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COINCIDENCE AND FIXED POINT FOR WEAKLY RECIPROCALLY CONTINUOUS SINGLE-VALUED AND MULTI-VALUED MAPS

Abstract. In the present paper, we extend the concept of Weak Reciprocal Continuity for a hybrid pair of single-valued and multi-valued maps and introduce (T, f)-completeness of the space. Further, we establish some results on the existence of coincidence and fixed points for the hybrid pair of maps. Our results generalize several well known results available in the literature.

1. Introduction

Sessa [14] introduced the weak commutativity condition for a pair of single valued maps. In 1986, Jungck [4] generalized the concept of weak commutativity condition by introducing compatibility of maps. Kaneko [6] defined weak commutativity condition for a pair of multi-valued maps. Kaneko and Sessa [7] extended the concept of compatibility condition for a hybrid pair of single-valued and multi-valued maps. Pant [9] initiated the study of noncompatibilty by introducing point wise R-weak commutativity. Shahzad and Kamran [15], and Singh and Mishra [16], independently, extended the idea of R-weak commutativity for a hybrid pair of single and multi-valued maps. Kamran [5] extended the concept of R-weak commutativity of type A_g [11] for multivalued maps and introduced R-weakly commuting mappings of type A_T. Recently, Al-Thagafi and Shahzad [1] introduced the notion of occasionally weakly compatible maps and employed the new notion to prove fixed point theorem under new condition. Here it seems important to mention that weak commutativity implies compatibility but the converse is not true. Weak commutativity implies R-weak commutativity but R-weak commutativity implies weak commutativity only when $R \leq 1$.

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In 1998, Pant [8] introduced reciprocal continuity (r.c) for the pair of single-valued maps which states that maps f and g are r.c. if and only if $lim_n gf(x_n) = gt$ and $lim_n fg(x_n) = ft$, whenever x_n is a sequence in X such that $lim_n f(x_n) = lim_n g(x_n) = t$ for some t in X. They also established some common fixed point theorems for reciprocally continuous maps. It is also proved that a pair of maps which is receprocally continuous need not be continuous even on their common fixed point [see example [8]].

Generalizing reciprocal continuity, Pant et al. [10] recently introduced Weak Reciprocal Continuity (w.r.c.) for a pair of single-valued maps as follows:

Definition 1. [10] Two self mappings f and g of a metric space (X, d) are called weakly reciprocally continuous if $lim_n fgx_n = ft$ or $lim_n gfx_n = gt$, whenever $\{x_n\}$ is a sequence in X such that $lim_n f(x_n) = lim_n g(x_n) = t$ for some t in X.

It is remarkable that reciprocal continuity implies weak reciprocal continuity but the converse is not true as shown in the following example.

Example 1. [10] Let $X = [2, 20]$ and d is a usual metric in X. Define $f, g :\to X$ as follows,

$$fx = 2 \text{ if } x = 2 \text{ or } x > 5,\ fx = 6 \text{ if } 2 < x \leq 5,$$
$$g2 = 2,\ gx = 12 \text{ if } 2 < x \leq 5,\ gx = (x + 1)/3 \text{ if } x > 5.$$

Then f and g are clearly weakly reciprocally continuous but not reciprocally continuous.

Pant et al. [10] also obtained some common fixed point theorems for the w.r.c pair of maps. For the rest of the part of the paper, we follow following notations and definitions.

$CB(X)$ (resp. $CL(X)$) denote the family of all closed and bounded (resp. closed) subsets of X. $C(X)$ represents set of all compact subsets of X. The Hausdorff distance for two subsets A, B of X is defined as $H(A, B) = max(\{sup\ d(a, B) : a \in A)\}, \{sup\ d(A, b) : b \in B\})$, where $d(a, B) = inf\{d(a, b) : b \in B\}$, $d(A, B) = inf\{d(a, b) : a \in A, b \in B\}$.

