tensile strength.

2. MATERIALS

Analysis of experimental data scattering, was performed on four type of materials, three stainless steel grades (noted S1- grade 430 annealed (12 samples), S2- grade 316L annealed (20 samples) and S3- grade 301HT cold rolled and untreated (26 samples)) and a glass/epoxy composite (5 samples). The steel samples were cut out from head and tail of coils with 0.1 mm thickness. In order to emphasize the effect of specimens volume on the tensile strength, from S2 grade, were tested another 8 samples with 0.3 mm thickness provided from coils in the same state of heat treatment.

Composite plate has been made by four unidirectional prepreg plies HexPly M10 (50% volume fraction – E glass fibers) with stacking sequence [0/90/90/0] and 5 samples (noted GS_0) were cut out along fibers direction from external plies.
All tensile tests, up to failure, were performed according to DIN EN ISO 6892-1/ ISO 10113/ ISO 10275 by using a 30kN Zwick Roell machine.
A brief analysis of correlation between sample volume and tensile strength is shown in Fig.1.

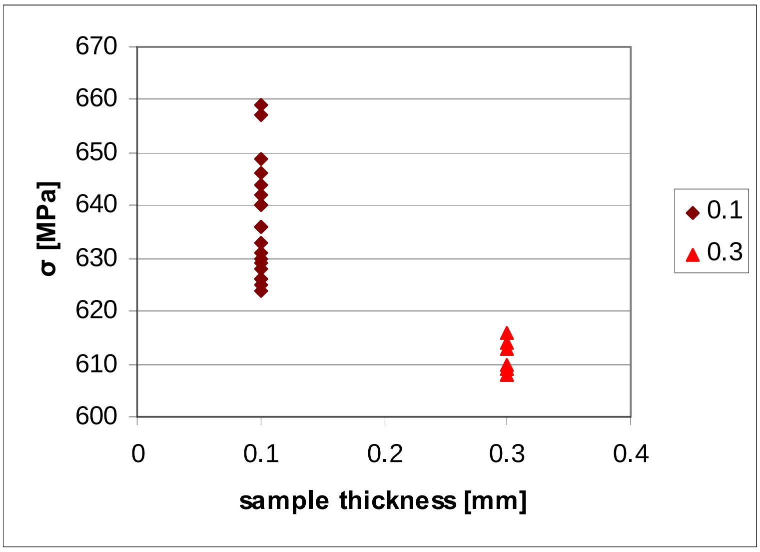

Figure 1. Ultimate tensile strength of samples with 0.1 and 0.3 thickness from grade S2

Even the difference is only 4% that it is not effect of imprecision measurement (normal scattering of experimental data due to measurement procedure or testing machine). It can be seen that even the maximum value of stress for thicker specimens is below the minimum value of thinner ones.

It should be specify clear that all the samples from grade S2 were annealed in same condition after plastic deformation. In explaining the differences, might be possible the influence of intense cold rolled hardening effect which is higher in case of thinner sheet.

The effects of cold rolling on thinner specimens may not be completely removed by annealing applied later. However in specimens with larger volume is higher the probability of defects occurrence which can reduce tensile strength. Experiments confirm the observations of researchers who have studied the influence of specimen volume, even in relatively homogeneous and isotropic material such as stainless steel.

3. WORK PROCEDURE

3.1 Estimation of Weibull parameters

In theory, for a total number of samples n<100, ranking the tensile strength test data according to the order from small to large, the cumulative failure probability is given by the Bernard formula [10]:

$$F(\sigma_i) = \frac{i - 0.3}{n + 0.4} \qquad (4)$$

Starting from relation (2) and setting the parameter σ_{lim}=0 (for brittle material there is not a limit of stress under the structure is safe - case of glass, ceramics and for steels we can make the same assumption in the absence of safety data) the Weibull distribution function becomes a two parameter function [11]:

$$F(\sigma_i;\sigma_0,\beta) = 1 - e^{-\left(\frac{\sigma_i}{\sigma_0}\right)^{\beta}} \qquad (5)$$

