Research Article

Wadei F. Al-Omeri*, O. H. Khalil, and A. Ghareeb

Degree of \((L, M)\)-fuzzy semi-precontinuous and \((L, M)\)-fuzzy semi-preirresolute functions

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Abstract: The aim of this paper is to present the degree of semi-preopenness, semi-precontinuity, and semi-preirresoluteness for functions in \((L, M)\)-fuzzy pretopology with the help of implication operation and \((L, M)\)-fuzzy semi-preopen operator introduced by [Ghareeb A., \(L\)-fuzzy semi-preopen operator in \(L\)-fuzzy topological spaces, Neural Comput. & Appl., 2012, 21, 87–92]. Further, we generalize the properties of semi-preopenness, semi-precontinuity and semi-preirresoluteness to \((L, M)\)-fuzzy pretopological setting relying on graded concepts. Also, we discuss their relationships with the corresponding degrees of semi-precompactness, semi-preconnectedness and semi-preseparation axioms.

Keywords: \((L, M)\)-fuzzy pretopology, \((L, M)\)-fuzzy semi-preopen operator, \((L, M)\)-fuzzy semi-precontinuous function, \((L, M)\)-fuzzy semi-preirresolute function

MSC: 03E72, 54A40, 54C20

1 Introduction

Study of fuzzy topological spaces was pioneered by Chang [2] in 1968. Chang’s fuzzy topology is a crisp subset of the family of \(I^X\) (the collection of all fuzzy sets defined on a non-empty set \(X\)) that fulfills the same conditions as in topology. This notion was extended to the \(L\)-fuzzy setting by Goguen [3], which currently holds the name “\(L\)-topological spaces”. However, in a completely different direction, Höhle [4] presented the notion of a fuzzy topology being viewed as an \(L\)-subset of a powerset \(2^X\). Ying [5] studied Höhle’s topology from a logical point of view and gave it the name “fuzzifying topology”. Šostak [6] and Kubiak [7] independently extended Höhle’s fuzzy topology to \(I\)-subsets of \(I^X\). Subsequently, Ghanim et al. [8] introduced the degree of supra openness of a fuzzy set on \(X\) which is now called \(I\)-fuzzy pretopological spaces.

The development of fuzzy topology was accompanied with many topological properties that were endowed with degrees, like connectedness, separability, compactness, and filter convergence. In 1988, Šostak [9, 10] introduced the degree of connectedness via the level \(I\)-topological spaces. Yue and Fang [11] introduced the connectivity for the whole \(L\)-fuzzy topological space. In [12], Pang defined the openness, closeness, and continuity degrees of functions in \(L\)-fuzzifying topology and generalized some results in general topology to \(L\)-fuzzifying topology. Recently, Liang and Shi [13] introduced the degrees to which a function is continuous, open or closed by using implication operation, and extended most of their elementary properties in topological spaces to \((L, M)\)-fuzzy topological spaces by means of graded concepts. In 2009, Shi [14] presented

*Corresponding Author: Wadei F. Al-Omeri: Department of Mathematics, Al-Balqa Applied University, Salt 19117, Jordan;
E-mail: wadeimoon1@hotmail.com, wadeialomeri@yahoo.com

O. H. Khalil: Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt,
Current address: Department of Mathematics, Faculty of Science in Al-Zulfi, Majmaah University, 1952, Saudi Arabia;
E-mail: o.khalil@mu.edu.sa

A. Ghareeb: Department of Mathematics, Faculty of science, South Valley University, Qena, Egypt,
Current address: Department of Mathematics, College of science, Al-Baha University, Al-Baha, Saudi Arabia;
E-mail: a.ghareeb@sci.svu.edu.eg, nasserfuzt@hotmail.com

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separatedness and connectedness degrees of $L$-subset based on $L$-fuzzy closure operator. Recently, Shi [15] used $L$-fuzzy semiopen and $L$-fuzzy preopen operators in $L$-fuzzy pretopological spaces as a tool to measure the semiopenness and preopenness degrees of an $L$-subset, respectively. Based on Shi’s operator, the notion of semicompactness is introduced and characterized in [16, 17]. Moreover, the preconnectedness degree presented by Ghareeb [18], was defined using $L$-fuzzy preopen operator. In 2012, Ghareeb [1] used $L$-fuzzy preopen operator to present a new operator to measure the degree of semi-preopenness of an $L$-subset. The notions of semi-precompactness degree, semi-preseparatedness degree and semi-preconnectedness degree of an $L$-subset are discussed through graded concepts. Furthermore, we study the relationship with semi-precompactness, semi-preconnectedness, and some semipre-separation axioms degree in $(L, M)$-fuzzy pretopological spaces.