In a recent paper, Singh and Mishra [17] extended the idea of reciprocal continuity for a hybrid pair of single-valued and multi-valued maps. The maps $T : X \to CL(X)$ and $f : X \to X$ are reciprocally continuous if and only if $fTx \in CL(X)$ for each $x \in X$ (resp., $fTt \in CL(x)$) and $lim_n fTx_n = fM$, $lim_n Tfx_n = Tt$, whenever $\{x_n\}$ is a sequence in X such that $lim_n Tx_n = M \in CL(X)$, $lim_n fx_n = t \in M$. Any continuous pair is reciprocally continuous but, as the following example shows, the converse is not true.

$$Tx = \left\{ \begin{array}{ll} [\frac{1}{2}, x+1], & \text{if } x > 0 \\ \{0\}, & \text{if } x = 0 \\ [x-1, -\frac{1}{2}], & \text{if } x < 0 \end{array} \right\}, \quad fx = \left\{ \begin{array}{ll} 2x+1, & \text{if } x > 4 \\ 0, & \text{if } x = 0 \\ 2x-1, & \text{if } x < 4 \end{array} \right\}.$$

Then T and f are r.c. at $x = 0$ (take $x_n = 0, n \in N$) but there is discontinuity at their common fixed point ($x = 0$).

Now we define weak reciprocal continuity for hybrid pair of single-valued and multi-valued maps which is an extension of the definition given by Pant et al. [10] for the hybrid pair of single-valued and multi-valued maps and generalization of the definition given by Singh and Mishra [17].

Definition 2. The maps $T : X \to CL(X)$ and $f : X \to X$ are weakly reciprocally continuous on X (resp., at $t \in X$) if and only if $fTx \in CL(X)$ for each $x \in X$ (resp., $fTt \in CL(X)$) and $lim_n fTx_n = fM$ or $lim_n Tfx_n = Tt$ whenever $\{x_n\}$ is a sequence in X such that $lim_n Tx_n = M \in CL(X)$, $lim_n fx_n = t \in M$.

It is to be noted that the reciprocal continuity implies weak reciprocal continuity but the converse is not true [10]. Following example shows that this statement is also valid for hybrid pair of maps.

Example 3. Let $X = [0.\infty)$ with usual metric and

$$Tx = \left\{ \begin{array}{ll} [0, \quad x], & \text{if } x < 4 \\ [4, 2+x], & \text{if } x \geq 4 \end{array} \right\}, \qquad fx = \left\{ \begin{array}{ll} x, & \text{if } x \leq 4 \\ 6, & \text{if } x > 4 \end{array} \right\}.$$

Then T and f are weakly reciprocal continuous (take $x_n = 4 - \frac{1}{n}$) but not reciprocally continuous. Notice that there is discontinuity at their common fixed point ($x = 4$).

The following definition is due to Kaneko and Sessa [7].

Definition 3. The maps $T : X \to CL(X)$ and$f : X \to X$ are compatible if and only if $fTx \in CL(X)$ for each $x \in X$ and $H(Tfx_n, fTx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_n Tx_n = M \in CL(X)$, $lim_n fx_n = t \in M$.

Here it is to be noted that r.c. and compatibility are independent (see Example 2.2 [17]). Further, reciprocal continuity implies w.r.c., so by same example (see Example 2.2 [17]) it is easily verified that w.r.c. and compatibility are also independent concepts. Singh and Mishra [17] also gave an example (Example 2.6) where the requirement of compatibility is vacuously satisfied. Singh and Mishra [17] established a result stating nonvacuously compatible and reciprocally continuous hybrid pair of maps on a metric space have a coincidence point. Further, they also proved that with certain other conditions, the pair has a common fixed point. Here it is remarkable that if in Theorem 2.8 of Singh and Mishra [17], we replace reciprocal continuity by weak reciprocal continuity then the hybrid pair of maps not necessarily needs to possess a coincidence point i.e. nonvacously compatible and w.r.c., hybrid pair of maps not necessarily needs to possess a coincidence point. Following example proves this fact.