If in the formula of Weibull distribution function is applied the double logarithm on both sides that is transformed into a linear relation such as equation (8).

$$1 - F(\sigma_i) = e^{-\left(\frac{\sigma_i}{\sigma_0}\right)^{\beta}} \qquad (6)$$

$$\ln(1 - F(\sigma_i)) = -\left(\frac{\sigma_i}{\sigma_0}\right)^{\beta} \qquad (7)$$

$$y = \ln\left(\ln\left(\frac{1}{1 - F(\sigma_i)}\right)\right) = \beta\ln(\sigma_i) - \beta\ln(\sigma_0) = \beta \cdot x + c \qquad (8)$$

In order to estimate the parameters of function (β- slope of the curve and a measure of data scattering and the scale parameter σ_0) the calculus was performed according to procedure shown in Table 1, which is specific for those n=12 samples from S1 steel. For all tested materials has used the same procedure.

Table 1. Weibull parameters estimation (stainless steel grade S1)

Sample no. (i)	Ultimate tensile strength σ_i [MPa]	Probability of fracture $F(\sigma_i) = \frac{i - 0.3}{n + 0.4}$	$y = \ln\left(\ln\left(\frac{1}{1 - F(\sigma_i)}\right)\right)$	$x = \ln(\sigma_i)$
1	490	0.0565	-2.8455	6.1944
2	491	0.1371	-1.9142	6.1964
-	-	-	-	-
11	518	0.8629	0.6867	6.2500
12	523	0.9435	1.0558	6.2596

In Figure 1 is shown plot of the function in reciprocal double log scale on one axis (y) and a log scale on the other for three grade of steel and a stratified bidirectional glass/epoxy composite.

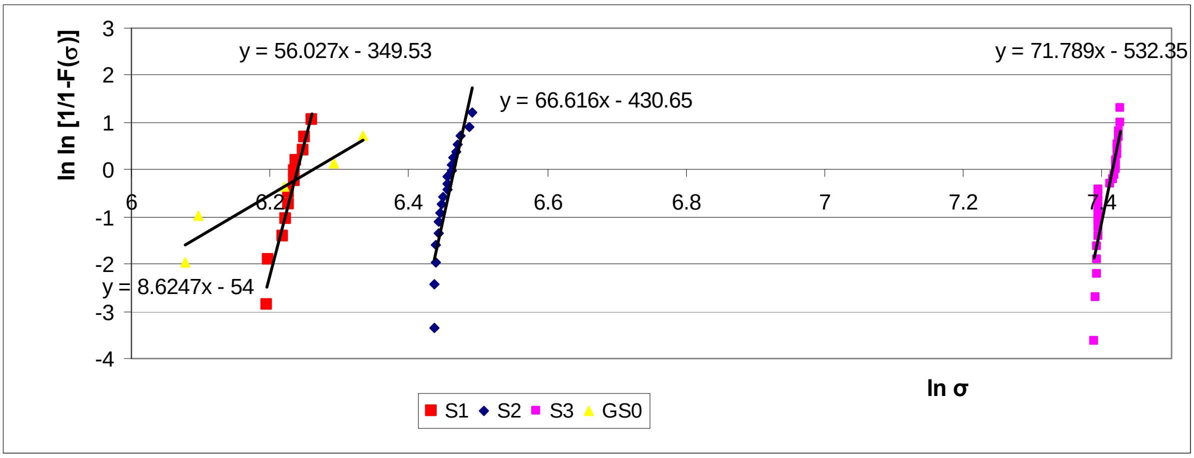

Figure 2. Weibull probability curves

Regression of experimental data has been performed and if trend line is a straight line, the material strength distribution can be described by using the two parameter Weibull distribution.

In order to compare the obtained parameter, were calculated according to normal (Gaussian) distribution the average, variance and standard deviation, results which are shown in Table 2.