The main purpose of this paper is to present the degree of semi-preopenness, semi-precontinuity and semi-preirresoluteness for functions in $(L, M)$-fuzzy pretopological spaces relied on $(L, M)$-fuzzy semi-preopen operator [1] and implication operation. The characteristic properties of the new notions will be discussed through graded concepts. Furthermore, we study the relationship with semi-precompactness, semi-preconnectedness, and some semipre-separation axioms degree in $(L, M)$-fuzzy pretopological spaces.

2 Preliminaries

Throughout this paper, unless otherwise stated, both $L$ and $M$ refer to completely distributive De Morgan algebras and $X$ is a nonempty set. The smallest and greatest members in $L$ and $M$ are denoted by $0_L$, $1_L$ and $0_M$, $1_M$, respectively. For any $a$, $b \in L$, we say that the element $a$ is wedge below $b$ [21], in symbols $a \triangleright b$, if for all $A \subseteq L$, $\bigvee A \geq b$ it follows that $c \triangleright a$ for $c \in A$. The complete lattice $L$ is said to be a completely distributive lattice iff $b = \bigvee_{a \leq b} a$ for all $b \in L$. The element $b \in L$ is called co-prime if $a \leq b \vee c$ it follows that $a \leq b$ or $a \leq c$.

The set of all non-zero co-prime members in $L$ is denoted by $P(L)$. By $L^X$, we denote the family of all $L$-subsets defined on $X$. The smallest and the greatest $L$-subsets in $L^X$ are denoted by $0_{L^X}$ and $1_{L^X}$, respectively. Further, $2^X$ refers to the family of all finite sub-families of $B$ of $L^X$. It is clear that $L^X$ forms a completely distributive De Morgan algebra. Moreover, $\{xa\}_{a \in P(L)}$ is a family of non-zero co-primes in $L^X$. The implication operation $\Rightarrow : M \times M \rightarrow M$ is defined on any completely distributive De Morgan algebra $M$ by $a \Rightarrow b = \bigvee_{a \leq c \leq b} c$. Moreover, the operation “$\Leftrightarrow$” is defined also on $M$ based on “$\Rightarrow$” by $a \Leftrightarrow b = (a \Rightarrow b) \land (b \Rightarrow a)$.

An $L$-fuzzy inclusion [22, 23] on $X$ is a mapping $\tilde{\cap} : L^X \times L^X \rightarrow L$ defined by the equality $\tilde{\cap}(G_1, G_2) = \bigwedge_{x \in X} \{G_1(x) \vee G_2(x)\}$.

It is customary to denote the $L$-fuzzy inclusion $\tilde{\cap}(G_1, G_2)$ by the symbol $[G_1 \tilde{\cap} G_2]$.

**Lemma 2.1.** [24] For any completely distributive lattice $(M, \lor, \land)$ and an implication operation $\Rightarrow$ defined on $M$, we have:

1. $(a \Rightarrow b) \geq c \iff a \land c \leq b$.
2. $a \leq b \iff a \Rightarrow b = 1_M$.
3. $a \Rightarrow (b \Rightarrow c) = (a \land b) \Rightarrow c$.
4. $(c \Rightarrow a) \land (a \Rightarrow b) \leq c \Rightarrow b$.
5. $c \Rightarrow a \leq (a \Rightarrow b) \Rightarrow (c \Rightarrow b)$.
6. $a \Rightarrow \bigwedge_{i \in I} a_i = \bigwedge_{i \in I} (a \Rightarrow a_i)$, hence $a \Rightarrow b \leq a \Rightarrow c$ whenever $b \leq c$.
(7) \( \bigvee_{i \in I} a_i \mapsto b = \bigwedge_{i \in I} (a_i \mapsto b) \), hence \( a \mapsto c \geq b \mapsto c \) whenever \( a \leq b \).

Where \( a, b, c \in M, \{a_i\}_{i \in I}, \) and \( \{b_i\}_{i \in I} \subseteq M \).