Example 4. Let $X = [0, \infty)$ be endowed with the usual metric and

$$Tx = \left\{\begin{array}{lll} [0, & \frac{x}{2}], & \text{if } \quad x \leq 2 \\ [\frac{3}{2}, & 2], & \text{if } 2 < x \leq 3 \\ \{3\}, & & \text{if } 3 < x \leq 4 \\ [4, & x], & \text{if } \quad x > 4 \end{array}\right\}, \quad fx = \left\{\begin{array}{ll} \frac{3}{2}, & \text{if } \quad x \leq 2 \\ x+1, & \text{if } 2 < x < 3 \\ x, & \text{if } 3 \leq x < 4 \\ x+6, & \text{if } \quad x \geq 4 \end{array}\right\}.$$

If we take $\{x_n\} = (3 + \frac{1}{n}\ )$

$$\lim_n T\left[3 + \frac{1}{n}\right] = \{3\}, \qquad \lim_n f\left[3 + \frac{1}{n}\right] = 3,$$

$$\lim_n fT\left[3 + \frac{1}{n}\right] = \{3\} = f\{3\}, \quad \lim_n Tf\left[3 + \frac{1}{n}\right] = \{3\} \neq T(3).$$

Since $lim_n fTx_n = fM$ but $lim_n Tfx_n \neq Tt$, the pair (T, f) is not reciprocally continuous but weakly reciprocally continuous and compatible. It is to be noted that the T and f do not have coincidence point.

A map $T : X \to X$ is said to be nonexpansive if $d(Tx, Ty) \leq d(x, y)$ for all x, y in X. Ciric [3] investigated a class of nonexpansive type condition and established a result for the existence of fixed points. Chandra et al. [2] generalized the condition of Ciric [3] for single valued as well as for hybrid types of maps and gave the following nonexapansive type condition. Let T be a multivalued map from metric space X to the collection of nonempty subsets of X and f a self map of X,

$$\begin{aligned} (1) \qquad H(Tx, Ty) \leq a(x, y)d(fx, fy) + b(x, y) \max\{d(fx, Tx), d(fy, Ty)\} \\ + c(x, y)[d(fx, Ty) + d(fy, Tx)]. \end{aligned}$$

Where a, b, c are nonnegative functions from $X \times X \to [0, 1)$ such that $\beta = inf_{x,y \in X} b(x, y) > 0$, $\gamma = inf_{x,y \in X} c(x, y) > 0$, and

$$\sup_{x,y \in X} [a(x, y) + b(x, y) + c(x, y)] = 1.$$

Here it is remarkable that taking T, a single-valued map in above condition, the condition given by Ciric [3] is generalized. Using above nonexapansive type condition and concept of weak reciprocal continuity, we prove following result.

Theorem 1. *Let $T : X \to C(X)$ and $f : X \to X$ are weakly reciprocally continuous and nonvacuously compatible maps of a metric space (X, d) satisfying condition (1) and $T(X) \subseteq f(X)$. Then T and f have a coincidence point. Further, if $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.*

Proof. Since $T(X) \subseteq f(X)$, we construct a sequence $\{x_n\}$, $\{fx_n\}$ and $\{Tx_n\}$ as follows:

Choose $x_0 \in X$ and $x_1 \in X$ such that $fx_1 \in Tx_0$. In general, we choose x_{n+1} such that $fx_{n+1} \in Tx_n$. Since f and T are nonvacuously compatible, the sequence $\{x_n\}$ satisfies the compatibility condition i.e., $\{fx_n\}$ and $\{Tx_n\}$ converges to $t \in X$ and $M \in C(X)$, respectively, such that $t \in M$ and $H(Tfx_n, fTx_n) = 0$. Now, f and T are weakly reciprocally continuous and according to the definition of w.r.c either $lim_n fTx_n = fM$ or $lim_n Tfx_n = Tt$.

Case I. Let $lim_n fTx_n = fM$, by compatibility of f and T, we have $H(Tfx_n, fTx_n) = 0$. Hence $lim_n Tfx_n = fM$.