It is obvious that the Weibull modulus β in case of composite (8.6247) is much lower than moduli for steels. The difference in experimental data scattering are due to composite material anisotropy, lack of homogeneity (two distinct phases: the glass reinforcement and plastic matrix material), imperfect interfaces, presence of air voids during the resin cure.

The probability, in case of composite, to propagate and accumulate flaws that may cause breaking at stress values different from the average value is much higher than in homogeneous materials such as steels. The value of standard deviation associated with β_{GS0} =8.6247, is σ_{GS0}= 50.8Mpa (10.2% from mean value) in comparison with values calculated for steels (e.g. β_{S1}=56.027, σ_{S1}= 9.56MPa, 1.88%).

The differences between the average calculated by estimating the Weibull function parameters and corresponding classical average for normal distribution are smaller than the standard deviation.

Table 2. Parameters of Weibull distribution and normal distribution

Parameter/Material		Symbol	S1	S2	S3	GS_0
Weibull distribution	shape parameter	β	56.027	66.616	71.789	8.6247
	constant term	c	394.53	430.65	532.35	54
	scale parameter	$\sigma_0 = e^{\frac{c}{\beta}}$	507.31	642.04	1661.51	523.78
Normal distribution	mean	μ	507.41	636.85	1648.96	497.4
	variance	σ^2	91.57	100.22	535.34	2580.64
	standard deviation	σ	9.56	10.01	23.13	50.8

3.2 Hypothesis distribution inspection

Making a first approximation we have the Weibull cumulative distribution function described in relations (9-12):

$$F_{S1}(\sigma;\sigma_0,\beta) = 1 - e^{-\left(\frac{\sigma}{507.31}\right)^{56.027}} \quad (9)$$

$$F_{S2}(\sigma;\sigma_0,\beta) = 1 - e^{-\left(\frac{\sigma}{642.04}\right)^{66.616}} \quad (10)$$

$$F_{S3}(\sigma;\sigma_0,\beta) = 1 - e^{-\left(\frac{\sigma}{1661.51}\right)^{71.789}} \quad (11)$$

$$F_{GS0}(\sigma;\sigma_0,\beta) = 1 - e^{-\left(\frac{\sigma}{523.78}\right)^{8.6247}} \quad (12)$$

In order to validate the form of function described above we must perform hypothesis distribution inspection. Generally are used nonparametric methods for comparing two set of distribution, in our case Bernard approximation and Weibull distributions (9-12).

One method of analysis is Kolmogorov-Smirnov test (the best option mainly due to its sensitivity with small sample size) in which maximum difference between the empirical distribution function of the sample (Weibull) and the cumulative distribution function of the reference distribution (Bernard) have to be lower than KS constant (for a given number of samples and a given significance level).

$$\max\left|F_W(\sigma_i) - F_B(\sigma_i)\right| < D_{(n,\alpha)} \quad (13)$$

The significance level is usually set to 0.05 which means a 95% confidence interval.

The procedure is illustrated for S1 stainless steel in Table 3.

Table 3. Kolmogorov –Smirnov test

Sample no. (i)	Ultimate tensile strength σ_i [MPa]	Probability of fracture $F_B(\sigma_i) = \frac{i-0.3}{n+0.4}$	Probability of fracture $F_W(\sigma_i) = 1 - e^{-\left(\frac{\sigma}{507.31}\right)^{56.027}}$	$D_i = \left|F_W(\sigma_i) - F_B(\sigma_i)\right|$
1	490	0.0565	0.133105	0.076653
2	491	0.1371	0.147959	0.010863
-	-	-	-	-
11	518	0.8629	0.959785	0.096881
12	523	0.9435	0.995932	0.052383
			Max (D_i)	0.207643
			Kolmogorov-Smirnov statistic $D_{(12,0.05)}$	0.375 [12]

4. RESULTS AND DISCUTION

The test results (cumulative distribution function CDF) are also shown in Fig.3. The error bars, added to Weibull distribution function values, have fixed value equal with Kolmogorov - Smirnov statistic.