**Lemma 2.2.** [25] Let \( f : X \rightarrow Y \) be a mapping. Then for each \( \mathcal{H} \subseteq L^Y \), we have

\[
\bigwedge_{y \in Y} \left\{ f_L^{-1}(G)(y) \lor \bigvee_{H \in \mathcal{H}} H(y) \right\} = \bigwedge_{x \in X} \left\{ G(x) \lor \bigvee_{H \in \mathcal{H}} f_L^{-1}(H)(x) \right\}.
\]

**Definition 2.3.** For any non-empty set \( X \), a function \( \mathcal{T} : L^X \rightarrow M \) which satisfies the following statements:

1. \( \mathcal{T}(1) = \mathcal{T}(0) = 1_M \).
2. \( \mathcal{T} \left( \bigvee_{i \in I} G_i \right) \geq \bigwedge_{i \in I} \mathcal{T}(G_i) \) for all \( \{G_i\}_{i \in I} \subseteq L^X \),

is called an \((L, M)\)-fuzzy pretopology on \( X \) [8]. The pair \((X, \mathcal{T})\) is called \((L, M)\)-fuzzy pretopological space \(((L, M)-fpts, \text{for short})\). An \((L, M)\)-fuzzy pretopology \( \mathcal{T} \) is said to be \((L, M)\)-fuzzy topology \(((L, M)-ft, \text{for short})\) on \( X \) [6, 7, 26, 27] if it satisfies the following additional statement:

3. \( \mathcal{T}(G_1 \land G_2) \geq \mathcal{T}(G_1) \land \mathcal{T}(G_2) \) for all \( G_1, G_2 \in L^X \).

The pair \((X, \mathcal{T})\) is called \((L, M)\)-fuzzy topological space \(((L, M)-fts, \text{for short})\). The gradation of openness and closeness of an \( L \)-subset \( G \) is given by \( \mathcal{T}(G) \) and \( \mathcal{T}^*(G) \) respectively, where \( \mathcal{T}^*(G) = \mathcal{T}(G') \). A mapping \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) is called an \((L, M)\)-fuzzy continuous if \( \mathcal{T}_1(f_L^{-1}(H)) \geq \mathcal{T}_2(H) \) for all \( H \in L^Y \).

**Definition 2.4.** [27] Let \((X, \mathcal{T})\) be an \((L, M)\)-pfts and \( G \in L^X \). An \((L, M)\)-fuzzy preopen operator \( \mathcal{P} : L^X \rightarrow M \) is given by

\[
\mathcal{P}(G) = \bigwedge_{x \in G} \bigvee_{x \in G} \left\{ \mathcal{T}(G_1) \land \bigwedge_{y \in G_1} \bigwedge_{y \in G_2} (\mathcal{T}(G_2))' \right\}.
\]

The value \( \mathcal{P}(G) \) interprets the degree to which \( G \) is preopen and \( \mathcal{P}^*(G) = \mathcal{P}(G') \) interprets the degree to which \( G \) is a preclosed \( L \)-subset.

**Definition 2.5.** [1] Let \((X, \mathcal{T})\) be an \((L, M)\)-pfts and \( G \in L^X \). An \((L, M)\)-fuzzy semi-preopen operator \( \mathcal{T}_{sp} : L^X \rightarrow M \) is given by

\[
\mathcal{T}_{sp}(G) = \bigvee_{G_1 \in G} \left\{ \mathcal{P}(G_1) \land \bigwedge_{x \in G_1} \bigwedge_{x \in G_2} (\mathcal{T}(G_2))' \right\},
\]

where \( \mathcal{T}_{sp}(G) \) and \( \mathcal{T}_{sp}(G') \) can be regarded as the semi-preopenness and the semi-precloseness degree of an \( L \)-subset \( G \), respectively.

**Lemma 2.6.** [19] For any \((L, M)\)-pfts \((X, \mathcal{T})\), an \((L, M)\)-fuzzy semi-preopen operator \( \mathcal{T}_{sp} \) verifies the following statements:

1. \( \mathcal{T}_{sp}(\mathcal{T}_1) = \mathcal{T}_{sp}(\mathcal{T}_2) = 1_M \).
2. \( \mathcal{T}_{sp}(\bigvee_{i \in I} G_i) \geq \bigwedge_{i \in I} \mathcal{T}_{sp}(G_i) \) for any \( \{G_i\}_{i \in I} \subseteq L^X \).

**Theorem 2.7.** [28] Let \((X, \mathcal{T})\) be an \((L, M)\)-pfts and \( \mathcal{P} \) be the corresponding \((L, M)\)-fuzzy preopen operator. Then \( \mathcal{T}(G) \leq \mathcal{P}(G) \) for any \( G \in L^X \).

