Abstract: A simple analytical model to estimate the longitudinal strength of ship hulls in composite materials under buckling, material failure and ultimate collapse is presented in this paper. Ship hulls are regarded as assemblies of stiffened panels which idealized as group of plate-stiffener combinations. Ultimate strain of the plate-stiffener combination is predicted under buckling or material failure with composite beam-column theory. The effects of initial imperfection of ship hull and eccentricity of load are included. Corresponding longitudinal strengths of ship hull are derived in a straightforward method. A longitudinally framed ship hull made of symmetrically stacked unidirectional plies under sagging is analyzed. The results indicate that present analytical results have a good agreement with FEM method. The initial deflection of ship hull and eccentricity of load can dramatically reduce the bending capacity of ship hull. The proposed formulations provide a simple but useful tool for the longitudinal strength estimation in practical design.

Keywords: Composites; Ship hull; stiffened panel; Ultimate longitudinal strength; analytical model

1 Introduction

Laminated fiber reinforced composites have been widely used in marine and ship structures. The application of such materials in ship structures dates back to the late 1970s. Over the time, the usage of composite materials in ship construction continues to grow due to the improvement of design, fabrication and mechanical performance of advanced composites. Currently, some large vessels like frigates and passenger ships are made of laminated composite materials. There are now all-composite naval ships in service.

Longitudinal bending moment carried by ship hull girder increasing with the increasing length of composite ship hulls, the ship safety related to longitudinal ultimate strength becomes more important at design. According to traditional ship design rules, the longitudinal strength of the ship hull built of steel with length exceeding 60 m must be assessed. The stiffness of composite materials is generally low. Longitudinal deformation of ship hulls in composite materials is usually larger than that built of steel. Recent years, with the rapid development trend and momentum of construction of large-scale and super large composite ship hulls, the length of ship hulls in composite materials are increasing so that the study of longitudinal ultimate strength of composite ship becomes increasingly important and urgent.

The ultimate strength of steel ships has been widely investigated by many researchers. Caldwell [1] was the first who estimated the ultimate strength of steel ships employing the fully plastic bending theory of beams. In his method, the hull girder was divided into a number of panels and the collapse load of each panel was calculated through strength reduction factor due to buckling for compressive loading. However his model did not take into account the post-buckling strength of the panels. Smith [2] proposed an approach for calculation of the ultimate strength of ship hulls. He divided the ship’s cross section into a series of stiffened panels and then performed a progressive collapse analysis under bending where the post-buckling behavior of the stiffened plates was considered. The collapse of a girder section is assumed to occur between two adjacent frames, being induced either by the inter-frame flexural beam-column collapse of panels under compression or by the inter-frame yielding of panels under tension. The behavior of the stiffened plates under compressive load can be characterized through so-called load-shortening curves. The resulting strains and stresses in the stiffened plates making up the cross section were calculated using the predefined load-shortening curves. Based on the general approach of Smith’s method, Dow et al. [3] and Ueda and Yao [4] developed a method for determination of load-shortening curves of beam-column ele-
ments, which similar to the Smith's beam-column method
with the difference that a large portion of the hull girder
plating with multiple stiffeners was considered. Viner [5]
proposed an expression for calculating the ultimate bend-
ing moment assuming that the elastic behavior is main-
tained up to the point where the compression flange reach
the collapse state resulting in immediate hull collapse.
Khedmati [6] made attempts for derivation of the average
stress-average strain relationships for stiffened plates sub-
ject to in-plane compression alone or in combination with
other loads using analytical approaches. Frieze and Lin [7]
expressed ultimate bending moment capacity of the ship
hull as a function of a normalized ultimate strength of the
compression flange. Paik and Mansour [8] presented an ex-
pression for predicting the ultimate strength of single and
double-hull ships under vertical bending moments. Gar-
batov [9] performed hull girder ultimate strength verifica-
tion according to the Class Society rules based on experi-
mental results and the dimensional theory. Some other ul-
timate strength approaches can be found in [10–12].