It can be seen that all the values provided by normal cumulative distribution function (according to Bernard's approximation) are inside the bars, which emphasize that relation (13) is respected.

In this condition, the tested distribution is accepted according to the KS method and isreasonable the asumption that, the tensile strength of S1 steel it can be described by using two parameter Weibull distribution.

In Table 4 are shown results of KS test applied for all steels and composite.

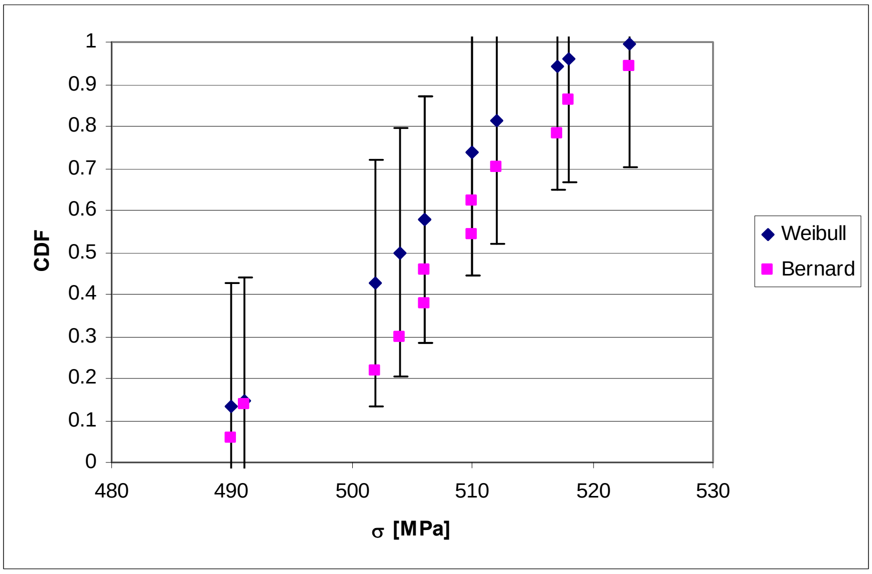

Figure 3 Hypothesis distribution inspection of empirical cumulative distribution function

Table 4.The results of Kolmogorov – Smirnov test

Material/Symbol	No. of samples	$\max\lvert F_W(\sigma_i) - F_B(\sigma_i)\rvert$	KS test (significance level $\alpha = 0.05$) [12]
Steel S1	12	0.207	$D_{(12,0.05)}$=0.375
Steel S2	20	0.104	$D_{(20,0.05)}$=0.294
Steel S3	26	0.266	$D_{(26,0.05)}$=0.259
Composite GS_0	5	0.134	$D_{(5,0.05)}$=0.563

As can be seen in Table 4, except S3 grade, we may consider that the distribution of experimental data in case of tensile strength measurement could be estimated by a two parameters Weibull distribution.

The hypothesis distribution inspection of functions shown in relations (9), (10), (12) reveals that, for each case, the Kolmogorov – Smirnov test value is lower than maximum difference between Weibull and Bernard cumulative distribution functions. For specimens of grade S3 (with cumulative distribution function shown in equation (11)) should be performed an extra analysis of that apparently different distributions from 'normal' distribution, because Kolmogorov – Smirnov test is not verified.