**Corollary 2.8.** For any \((L, M)\)-pfts \((X, \mathcal{T})\), if \( \mathcal{T}_{sp} \) is the corresponding \((L, M)\)-fuzzy semi-preopen operator, then \( \mathcal{T}(G) \leq \mathcal{T}_{sp}(G) \) for any \( G \in L^X \).
Definition 2.9. [1] Let \((X, \mathcal{T}_1)\) and \((Y, \mathcal{T}_2)\) be two \((L, M)\)-pfts. The mapping \(f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)\) is said to be:

1. \((L, M)\)-fuzzy semi-precontinuous mapping iff \(\mathcal{T}_1(H) \subseteq (\mathcal{T}_2)_\text{sp}(f^{-1}_L(H))\) for any \(H \in L^Y\).
2. \((L, M)\)-fuzzy semi-preirresolute mapping iff \((\mathcal{T}_2)_\text{sp}(H) \subseteq (\mathcal{T}_1)_\text{sp}(f^{-1}_L(H))\) for any \(H \in L^Y\).

Corollary 2.10. Let \(f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)\) be a mapping between two \((L, M)\)-pfts \((X, \mathcal{T}_1)\) and \((Y, \mathcal{T}_2)\). Then:

1. \(f\) is \((L, M)\)-fuzzy semi-precontinuous iff \(\mathcal{T}_1'(H) \subseteq (\mathcal{T}_2)_\text{sp}(f^{-1}_L(H))\) for all \(H \in L^Y\).
2. \(f\) is \((L, M)\)-fuzzy semi-preirresoluteiff \((\mathcal{T}_2)_\text{sp}(H) \subseteq (\mathcal{T}_1)_\text{sp}(f^{-1}_L(H))\) for all \(H \in L^Y\).

Theorem 2.11. Let \(f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)\) be a mapping between two \((L, M)\)-pfts \((X, \mathcal{T}_1)\) and \((Y, \mathcal{T}_2)\). Then:

1. \(f\) is \((L, M)\)-fuzzy continuous, then \(f\) is \((L, M)\)-fuzzy semi-precontinuous.
2. \(f\) is \((L, M)\)-fuzzy semi-preirresolute, then \(f\) is \((L, M)\)-fuzzy semi-precontinuous.

Proof. (1) If \(f\) is \((L, M)\)-fuzzy continuous mapping, then \(\mathcal{T}_2(H) \subseteq \mathcal{T}_1(f^{-1}_L(H))\) for any \(H \in L^Y\). By using Corollary 2.8, we have

\[ \mathcal{T}_2(H) \subseteq \mathcal{T}_1(f^{-1}_L(H)) \subseteq (\mathcal{T}_2)_\text{sp}(f^{-1}_L(H)), \]

for any \(H \in L^Y\). Hence, \(f\) is an \((L, M)\)-fuzzy semi-precontinuous.

(2) If \(f\) is \((L, M)\)-fuzzy semi-preirresolute mapping, then \((\mathcal{T}_2)_\text{sp}(H) \subseteq (\mathcal{T}_1)_\text{sp}(f^{-1}_L(H))\) for all \(H \in L^Y\). Based on Corollary 2.8, we get

\[ (\mathcal{T}_2)_\text{sp}(H) \subseteq (\mathcal{T}_1)_\text{sp}(f^{-1}_L(H)), \]

for any \(H \in L^Y\). Hence \(f\) is an \((L, M)\)-fuzzy semi-precontinuous.

Definition 2.12. A mapping \(f : (X, \mathcal{T}) \rightarrow (Y, \mathcal{T}_2)\) between two \((L, M)\)-pfts’s \((X, \mathcal{T}_1)\) and \((Y, \mathcal{T}_2)\) is said to be an \((L, M)\)-fuzzy semi-preopen mapping iff \(\mathcal{T}_1(G) \subseteq (\mathcal{T}_2)_\text{sp}(f^{-1}_L(G))\) for all \(G \in L^X\).

Definition 2.13. [1] An \((L, M)\)-fuzzy semi-preclosure operator on \(X\) is a function \(\text{spCl} : L^X \rightarrow M^{P(L^X)}\) satisfying the following statements:

1. \(\text{spCl}(G)(x_a) = \bigwedge_{b \in G} \text{spCl}(G)(x_b)\) for all \(x_a \in P(L^X)\).
2. \(\text{spCl}(0_L)(x_a) = 0_M\) for all \(x_a \in P(L^X)\).
3. \(\text{spCl}(G)(x_a) = 1_M\) for all \(x_a \subseteq G\).
4. For any \(b \in M_0, (\text{spCl}(\vee(\text{spCl}(G)))_{[b]})_{[b]} \subseteq (\text{spCl}(G))_{[b]}\).