Since $fx_n \in Tx_{n-1} \Rightarrow ffx_n \in fTx_{n-1}$. Furthermore, since $lim_n fTx_{n-1} = fM$, $ffx_n \in fTx_{n-1}$ and f and T are compatible, we have $lim_n ffx_n \in fM$.

Now using (1)

$$H(Tfx_n, Tt) \leq a(x,y)d(ffx_n, ft) + b(x,y)max\ d(ffx_n, Tfx_n), d(ft, Tt) \\ + c(x,y)[d((ffx_n, Tt) + d(ft, Tfx_n)].$$

Making $n \to \infty$

$$H(fM, Tt) \leq b(x,y)d(ft, Tt) + c(x,y)d(fM, Tt), \text{ or}$$
$$H(fM, Tt) \leq \frac{(b(x,y))}{(1 - c(x,y))} d(ft, Tt) = \theta d(ft, Tt).$$

By the condition $sup_{x,y \in X}[a(x,y) + b(x,y) + 2c(x,y)] = 1$, we get

$$\theta = \frac{(b(x,y))}{(1 - c(x,y))} < 1.$$

Now,

$$d(ft, Tt) \leq d(ft, fM) + d(fM, Tt) = d(fM, Tt) \leq H(fM, Tt) \leq \theta d(ft, Tt),$$

hence $d(ft, Tt) = 0 \Rightarrow ft \in Tt$, i.e. t is the coincidence point of f and T. Further, compatibility implies that f and T commute at their coincidence point, i.e. $fTt = Tft$. Taking $ffT = ft$ where t is a coincidence point of f and T, we get $ft \in fTt = Tft$, i.e. ft is a common fixed point of f and T.

Case II. Let $lim_n Tfx_n = Tt$. Compatibility of f and T implies $lim_n fTx_n = Tt$. Since $T(X) \subseteq f(X)$, there exists $z \in X$ such that $fz \in Tt$.

Now using (1)

$$H(Tfx_n, Tz) \leq a(x,y)d(ffx_n, fz) + b(x,y)\max\{d(ffx_n, Tfx_n), d(fz, Tz)\} \\ + c(x,y)[d(ffx_n, Tz) + d(fz, Tfx_n)].$$

Since $fx_n \in Tx_{n-1} \Rightarrow ffx_n \in fTx_{n-1}$. Further, $lim_n fTx_{n-1} = Tt$, $ffx_n \in fTx_{n-1}$ and f and T are compatible implies $lim_n ffx_n \in Tt$.

Hence making $n \to \infty$, we get

$$H(Tt, Tz) \leq b(x,y)d(fz, Tz) + c(x,y)d(Tt, Tz) \text{ or}$$

$$H(Tt, Tz) \leq \frac{(b(x,y))}{(1 - c(x,y))} d(fz, Tz)$$

$$= \theta d(fz, Tz), \quad \text{where } \theta = \frac{(b(x,y))}{(1 - c(x,y))} < 1.$$

Now $d(fz, Tz) \leq d(fz, Tt) + H(Tt, Tz)$ implies $d(fz, Tz) \leq H(Tt, Tz)$, which implies $d(fz, Tz) \leq \theta d(fz, Tz)$.

Hence $d(fz, Tz) = 0$ or $fz \in Tz$, i.e. z is the coincidence point of f and T. Compatibility of f and T implies that they commute at their coincidence point, i.e. $fTz = Tfz$. Again fz is a fixed point of f, i.e. $fz = ffz$ implies $fz = ffz \in fTz = Tfz$, i.e. fz is common fixed point of f and T. ∎

Now we cite following definition (Rhoades et al [13]).

Definition 4. [13] If, for a point x_0 in X, there exists a sequence $\{x_n\} \subset X$ such that $fx_{n+1} \in Tx_n$, $n = 0, 1, 2, ...$ then $O_f(x_0) = fx_n : n = 1, 2$ is an orbit of (T, f) at x_0. A space is called (T, f)-orbitally complete if every Cauchy sequence of the form $\{fx_{n_i} : fx_{n_i} \in Tx_{n_{i-1}})\}$ converges in X.