Very few publications can be found in the literature
on evaluating the ultimate strength of composite ships.
Chen et al. [13] firstly tried to estimate the ultimate strength
of composite ships. They proposed a simple analytical
method for calculating the ultimate strength of com-
posite vessels. But his model is too simple to handle the stak-
ing sequence of real composite materials and fail to give
procedures to handle the composite hat-stiffened panels.
Chen and Soares [14] conducted a calculation of ultimate
strength of composite ships under bending moment with
nonlinear finite element method. Two types of the failure
modes were considered in their study; the panel buckling
as well as fracture of the composite materials. Later, Chen
and Soares [15] used Smith's method to calculate the ul-
timate strength of composite vessels. Morshedsoluk [16]
applied a Coupled Beam Theory to calculate the ultimate
strength of composite ships taking into account the ef-
fect of the superstructure. The behaviour of the com-
posite panels in the ship structure is deduced from their mean
stress-mean strain curves. The progressive failure method
in conjunction with the nonlinear finite element method
is used to calculate mean stress- mean strain curves. The ef-
ciency of the composite superstructure in contribution
to the ultimate bending strength of the composite ships is
evaluated. Case studies showed that the length of super-
structure has significant effect on the ultimate strength of
the composite ship.

Application of composite materials to the construc-
tion of large ships is a relatively new and growing subject,
which needs correct assessment of the ultimate strength
of these types of the ships. Especially, a simple analyti-
cal method is needed for practical engineering at prelimi-
nary stage of ship design. For this purpose, an analyti-
cal approach to estimate the longitudinal strengths of ship
hull in composite materials was developed in this paper.
Ship hulls are modeled as assemblies of stiffened com-
posite panels. Stiffened panels are idealized as a series of
plate-stiffener combinations. Ultimate strain is predicted
under buckling or material failure with beam-column the-
ory, considering the effects of initial imperfection of ship
hull and eccentricity of load. The corresponding longitudi-
dlars of ship hulls are derived with a straightforward
analytical method.

The primary modes of failure for ship hull girder is col-
um or beam-column type collapse of plate-stiffener com-
bination as a representative of stiffened panel. Two failure
modes of beam-column in composite materials are consid-
ered in this paper. The first failure mode is buckling. The
tensile and compressive strengths of FRP laminates com-
monly employed in hull design are approximately equal
to the yield strength of mild steel, and the Young's modu-
lus of such laminates are only 5-10% of that of steel. Thus
buckling may be an important failure mode of ship hull
in composite materials. The second failure mode is mate-
rial failure. The material failure of a local area in ship hull
may occur with the increase of longitudinal bending mo-
ment, which will influence structural performances. Con-
sequently, it is necessary to take material failure as an im-
portant failure mode of ship hull in composite materials.

2 Bending stiffness of composite
ship hulls

Longitudinal framed structure system is usually used for
large ships. The deck plate, bilge plate and side plate are
strengthened with a series of longitudinal stiffeners (in-
cluding Stringer). They are all hat-stiffened laminates. Hull
girder consists of a number of structural elements of stiff-
ened plates (Figure 1). According to the Smith analysis
method [17], each stiffener of the hull girder (including
attached flange and plating) works almost independently
from other stiffeners. Consequently, each plate-stiffener
combination can be analyzed alone. The load capacity of
structural element of stiffened panel and ship hull girder
can be calculated with composite beam-column theories.

To determine the bending stiffness of ship hull girder,
the section is divided into a set of representative plate-
   stiffener combination (Figure 2). The plate-stiffener com-
bination is then divided into a set of slats [18]. Each strip
can be calculated as a laminate.
2.1 Stiffness of symmetrical balanced laminate

Symmetrical balanced laminate presents orthotropic properties. The planar stiffness coefficients are

\[(Et)_{11} = A_{11} - A_{12} \frac{A_{22}}{A_{11}}, \quad (Et)_{12} = A_{22} - A_{12} \frac{A_{11}}{A_{22}}; \quad (Gt)_{1} = A_{66} \]  

(1)

Where, \((Et)_{11}\) is effective tensile/compressive stiffness coefficient at \(x\) direction; \((Et)_{12}\) is effective tensile/compressive stiffness coefficient at \(y\) direction, \((Gt)_{1}\) is effective shear stiffness coefficient.

Bending stiffness coefficients of symmetrical balanced laminate are

\[D_{11} = D_{11} - D_{12} \frac{D_{12}}{D_{11}}; \quad D_{12} = D_{22} - D_{12} \frac{D_{11}}{D_{22}}; \quad D_{112} = D_{66} \]  

(2)

Where, \(D_{11}\) is effective bending stiffness at \(x\) direction; \(D_{12}\) is effective bending stiffness at \(y\) direction; \(D_{112}\) is torsional stiffness.