The value \(\text{spCl}(G)(x_a)\) represents the degree to which \(x_a\) belongs to the semi-preclosure of an \(L\)-subset \(G\).

Theorem 2.14. [1] If \(\mathcal{T}_\text{sp}\) is an \((L, M)\)-fuzzy semi-preopen operator on \(X\) and \(\text{spCl}\) is an \((L, M)\)-fuzzy semi-preclosure operator, then we have

\[ \text{spCl}(G)(x_a) = \bigwedge_{x_a \subseteq G} (\mathcal{T}_\text{sp}(G'_1))^{'}, \]

for all \(x_a \in P(L^X)\) and \(G \in L^X\).

Definition 2.15. An \((L, M)\)-fuzzy quasi pre-neighborhood system on \(X\) is a family \(\text{spQ} = \{\text{spQ}_{x_a} \mid x_a \in P(L^X)\}\) of mappings \(\{\text{spQ}_{x_a} : L^X \rightarrow M\}\) which satisfies the following axioms:

1. \(\text{spQ}_{x_a}(Q_{x_a}) = 0_M\).
2. \(\text{spQ}_{x_a}(G) \neq 0_M\), then \(x_a \not\subseteq G\).
3. \(\text{spQ}_{x_a}(G_1 \wedge G_2) \leq \text{spQ}_{x_a}(G_1) \wedge \text{spQ}_{x_a}(G_2)\).
Definition 2.18. Let \( \mathcal{T}_{sp} \) be the corresponding \((L,M)\)-fuzzy semi-preopen operator, and \( spQ^{\mathcal{T}_{sp}} = \{ spQ_{x_a} \mid x_a \in P(L^X) \} \) be the \((L,M)\)-fuzzy quasi semi-preneighborhood system induced by \( \mathcal{T}_{sp} \).

Theorem 2.16. Let \((X,T)\) be an \((L,M)\)-pfts, \( \mathcal{T}_{sp} \) be the corresponding \((L,M)\)-fuzzy semi-preopen operator, and \( spQ^{\mathcal{T}_{sp}} = \{ spQ_{x_a} \mid x_a \in P(L^X) \} \) be the \((L,M)\)-fuzzy quasi semi-preneighborhood system induced by \( \mathcal{T}_{sp} \). If the mapping \( spC^{\mathcal{T}_{sp}} : L^X \rightarrow M^{P(L^X)} \) is defined by

\[
spC^{\mathcal{T}_{sp}}(x_a) = \left( spQ_{x_a}(G') \right),
\]

then \( spC^{\mathcal{T}_{sp}} \) is an \((L,M)\)-fuzzy semi-preclosure operator on \( X \).

Definition 2.17. An \((L,M)\)-fuzzy semi-preneighborhood system on \( X \) is the family \( spN = \{ spN_{x_a} \mid x_a \in P(L^X) \} \) of mappings \( spN_{x_a} : L^X \rightarrow M \) which satisfy the following statements:

1. \( spN_{x_a}(\emptyset) = 0_M \).
2. \( spN_{x_a}(G) \neq 0_M \), then \( x_a \notin G \).
3. \( spN_{x_a}(G_1 \cap G_2) \leq spN_{x_a}(G_1) \land spN_{x_a}(G_2) \).
4. \( spN_{x_a}(G) = \bigvee_{x_a \in G} spN_{x_b}(G_1) \).

Definition 2.18. An \((L,M)\)-fuzzy semi-preinterior operator on \( X \) is a mapping \( spI : L^X \rightarrow M^{P(L^X)} \) which satisfy the following statements:

1. \( spI(G(x_a)) = \bigwedge_{b \subseteq a} spI(G(x_b)) \) for all \( x_a \in P(L^X) \).
2. \( spI(\mathbb{1}_X)(x_a) = 1_M \) for all \( x_a \in P(L^X) \).
3. \( spI(G(x_a)) = 0_M \) for all \( x_a \notin G \).
4. For any \( b \in M_0 \), \( spI(b) \subseteq (spI(\bigvee spI(G)))(b) \).