Now we introduce (T, f)-completeness as follows:

Definition 5. A metric space (X, d) is (T, f)-complete if there exists an orbit of (T, f) at point x_0 for which the Cauchy sequences $\{fx_n\}$ and $\{Tx_n\}$ converges to t and M such that $t \in M \subset X$. From the definition it is clear that (T, f)-completeness with compatibility yield nonvacuous compatibility. Further, following example shows that (T, f) completeness does not guarantee the completeness of the space.

Example 5. Let $X = (0, \infty)$ be endowed with the usual metric and

$$Tx = \left\{ \begin{array}{lll} [0, & 2], & \text{if} \quad x \leq 3 \\ [3, & 4], & \text{if } 3 < x \leq 4 \\ [4, & x], & \text{if} \quad x > 4 \end{array} \right\}, \quad fx = \left\{ \begin{array}{ll} x+1, & \text{if} \quad x < 3 \\ x, & \text{if } 3 < x \leq 4 \\ x+6, & \text{if} \quad x > 4 \end{array} \right\}.$$

For the sequence $\{x_n\} = \{3 + \frac{1}{n}\}$ such that $fx_n \in Tx_{n-1}$, further $fx_n \to 3$, $Tx_n \to [3, 4]$ and $3 \in [3, 4]$ implies that the space X is (T, f)-complete but not complete.

Theorem 2. *Let $T : X \to C(X)$ and $f : X \to X$ be weakly reciprocally continuous and compatible mappings of a (T, f)-complete metric space (X, d) such that $T(X) \subseteq f(X)$ and (1) is satisfied. Then T and f have a coincidence point. If $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.*

Proof. As in [2], conditions $T(X) \subseteq f(X)$ and (1) are enough for the existence of $\{fx_n\}$ and $\{Tx_n\}$ as a Cauchy sequence. Since X is (T, f)-complete, let $lim fx_n = t$ and $lim Tx_n = M$, then $t \in M$. Thus for the sequence $\{x_n\}$ in X, we have $\{Tx_n\}$ and $\{fx_n\}$ converging, respectively, to M and $t \in M$, hence compatibility of f and T implies $H(Tfx_n, fTx_n) = 0$. The rest of the proof is the same as in Theorem 1. ∎

If in place of $C(X)$, we take $CL(X)$, we get the following result as corollary.

Corollary 1. *Let $T : X \to CL(X)$ and $f : X \to X$ be either weakly reciprocally continuous and nonvacuously compatible mappings of a metric space (X, d) or weakly reciprocally continuous and compatible maps of a (T, f)-complete metric space (X, d) such that $T(X) \subseteq f(X)$ and (1) with $sup_{x,y \in X}[a(x, y) + b(x, y) + 2c(x, y)] < 1$ is satisfied. Then T and f have a coincidence point. If $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.*

Remark. In place of nonexpansive type maps, if we take most general contractive condition (Condition 21, Rhoades [12]), we get following corollary. It is remarkable that the following corollary extend and generalize several well known results.

Corollary 2. *Let $T : X \to CL(X)$ and $f : X \to X$ be either weakly reciprocally continuous and nonvacuously compatible maps of a metric space (X, d) or weakly reciprocally continuous and compatible maps of a (T, f)-complete metric space (X, d) such that*

(I) $T(X) \subseteq f(X)$,

(II) $H(Tx, Ty) \leq q \max\{d(fx, fy), d(fx, Tx), d(fy, Ty), 1/2[d(fx, Ty) + d(fy, Tx)]\}$,

where q is a positive number such that $q < 1$. Then T and f have a coincidence point. If $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.

Kamran [5] extended the concept of R-weak commutativity maps of type A_g [11] for multivalued maps and introduced R-weakly commuting mappings of type A_f. Maps $T : X \to CB(X)$ and $f : X \to X$ are R-weakly commuting mappings of type A_f at $x \in X$, if there exists some positive real number R

such that

$$D(ffx, Tfx) \leq RD(fx, Tx).$$

Now we prove some results for R-weak commutative mappings of type A_f.