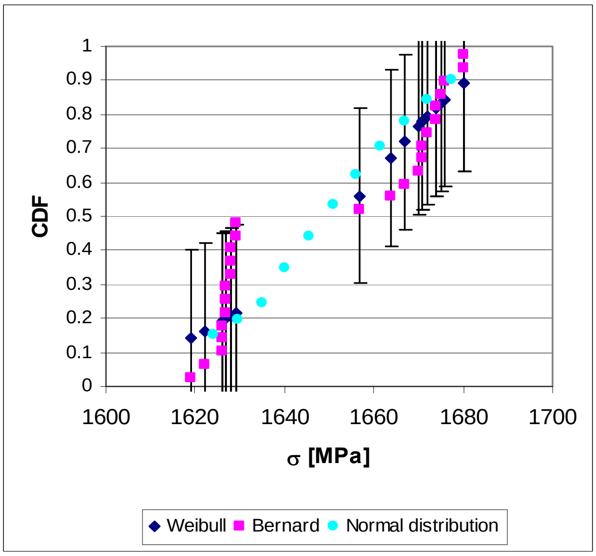

Figure 4. Weibull, real and normal cumulative distributions

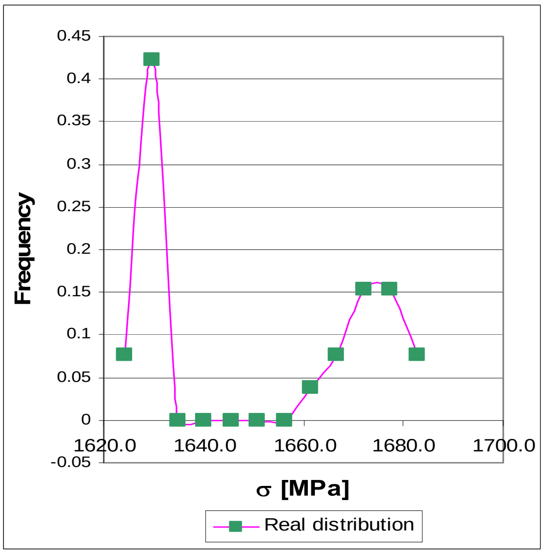

Figure 5. Real distribution

First of all, the experimental data were divided in intervals and have been calculated the frequencies for each interval according to procedure described in [13].
As shown in Fig.4, the Weibull distribution is close to normal one, but the values of cumulated frequency calculated with Bernard relation seems to be far away for some stress values situated at the beginning of the stress range. It can be seen that the real distribution in case of S3 grade steel is a bimodal distribution, shown in Fig.5.

In case of S3 grade the “abnormal” distribution has a simple origin.
This is due to sample prelevation place (these are cut out from head and tail of coils). We can consider that there is a continuous variation of tensile strength along the coil length but in our tests the values around average are missing.

The data should be a statistically random sample of the population, but in case of real cumulative distribution function curve the value 0.5 which is usualy specific for average of data set, is specific for the second frequency interval (the total number of samples is 26).

We can consider that the Weibull distribution function could describe the tensile strength distribution even in case of S3 grade (ewuation has valability in case of an entire coil), but Kolmogorov - Smirnov test in our particular situation does not verify hypothesis distribution because the particularly values for specimens tested are just a part of a statistically random sample of entire population (specimens are characteristic just for coil extremities).

5. CONCLUSIONS

Tensile strength distribution (the scatter of experimental data) could be described by using two parameter Weibull function.

It is obvious that the Weibull modulus β in case of composite (8.6248) is much lower than moduli for steels (56, 66.6, respective 71,7).

A large β is associated with a narrow interval for experimental data scattering. The standard deviation in case of glass/epoxy composite is 50.8, in comparison with those in case of steels (9.56, 10.01 and 23.13).

The difference in experimental data scattering are due to composite material anisotropy, a lack in homogeneity (two distinct phases: the glass reinforcement and plastic matrix material), imperfect interfaces, presence of air voids during the resin cure. Fiber strenght and orientation could have also an statistical effect in composite and that will be the subject of future work.

In case of samples from steel plate, the probability of defects presence increases with thickness of plate.

The comparison between predicted value and experimental data reveals that Weibull distribution provide an accurate instrument for analysis of data variability, despite fact that for S3 grade KS test reveal an nonconformity.

Because the samples are cut out from head and tail of coils, non-normal distributions (bimodal distribution) is possible.

That is just the effect of limitation in chosing of sampling site. Between head and tail of a coil is a continuous series of tensile strength values and is reasonable to believe that the mean for entire coil is specific for samples situated in the middle of the coil.

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