The value \( spI(G(x_a)) \) represents the degree to which \( x_a \) belongs to the semi-preinterior of \( G \).

Theorem 2.19. Let \((X,T)\) be an \((L,M)\)-pfts and \( spN^T = \{ spN_{x_a}^T \mid x_a \in P(L^X) \} \) be the corresponding \((L,M)\)-fuzzy semi-preneighborhood system. If the functions \( spI^{T}_{sp} : L^X \rightarrow M^{P(L^X)} \) is defined by

\[
spI^{T}_{sp}(G(x_a)) = spN_{x_a}^{T}(G),
\]

then \( spI^{T}_{sp} \) is an \((L,M)\)-fuzzy semi-preinterior operator on \( X \).

Definition 2.20. \cite{19} For an \((L,M)\)-pfts \((X,T)\) and \( G \in L^X \), let

\[
spCon(G) = \bigwedge_{G_1, G_2 \subseteq L^X} \left\{ \bigvee_{x_a \in G_1} spCl(G_2)(x_a) \cup \bigvee_{y_b \in G_2} spCl(G_1)(y_b) \right\}.
\]

Then \( spCon(G) \) is called the degree of fuzzy semi-preconnectedness of an \( L \)-subset \( G \).

Theorem 2.21. \cite{19} Let \((X,T)\) be an \((L,M)\)-pfts and \( G \in L^X \). Then

\[
spCon(G) = \bigwedge_{G_1, G_2 \subseteq L^X} \left\{ (\mathcal{T}_{sp}(G_1))' \cup (\mathcal{T}_{sp}(G_2))' \right\}.
\]

In \cite{1}, Ghareeb introduced the notion of semi-precompactness in \((L,M)\)-fuzzy pretopology based on the \((L,M)\)-fuzzy semi-preopen operator. In the following definitions, we define the degree
of semi-precompactness, semipre-T₁, and semipre-T₂ in (L, M)-fuzzy pretopology by using the implication operation.

**Definition 2.22.** Let (X, τ) be an (L, M)-pfts, L = M, and

\[ \text{spcom}_{\bar{\text{sp}}} (G) = \bigwedge_{A \subseteq \bar{L}^{X}} \left\{ \tau_{\bar{\text{sp}}}(A) \rightarrow \left( \bigcap_{G \subseteq \bar{A}} \left[ G \subseteq \bigvee B \right] \right) \right\} \]

\[ = \bigwedge_{A \subseteq \bar{L}^{X}} \left\{ \bigwedge_{G \in A} \left( \tau_{\bar{\text{sp}}}(G) \rightarrow \bigwedge_{x \in X} \left( G \subseteq \bigvee G_{1} \right)(x) \right) \right\}, \]

for each \( G \in L^{X} \). Then \( \text{spcom}_{\bar{\text{sp}}} (G) \) is called the semi-precompactness degree of \( G \) with respect to \( \tau_{\bar{\text{sp}}} \).

In this paper, we extend **Semipre-T₁** and **Semipre-T₂** in our sense.

**Definition 2.23.** If (X, τ) be an (L, M)-pfts, then

1. The degree semipre-T₁(X, τ) to which \( \tau \) is semipre-T₁ is given by:

\[ \text{semipre-T₁}(X, \tau) = \bigwedge_{\alpha_{1} \leq \alpha_{2}} \bigvee \tau_{\bar{\text{sp}}} (G'). \]

2. The degree semipre-T₂(X, τ) to which \( \tau \) is semipre-T₂ is given by:

\[ \text{semipre-T₂}(X, \tau) = \bigwedge_{\alpha_{1} \leq \alpha_{2}} \bigvee \left\{ \tau_{\bar{\text{sp}}}(G') \land \tau_{\bar{\text{sp}}}(G'') \mid \alpha_{1} \leq G_{1} \geq G_{2} \geq \alpha_{2} \right\}. \]

### 3 Semi-preopenness, semi-precontinuity and semi-preirresoluteness degree of mappings based on (L, M)-fuzzy semi-preopen operator

We introduce semi-preopenness, semi-precontinuity, and semi-preirresoluteness degree of mappings in (L, M)-fuzzy pretopology. Moreover, we characterize it by using (L, M)-fuzzy quasi-semipreneighborhood systems, (L, M)-fuzzy semi-semipreneighborhood systems, (L, M)-fuzzy semi-preinterior operators and (L, M)-fuzzy semipre-closure operators.