Effective elastic modulus of symmetrical balanced laminate is

\[E_{11} = \frac{1}{t} \left( A_{11} - A_{12} \frac{A_{22}}{A_{11}} \right), \quad E_{12} = \frac{1}{t} \left( A_{22} - A_{12} \frac{A_{11}}{A_{22}} \right), \]  

(3)

2.2 Stiffness of laminate plate-stiffener combination

The stiffened panel is idealized as a number of plate-stiffener combination (stiffener including attached flange) or a beam-column. Typical section of a beam-column is shown in Figure 2.

Beam-column is divided into a set of strips. Assuming these strips to possess the same longitudinal strain, which are equal to that of stiffened laminate, we have

\[E = \frac{P}{(EA)_{ps}} = \frac{P_i}{E_{11i}A_i} \]  

(4)

Where, \(P\) is axial force of beam-column; \(P_i\) is axial force of each slat, \((EA)_{ps}\) is tensile/compressive stiffness of beam-column; \(E_{11i}\) is the effective elastic modulus of each slat; \(A_i\) is the sectional area of each strip.

\[P = \sum P_i = \frac{P}{(EA)_{ps}} \sum E_{11i}A_i = \frac{P}{(EA)_{ps}} \sum (E_{11i}b_i) \]  

(5)

Where \(t_i\) is thickness of strip; \(b_i\) is width of strip. The tensile/compressive stiffness of beam-column is

\[(EA)_{ps} = \sum (E_{11i}t_i)b_i = \sum \left( A_{11i} - A_{12i} \frac{A_{22i}}{A_{22i}} \right) b_i \]  

(6)

Where, \(E_0\) is the elastic modulus of an arbitrarily selected standard reference material and

\[\lambda_i = \left( A_{11i} - A_{12i} \frac{A_{22i}}{A_{22i}} \right) / E_0 \]  

(7)
Similarly,
\[(GA)_{ps} = \sum A_{66i}b_i\]  
(8)

The sectional area of beam-column is
\[A_{ps} = \sum A_i = \sum b_i t_i\]  
(9)

The effective elastic modulus of beam-column is
\[E_{ps} = \sum \lambda_i b_i E_0\]  
(10)

Assuming the bending deformation of each strip keeps consistent with that of beam-column
\[\varphi = \frac{MI}{(EI)_{ps}} = \frac{M_i l}{(EI)_{li}}\]  
(11)

Where, \(M\) is the bending moment of beam-column, \((EI)_{ps}\) is the bending stiffness of beam-column; \(M_i\) is the bending moment of each slat; \((EI)_{li}\) is the bending stiffness of each slat; \(l\) is the length of beam-column; \(\varphi\) is the relative rotation of beam-column end sections.

\[M = \sum M_i = \frac{M}{(EI)_{ps}} \sum (EI)_{li}\]  
(12)

The bending stiffness of beam-column is obtained from
\[(EI)_{ps} = \sum (EI)_{li}\]  
(13)

At local reference coordinates system shown in Figure 2, the neutral axis of beam-column is determined from
\[z_{cps} = \frac{\sum E_{i1i} A_i z_i}{\sum E_{i1i} A_i}\]  
(14)

\[= \frac{\left( A_{11i} - \frac{A_{12i}}{A_{22i}} \right) b_i z_i}{\sum \left( A_{11i} - \frac{A_{12i}}{A_{22i}} \right) b_i} = \sum \lambda_i b_i z_i \]

\[= \sum \lambda_i b_i z_i \]

Where, the \(z_{cps}\) is the coordinates of neutral axis of beam-column; \(z_i\) is the centroidal coordinate of each slat.

When the width of slat is parallel to the neutral axis of beam-column section, bending stiffness of the slat about its own neutral axis is in form of
\[(EI_y)_{li} = \left( D_{11i} - \frac{D_{12i}^2}{D_{22i}} \right) b_i\]  
(15)

When the width of slat is at an inclined direction which is not parallel to the neutral axis of beam-column section, bending stiffness of the slat about its own neutral axis is expressed as
\[(EI_y)_{li} = \frac{1}{12} E_{i1i} l_i (b_i \sin \alpha)^3\]  
(16)

Where, \(\alpha\) is the angle between the slat width and the neutral axis.

