Calculation and 3D analyses of ERR in the band crack front contained in a rectangular plate made of multilayered material

Abstract: An investigation into the values of the Energy Release Rate (ERR) at the band crack's front in the rectangular plate made of multilayered composite material is carried out for the opening mode. The corresponding boundary-value problem is modelled by using three-dimensional linear theory and solved numerically by using 3D FEM (Three Dimensional Finite Element Method). The main purpose of the current investigation is to study the influence of mechanical and geometrical parameters on the Energy Release Rate (ERR) at this crack front. The numerical results related to the ERR, and the effect of the mechanical and other problem parameters on the ERR are presented and discussed.

Keywords: Energy Release Rate (ERR); band crack; 3D FEM (Three Dimensional Finite Element Method); rectangular plate.

1 Introduction

Since material defects such as cracks have a significant role in the processes of fractures, fracture mechanics investigates the mechanical behavior of fracture parameters (Stress Intensity Factors (SIF) or Energy Release Rates (ERR)) from various aspects such as load, material property and crack length. From recent research into this area, it can be seen that many of them have focused on determining the effects of anisotropy and other problem parameters on the SIF or ERR within the framework of the two-dimensional (2D) problem formulation [1-12]. Up to now, there have not been any investigations regarding the corresponding 3D macro-crack problems for anisotropic bodies.

In connection with this, the purpose of this current paper is to investigate the effects of anisotropic properties of the plate material and other problem parameters on the ERR in the macro-band crack contained in an anisotropic plate.

2 Problem Formulation and Solution Method

Let us consider a thick rectangular plate (Figure 1) occupying the domain \( \Omega = \{0 \leq x_1 \leq \ell_1, 0 \leq x_2 \leq h, 0 \leq x_3 \leq \ell_3 \} \) in the Cartesian Coordinate system \( \Omega x_1 x_2 x_3 \), which contains a band crack located in the region \( \Omega' = \{ (\ell_1 / 2 - \ell_0 / 2) \leq x_1 \leq (\ell_1 / 2 + \ell_0 / 2), x_2 = h, 0 \leq x_3 \leq \ell_3 \} \). Assume that on the crack's edge planes, the opening uniformly distributed normal forces with intensity \( p \) act.
The field equations and boundary conditions for the case under consideration are:
\[
\frac{\partial \sigma_{ij}}{\partial x_i} = 0, \quad \sigma_{ii} = A_{ij} \varepsilon_{ji}, \quad A_{ij} = A_{ji}, \quad \sigma_{ij} = 2\mu_{ij}, \quad \text{at } i \neq j, \quad \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),
\]
where
\[
\varepsilon_{ij} = \frac{\sigma_{ij}}{E_{ij}}, \quad \text{and} \quad E_{ij} = E_{ji}.
\]

To solve the foregoing problem (1), we employ the 3D FEM and for this purpose we introduce the following functional:
\[
\Pi = \frac{1}{2} \iint \sigma_{ij} \varepsilon_{ij} d\Omega + \iiint f_i p u_i d\Omega,
\]
where
\[
\Pi = \iint \sigma_{ij} \varepsilon_{ij} d\Omega + \iiint f_i p u_i d\Omega.
\]

The numerical results are obtained by using exact solution for an infinite plate, for the approximate series as detailed in the handbook [15]. In this case, based on mechanical considerations, with increasing of the ratio \( \ell_3 / \ell_1 \), the effective anisotropic material constants \( A_{ij} \) are determined according to the expressions given in [14].

The numerical investigations are performed for the case where \( \eta_2 = 1 - \eta_1 = 0.5 \), \( \nu_2 = \nu_1 = 0.3 \) and \( h / \ell = 0.20 \). Since the plate is symmetric with respect to the planes \( x_1 = \ell / 2 \) and \( x_3 = \ell / 2 \), then FEM solutions are obtained in a quarter part of the domain. This domain was divided into 30, 12, and 30 brick elements along the \( Ox_1 \times x_2 \times x_3 \) axes.

Before obtaining numerical results, the PC programs composed and used by the authors, were tested with the known results given in [15].

Table 1 shows the values of \( K_{f} / K_{I} \) and \( K_{S} / K_{I} \) (where \( K_{I} = \int p_{u} d\Omega \)) for various values of the ratios \( \ell_0 / \ell_1 \), \( \ell_0 / h \) and \( \ell_3 / \ell_1 \). Note that in Table 1, the values of \( K_{f} / K_{I} \) and \( K_{S} / K_{I} \) are SIF of Mode I; \( K_{f} / K_{I} \) is relevant to the plane strain state; and \( K_{S} / K_{I} \) is calculated by using exact solution for an infinite plate, for the approximate series as detailed in the handbook [15]. In this case, based on mechanical considerations, with increasing of the ratio \( \ell_3 / \ell_1 \), the effective anisotropic material constants \( A_{ij} \) must approach the corresponding values obtained in [15] for the plane strain state. This prediction is confirmed by the data given in Table 1. Moreover, our results agree with the mechanical consideration according to which the values of \( K_{f} / K_{I} \) must approach 1 with increasing of the ratios \( \ell_3 / \ell_1 \) and decreasing of the ratios \( \ell_0 / \ell_1 \) and \( \ell_0 / h \). This comparison shows good agreement and confirms the validity of the algorithm and programs used in the present investigations.

Now, we analyze the numerical results which illustrate the influence of the problem parameters on the values of the ERR. Table 2 shows the influence of the ratios \( s / \ell_1 \) and \( E_2 / E_1 \) on the ERR obtained in the case where \( \ell_0 / 2 \ell_1 = 0.15 \) and \( h_0 / \ell_1 = 0.1 \) (i.e. \( h_0 = h / 2 \)). It follows from Table 2, as can be predicted, that the absolute maximum value of the ERR occurs at \( s / \ell_1 = 0 \) and the values of the ERR monotonically decrease with \( s / \ell_1 \). Moreover, it follows from Table 2, that the magnitude of the
influence of the material anisotropy on the ERR increases with a decrease in the values of the ratio \( \frac{21}{EE} \). Table 3 illustrates the influence of the ratios \( \frac{31}{\lambda_1} \) on the ERR for various values of \( \frac{21}{EE} \) obtained in the case where \( \lambda_0 / \lambda_1 = 0.15 \), \( \lambda_1 = \lambda_s \) and \( \lambda_1 / \lambda_1 = 0.1 \) (i.e. \( h_0 / \lambda_1 = h / 2 \)). As can be seen, the values of the ERR monotonically increase with \( \frac{31}{\lambda_1} \). This effect becomes more pronounced with decreasing of the ratio \( \frac{21}{EE} \).

Table 4 shows the effect of the crack’s location and crack length on the ERR for various values of \( \frac{21}{EE} \) in the case where \( \lambda_0 / \lambda_1 = \lambda_s \). It can be concluded that the values of the ERR increase as the crack location approaches the upper face plane of the plate i.e. as the values of \( \lambda_0 / \lambda_1 \) decrease. This effect becomes significant with crack length, i.e. \( \lambda_0 / 2 \lambda_1 \).

### 4 Conclusion

In the current paper, the ERR in the band crack’s front which is contained in the thick rectangular plate made of multilayered composites is investigated by employing 3D FEM. The opening mode case which is considered is based on the numerical results of the influence of the plate material anisotropy and crack’s location geometry on the ERR, which are presented and discussed. At the same time, the approach used here can be applied to the related materials considered in works [16,17].
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References