**Definition 3.1.** If \( f : (X, \tau_{1}) \rightarrow (Y, \tau_{2}) \) is a mapping between two (L, M)-pfts (X, \( \tau_{1} \)) and (Y, \( \tau_{2} \)), then:

1. The degree \( \text{SPc}(f) \) to which \( f \) is semi-precontinuous is given by

\[ \text{SPc}(f) = \bigwedge_{H \in L^{Y}} \left\{ \tau_{1}(H) \rightarrow (\tau_{\bar{\text{sp}}}f_{L}^{-}\cdot H) \right\}. \]

2. The degree \( \text{SPo}(f) \) to which \( f \) is semi-preopen is given by

\[ \text{SPo}(f) = \bigwedge_{G \in L^{X}} \left\{ (\tau_{\bar{\text{sp}}}f_{L}^{-}\cdot G) \rightarrow (\tau_{\bar{\text{sp}}}f_{L}^{-}\cdot G) \right\}. \]

3. The degree \( \text{SPI}(f) \) to which \( f \) is semi-preirresolute is given by

\[ \text{SPI}(f) = \bigwedge_{H \in L^{Y}} \left\{ (\tau_{\bar{\text{sp}}}f_{L}^{-}\cdot H) \rightarrow (\tau_{\bar{\text{sp}}}f_{L}^{-}\cdot H) \right\}. \]
**Definition 3.2.** If \( f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2) \) is a bijective mapping, then the degree **Semipre-Hom\((f)\)** to which \( f \) is semi-prehomomorphism, is defined by

\[
\text{Semipre-Hom}\((f)\) = \text{SPI}(f) \land \text{SPO}(f),
\]

and

\[
\text{Semipre-Hom}\((f)\) = \text{SPc}(f) \land \text{SPO}(f).
\]

**Remark 3.3.** By Lemma 2.1 (2), if \( \text{SPc}(f) = 1_M \), we have \( (\mathcal{T}_{sp})_1(f^-_L(H)) \leq \mathcal{T}_2(H) \) for any \( H \in L^Y \). This is just the definition of semi-precontinuous function between two \((L, M)\)-fpts. Similarly for the cases \( \text{SPO}(f) = 1_M \) and \( \text{SPI}(f) = 1_M \). If \( id : (X, \mathcal{T}_1) \to (X, \mathcal{T}_1) \) is the identity function, then \( \text{SPI}(id) = \text{SPO}(id) = \text{Semipre-Hom}(id) = 1_M \). Moreover, if \( L = \{0_L, 1_L\} \), then we get the same degrees but between \( M \)-fuzzifying pretopological spaces.

The following corollaries are direct results from Definition 3.4 and Corollary 2.10.

**Corollary 3.4.** If \( f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2) \) is a mapping between two \((L, M)\)-fpts’s \((X, \mathcal{T}_1) \) and \((Y, \mathcal{T}_2) \), then:

1. The degree \( \text{SPc}(f) \) to which \( f \) is semi-precontinuous, is given by

\[
\text{SPc}(f) = \bigwedge_{H \in L^Y} \{ (\mathcal{T}_{sp})_2(H) \to (\mathcal{T}_{sp})_1(f^-_L(H)) \}.
\]

2. The degree \( \text{SPI}(f) \) to which \( f \) is semi-preirresolute, is given by

\[
\text{SPI}(f) = \bigwedge_{H \in L^Y} \{ (\mathcal{T}_{sp})_2(H) \to (\mathcal{T}_{sp})_1(f^-_L(H)) \}.
\]

**Definition 3.5.** If \( f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2) \) is a mapping between two \((L, M)\)-fpts’s \((X, \mathcal{T}_1) \) and \((Y, \mathcal{T}_2) \), then the degree \( \text{SPc}(f) \) to which \( f \) is semi-preclosed, is defined by

\[
\text{SPc}(f) = \bigwedge_{H \in L^Y} \{ (\mathcal{T}_{sp})_1(H) \to (\mathcal{T}_{sp})_2(f^-_L(H)) \}.
\]

**Theorem 3.6.** Let \((X, \mathcal{T}_1), (Y, \mathcal{T}_2)\) and \((Z, \mathcal{T}_3)\) be \((L, M)\)-fpts’s, \( f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2) \) and \( g : (Y, \mathcal{T}_2) \to (Z, \mathcal{T}_3) \) be two mappings. Then the following properties are satisfied:

1. \( \text{SPI}(f) \land \text{SPI}(g) \leq \text{SPI}(g \circ f) \).
2. \( \text{SPO}(f) \land \text{SPO}(g) \leq \text{SPO}(g \circ f) \).
3. \( \text{SPc}(f) \land \text{SPI}(g) \leq \text{SPc}(g \circ f) \).