Bending stiffness of shifting neutral axis of slat to that of beam-column is
\[(EI_y)_{li}'' = E_{i1i} l_i b_i (z_i - z_{cps})^2 = E_0 \lambda_i b_i (z_i - z_{cps})^2\]  
(17)

Bending stiffness of beam-column is expressed as
\[(EI_y)_{ps} = \sum [(EI)'_{li} + (EI)''_{li}]\]  
(18)

Similarly, bending stiffness of beam-column about its symmetric axis is expressed as
\[(EI_z)_{ps} = \sum [(EI)''_{li} + (EI)''_{li}]\]  
(19)

where,
\[\begin{align*}
(EI)''_{li} & = \frac{1}{12} (Et_i)(b_i \cos \alpha)^3 = \frac{1}{12} E_0 \lambda_i (b_i \cos \alpha)^3 \\
(EI)_z'' & = (EA_i)(y_i)^2 = E_0 \lambda_i b_i (y_i)^2
\end{align*}\]  
(20)

### 2.3 Bending stiffness of composite ship hull girder

The sectional area of ship hull girder is sum of that of all beam-column
\[A = \sum A_{ps}\]  
(21)

The composite beams are mainly distinguished from traditional beams in structure. Different types and specifications of reinforcement as well as different layups are utilized in different parts of a composite beam. Composite beam theory should be applied. Area conversion methods should be used to deal with composite beams [19, 20], namely, an elastic modulus of an arbitrarily selected materials as standard reference modulus, denoted \(E_0\), and cross-sectional area of other materials are transformed in the coefficient
\[\gamma_i = \frac{E_{psi}}{E_0}\]  
(22)

The axial stiffness of ship hull girder is
\[(EA)_{gir} = \sum (EA)_{psi} = E_0 \sum (\gamma_i A_{psi})\]  
(23)

Setting a global reference coordinates at the bottom of ship hull, the neutral axis is determined as
\[z_{cgir} = \frac{\sum (EA)_{psi} z_{psi}}{\sum (EA)_{psi}} = \frac{\sum (\gamma_i A_{psi} z_{psi})}{\sum (\gamma_i A_{psi})}\]  
(24)

\(z_{cgir}\)-centroidal coordinates of ship hull girder section.
Material failure

The lamina stresses in principal material direction under uniaxial stress are

\[
\begin{align*}
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T_\theta] \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix} \\
\begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m^2 \\ n^2 \\ -mn \end{bmatrix} \sigma_x
\end{align*}
\]

Or simply

\[
\phi_1 \sigma_{mf}^2 + \phi_2 \sigma_{mf} - 1 = 0
\]

Where

\[
\begin{align*}
\phi_1 &= F_{11} m^4 + 2F_{12} m^2 n^2 + F_{22} n^4 + F_{66} m^2 n^2 \\
&- 2F_{16} m^3 n - 2F_{26} m n^3 \\
\phi_2 &= F_1 m^2 + F_2 n^2 - F_6 m n
\end{align*}
\]

The ultimate strength of composite stiffened panel under material failure is given by

\[
\sigma_{mf} = \frac{-\phi_2 + \sqrt{\phi_2^2 + 4 \phi_1}}{2}
\]

The material failure strain is

\[
\varepsilon_{mf} = \frac{\sigma_{mf}}{E_{ps}}
\]

Column buckling

The column buckling load of plate-stiffener combination with uniform cross-section is given by

\[
P_{cr} = \frac{\pi^2 (EI)_{ps}}{l^2}
\]

The column buckling stress is

\[
\sigma_{cr} = \frac{\pi^2 (EI)_{ps}}{A_{ps} l^2}
\]

The critical buckling stress including shear deformation is in form of [22]

\[
\sigma_{cr} = \frac{\pi^2 (EI)_{ps}}{A_{ps} (\mu l)^2} \frac{1 + \mu (EI)_{ps}}{G A_{ps} (\mu l)^2}
\]

The ultimate buckling strain of stiffened composite panels is

\[
\varepsilon_{bf} = \frac{\sigma_{cr}}{E_{ps}}
\]

Initial imperfection and eccentricity of load

The initial deflection of beam-column is assumed to be

\[
w_1 = w_0 \sin \frac{\pi x}{T}
\]
Where, \( w_0 \) is the amplitude of initial deflection. \( l \) is length of beam-column. \( x \) is longitudinal coordinates.