Theorem 3. *Let $T : X \to C(X)$ and $f : X \to X$ be weakly reciprocally continuous mappings of a (T, f)-complete metric space (X, d) such that $T(X) \subseteq f(X)$, satisfying condition (1). Then T and f have a coincidence point if f and T are R-weakly commuting mappings of type A_f. Further, if $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.*

Proof. For a point $x_0 \in X$, we construct a sequence $\{fx_n\}$ and $\{Tx_n\}$ as follows. Since $T(X) \subseteq f(X)$, we choose $x_1 \in X$ such that $fx_1 \in Tx_0$. If $Tx_0 = Tx_1$, choose $x_2 \in X$ such that $fx_2 \in Tx_1$ and $fx_1 = fx_2$. If $Tx_0 \neq Tx_1$, choose x_2 such that $fx_2 \in Tx_1$ and $d(fx_1, fx_2) \leq H(Tx_0, Tx_1)$. In general, choose $fx_{n+2} \in Tx_{n+1}$ such that $d(fx_{n+1}, fx_{n+2}) \leq H(Tx_n, Tx_{n+1})$. Following the proof of Theorem 2.2 [2], we get $\{fx_n\}$, a Cauchy sequence. In a similar process, we can show that $\{Tx_n\}$ is also a Cauchy sequence. The space X is (T, f)-complete so the sequences $\{fx_n\}$ and $\{Tx_n\}$ converge to t and M, respectively, such that $t \in M$.

Weak reciprocal continuity of f and T implies $lim_n fTx_n = fM$ or $lim_n Tfx_n = Tt$. Let $lim_n fTx_n = fM$. Since $ffx_n \in fTx_{n-1}$ and $lim_n fTx_{n-1} = fM$, $lim_n ffxn \in fM$. f and T are R-weakly commuting of type A_f, hence $d(ffx_n, Tfx_n) \leq Rd(fx_n, Tx_n)$, where R is a positive real number. Letting $n \to \infty$, we get $lim_n Tfx_n \in fM$. The rest of the proof follows from Theorem 1. ∎

Further, for $lim_n Tfx_n = Tt$, R commutativity of type A_f of f and T yields $lim_n ffx_n = Tt$, the rest is same as in Theorem 1.

In Theorem 3, if we take $CL(X)$ in place of $C(X)$, we get the following result as a corollary:

Corollary 3. *Let $T : X \to CL(X)$ and $f : X \to X$ are weakly reciprocally continuous mappings of a (X, d) such that $T(X) \subseteq f(X)$ and condition (1) is satisfied. Then T and f have a coincidence point if T and f are R-weakly commuting mappings of type A_f. Further, if $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.*

Proof. The only change in the construction of the sequence in the proof of Theorem 3 is that in place of $d(fx_{n+1}, fx_{n+2}) \leq H(Tx_n, Tx_{n+1})$ we take $d(fx_{n+1}, fx_{n+2}) \leq \lambda H(Tx_n, Tx_{n+1})$, where $\lambda > 1$ and $\lambda\delta < 1$. ∎

Changing the contraction condition in Theorem 3, we get the following result as a corollary.

Corollary 4. *Let $T : X \to C(X)$ or $T : X \to CL(X)$ and $f : X \to X$ are weakly reciprocally continuous mappings of a (T, f)-complete metric space (X, d) such that*

(I) $T(x) \subseteq f(X)$,

(II) $H(Tx, Ty) \leq q \max\{d(fx, fy), d(fx, Tx), d(fy, Ty), \frac{1}{2}[d(fx, Ty) + d(fy, Tx)]\}$,

where q is a positive number $q < 1$. Then T and f have a coincidence point if f and T are R-weakly commuting of type A_f. Further if $fft = ft$ for some $t \in C(T, f)$ then f and T have a common fixed point.

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