The total deflection is given by \( w_t = w_1 + w \), where \( w \) is bending deflection.

The differential equation of beam-column is given by
\[
(El)^{ps} \frac{d^2 w}{dx^2} + Pw = -P \left( w_0 \sin \frac{\pi x}{l} + e \right)
\]  
(40)

Where, \( P \) is axial compressive load; \((El)^{ps} = D_{ps}\) is the flexural stiffness of plate-stiffener combination; \( e \) is the eccentricity of load.

Particular solution of Eq. (40) is given by
\[
w_p = C \sin \frac{\pi x}{l} + D \cos \frac{\pi x}{l} - e
\]  
(41)

Substituting into Eq. (40), we obtain
\[
\left[ C \left( k^2 - \frac{\pi^2}{l^2} \right) + k^2 w_0 \right] \sin \frac{\pi x}{l} + D \left( k^2 - \frac{\pi^2}{l^2} \right) \cos \frac{\pi x}{l} = 0
\]

where, \( k^2 = P/(El)^{ps} \). Solving for \( C \) and \( D \), we have
\[
C = w_0 f \left( \frac{\pi^2}{k^2} - 1 \right) = w_0 f \left( \frac{P_{cr}}{P} - 1 \right) = \eta w_0 / (1 - \eta);
\]
\[
D = 0
\]

Where, \( \eta = P/P_{cr} \).

General solution of Eq. (40) is in form of
\[
w = A \sin kx + B \cos kx + \eta w_0 / (1 - \eta) \sin \frac{\pi x}{l} - e
\]  
(42)

Boundary conditions are
\[
x = 0, \quad l \quad w = 0
\]  
(43)

Substituting into Eq. (42), we get
\[
A = \frac{e}{\sin kl} - e \cot kl; \quad B = e
\]

Solution of Eq. (40) is written as
\[
w = \eta w_0 / (1 - \eta) \sin \frac{\pi x}{l} + e \cos kx - e \cos kx \sin \frac{\pi x}{l} - e
\]
\[
= \eta w_0 / (1 - \eta) \sin \frac{\pi x}{l} + \frac{e}{\cos \frac{\pi x}{l}} \cos \left( kx - \frac{kl}{2} \right) - e
\]

Bending moment is determined as
\[
M = P(w + w_1 + e)
\]
\[
= Pw_0 / (1 - \eta) \sin \frac{\pi x}{l} + Pe \frac{P}{\cos \frac{\pi x}{l}} \cos \left( kx - \frac{kl}{2} \right)
\]

Maximum bending moment is
\[
M_{max} = Pw_0 / (1 - \eta) + Pe + Pe \frac{k^2 l^2}{8}
\]
\[
= Pw_0 / (1 - \eta) + Pe + Pe \frac{P}{8} \frac{P_{cr}}{P_{cr}}
\]

The maximum stress is
\[
\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{W} = \frac{P}{A} \left( 1 + \frac{M_{max}}{P} \right)
\]  
(44)

with
\[
W = \frac{(EI)^{ps}}{E_1 l^2} \quad \rho = W/A
\]

where \( z \) is the maximum vertical coordinates measured from neutral axis, \( A \) is the cross-sectional area.

Adopting the approximation:
\[
P_{cr} = 1 + \frac{P}{P_{cr}}
\]

Substituting into Eq. (44) yields:
\[
\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{e + w_0}{\rho} + \left( \frac{w_0 + \frac{\pi^2 e}{8}}{\rho P_{cr}} \right) \frac{P}{\rho P_{cr}} \right]
\]  
(45)

Ultimate strength is reached when material failure occurs:
\[
\sigma_u \left[ 1 + \frac{e + w_0}{\rho} + \left( \frac{w_0 + \frac{\pi^2 e}{8}}{\rho P_{cr}} \right) \frac{\sigma_u}{\rho \sigma_{cr}} \right] = \sigma_{mf}
\]  
(46)

or simply
\[
\sigma_u^2 + \mu \sigma_{cr} \sigma_u - \omega (\sigma_{cr})^2 = 0
\]  
(47)

where,
\[
\mu = \frac{\rho + e + w_0}{w_0 + \frac{\pi^2 e}{8}}; \quad \omega = \frac{\rho \sigma_{mf}/ \sigma_{cr}}{w_0 + \frac{\pi^2 e}{8}}
\]  
(48)

The ultimate strength of composite stiffened panel with initial imperfection and eccentricity of load is given by
\[
\sigma_u = -\mu + \sqrt{\mu^2 + 4\omega} \sigma_{cr}
\]  
(49)

The ultimate strain is
\[
\varepsilon_{ume} = \frac{\sigma_u}{E_{ps}}
\]  
(50)

6 Longitudinal strength of composite ship hulls

To simplify the problem, the following hypotheses are made:
1. The hull girder is assumed as an Euler-Navier beam. The transverse cross-sections of ship remain plane when subjected to bending moments and the influence of transverse restraint on longitudinal stress is negligible.
2. Longitudinal failure only occurs between two adjacent transverse frames.
3. The failure of each stiffened composite panel is assumed to occur individually and independently.

The normal strain $\varepsilon$ varies linearly across the perpendicular direction of cross-section.

$$
\varepsilon_i = z_i \kappa = \frac{2 M}{D_{gir}}
$$

where $z$ is the coordinates measured from neutral axis; $\kappa$ is the curvature; $M$ is bending moment.

The ultimate strength of ship hull is defined as

$$
M_u = \frac{D_{gir} E_u}{Z_{ps \text{ max}}}
$$

### 7 Numerical example

A longitudinally framed structure ship hull built of fiber-reinforced plastic was considered. The cross-section of mid-ship is shown in Fig. 1. Breadth moulded is 9 m; Depth moulded is 6 m. The thicknesses of the plate are 4.1 mm at deck and side, and 7.2 mm at the bilge, respectively. The geometry of the stiffened composite panels at mid-ship section is shown in Fig. 2. The dimensions of the stiffened panels at deck and side are shown in Table 1. The dimensions at bilge are shown in Table 2. The skin of deck is made of 32 symmetrically stacked unidirectional plies $[90/45/90/-45/0/45/90/90/-45/0/45/90/90/-45/0/-45]s$. The measured ply thickness is 0.13 mm. The stacking sequence of stringer is $[90/45/90/-45/0/45/90/90]$ and the ply thickness is 0.13 mm. The head of the stiffener is reinforced and made of 17 plies $[90/45/90/-45/0/45/90/90/-45/0/45/90/90]$s. The ship is made of one kind of composite material, and the material properties are shown in Table 3.

The strength coefficients for Tsai-Wu failure criterion can be computed from the failure strengths in Table 3. When the ship is under sagging, the stiffened panels in the deck are in compression and those in the bilge are in tension. Tables 1 and 2 show the dimensions of stiffened panels at bilge are larger than those at deck. It is certain that the buckling and material failure of the stiffened panels at deck will be prior to that at bilge. So the first step is to calculate the buckling and material failure strain of the stiffened panels at deck.

The sectional properties of plate-stiffener combination and ship hull are computed and shown in table 4. The plate-stiffener combination is assumed to be simply supported at the ends. Initial deflection $w_0$ is taken as 10 mm and eccentricity of load $e$ as 5 mm. The buckling failure strain $\varepsilon_{bf}$ and material failure strain $\varepsilon_{mf}$ at deck and ultimate strain with initial imperfection and load eccentricity $\varepsilon_{ime}$ are evaluated. Then the buckling failure strength $M_{bf}$, material failure strength $M_{mf}$ and ultimate strength $M_{ime}$ of ship hull under sagging are obtained from Eq. (52). The results from present analytical solution are shown in table 5. It indicates that the initial deflection of ship hull and the eccentricity of load can remarkably decrease the bending capacity of ship hull. The present analytical method has been correlated with simulations using the nonlinear finite element method described in [14]. The results indicate that it gives very similar results to that of the FEM.

### 8 Conclusions

An analytical model accounting for the effects of initial imperfection of ship hull and eccentricity of load is presented in this paper to estimate the longitudinal strength of ship hull in composite materials under buckling failure, material failure and ultimate collapse of the deck, based on
Table 3: Material properties of composite material

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus in 1 principal material direction $E_1$</td>
<td>15.7 GPa</td>
</tr>
<tr>
<td>Elastic modulus in 2 principal material direction $E_2$</td>
<td>14.8 GPa</td>
</tr>
<tr>
<td>Poisson’s ratios $\nu_{12}$</td>
<td>0.127</td>
</tr>
<tr>
<td>Shear modulus in 2-3 principal material plane $G_{23}$</td>
<td>0.34 GPa</td>
</tr>
<tr>
<td>Shear modulus in 1-3 principal material plane $G_{13}$</td>
<td>0.34 GPa</td>
</tr>
<tr>
<td>Shear modulus in 1-1 principal material plane $G_{12}$</td>
<td>0.34 GPa</td>
</tr>
<tr>
<td>Tensile strength in 1 principal material direction $X_T$</td>
<td>238 MPa</td>
</tr>
<tr>
<td>Compressive strength in 1 principal material direction $X_C$</td>
<td>204 MPa</td>
</tr>
<tr>
<td>Tensile strength in 2 principal material direction $Y_T$</td>
<td>210 MPa</td>
</tr>
<tr>
<td>Compressive strength in 2 principal material direction $Y_C$</td>
<td>224 MPa</td>
</tr>
<tr>
<td>Shear strength in 2-3 principal material plane $R$</td>
<td>23.5 MPa</td>
</tr>
<tr>
<td>Shear strength in 1-3 principal material plane $S$</td>
<td>23.5 MPa</td>
</tr>
<tr>
<td>Shear strength in 1-2 principal material plane $T$</td>
<td>104 MPa</td>
</tr>
</tbody>
</table>

Table 4: Sectional properties of plate-stiffener combination and ship hull

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sectional area of plate-stiffener $A_{ps}$</td>
<td>$1.73 \times 10^{-3}$ m$^2$</td>
</tr>
<tr>
<td>Bending stiffness of plate-stiffener ($EI_{ps}$)</td>
<td>$1.31 \times 10^4$ Nm$^2$</td>
</tr>
<tr>
<td>Shear stiffness of plate-stiffener ($GA_{ps}$)</td>
<td>$6.5 \times 10^5$ N</td>
</tr>
<tr>
<td>Effective elastic modulus of plate-stiffener $E_{ps}$</td>
<td>15.5 GPa</td>
</tr>
<tr>
<td>Neutral axis Coordinates of mid-ship hull $z_{c_{gir}}$</td>
<td>2.917 m</td>
</tr>
<tr>
<td>Flexural stiffness of hull girder $D_{gir}$</td>
<td>$1.736 \times 10^{10}$ Nm$^2$</td>
</tr>
</tbody>
</table>

Table 5: Results of failure strain and ultimate strength

<table>
<thead>
<tr>
<th></th>
<th>failure strain ($\times 10^{-3}$)</th>
<th>ultimate strength ($\times 10^7$ Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>buckling</td>
<td>material failure</td>
</tr>
<tr>
<td>Present analytical</td>
<td>8.63</td>
<td>13.8</td>
</tr>
<tr>
<td>Finite element</td>
<td>9.4</td>
<td>14.2</td>
</tr>
</tbody>
</table>

accurate strain computation of stiffened composite panels with a simple analytical method. In the present model, detailed procedures to evaluate the stiffness of symmetrical balanced laminate, composite beam-columns and ship hull girder is described. Because of the special characteristics of composite material, the buckling and material failure may be the important failure modes of ship hull in composite materials. The Tsai-Wu criterion is adopted to identify the material failure of stiffened composite panels. Because of the lower stiffness of composite materials at transverse direction, the critical buckling stress including shear deformation is adopted. A longitudinally framed structure ship hull in composite materials made of symmetrically stacked unidirectional plies under sagging is analyzed as an example application. The longitudinal ultimate strength was investigated. The results show that the initial deflection of ship hull and the eccentricity of load can dramatically decrease the bending capacity of the ship hull. Owing to lack of information on ultimate strength of composite ship hulls, the present method was compared to the finite element simulation. The results indicate present analytical results have a good agreement with FEM methods and the present approach provides a simple but useful tool for the longitudinal strength estimation of ship hull in composite materials.

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References