**Proof.** Proving (1) is enough since the other results can be proved similarly. By Lemma 2.1 and Definition 3.4, we have

\[
\text{SPI}(f) \land \text{SPI}(g) = \bigwedge_{H \in L^Y} \{ (\mathcal{T}_{sp})_2(H) \to (\mathcal{T}_{sp})_1(f^-_L(H)) \} \land \bigwedge_{W \in L^Z} \{ (\mathcal{T}_{sp})_3(W) \to (\mathcal{T}_{sp})_2(g^-_L(W)) \}
\]

\[
\leq \bigwedge_{W \in L^Z} \{ (\mathcal{T}_{sp})_2(g^-_L(W)) \to (\mathcal{T}_{sp})_1(f^-_L(g^-_L(W))) \} \land \bigwedge_{W \in L^Z} \{ (\mathcal{T}_{sp})_3(W) \to (\mathcal{T}_{sp})_2(g^-_L(W)) \}
\]

\[
= \bigwedge_{W \in L^Z} \{ (\mathcal{T}_{sp})_2(g^-_L(W)) \to (\mathcal{T}_{sp})_1((g_L \circ f_L)^-)(W)) \} \land \{ (\mathcal{T}_{sp})_3(W) \to (\mathcal{T}_{sp})_2(g^-_L(W)) \}
\]

\[
\leq \bigwedge_{W \in L^Z} \{ (\mathcal{T}_{sp})_3(g^-_L(W)) \to (\mathcal{T}_{sp})_1((g_L \circ f_L)^-)(W)) \}
\]

Thus we completed the proof. \( \square \)
The following corollaries are direct results from Definition 3.2 and Theorem 3.6.

**Corollary 3.7.** Let \( f : (X, \mathcal{I}_1) \rightarrow (Y, \mathcal{I}_2) \) and \( g : (Y, \mathcal{I}_2) \rightarrow (Z, \mathcal{I}_3) \) be two mappings between three \((L, M)\)-fpts's \((X, \mathcal{I}_1), (Y, \mathcal{I}_2), \text{and} (Z, \mathcal{I}_3)\). If \( f \) and \( g \) are two bijective mappings, then the following statement holds:

\[
\text{Semipre-Hom}(f) \wedge \text{Semipre-Hom}(g) \leq \text{Semipre-Hom}(f \circ g).
\]

**Theorem 3.8.** Let \( f : (X, \mathcal{I}_1) \rightarrow (Y, \mathcal{I}_2) \) and \( g : (Y, \mathcal{I}_2) \rightarrow (Z, \mathcal{I}_3) \) be two \((L, M)\)-fpts's on \( X \) and \( Y \). If \( f \) is a surjective function, then the following statements hold:

1. \( \text{SPo}(g \circ f) \wedge \text{SPI}(f) \leq \text{SPo}(g) \).
2. \( \text{SPcl}(g \circ f) \wedge \text{SPI}(f) \leq \text{SPcl}(g) \).

**Proof.** Proving (1) is enough since the other statements are similarly proved. Since \( f \) is surjective mapping, we obtain \( (g_L \circ f_L)^{-1}(f_L^-(H)) = g_L^-(H) \) for any \( H \in L^Y \). Based on Lemma 2.1 (4), we have

\[
\text{SPo}(g \circ f) \wedge \text{SPI}(f) = \bigwedge_{G \in L^X} \left\{ (\mathcal{I}_{sp}^1(G) \Rightarrow (\mathcal{I}_{sp}^3((g_L \circ f_L)^{-1}(G)))) \right\} \wedge \bigwedge_{H \in L^Y} \left\{ (\mathcal{I}_{sp}^2(H) \Rightarrow (\mathcal{I}_{sp}^3(f_L^-(H)))) \right\}
\]

Similarly, it can be verified that:

**Theorem 3.9.** Let \( f : (X, \mathcal{I}_1) \rightarrow (Y, \mathcal{I}_2) \) and \( g : (Y, \mathcal{I}_2) \rightarrow (Z, \mathcal{I}_3) \) be two \((L, M)\)-fpts's on \( X \), \( Y \), and \( Z \). If \( f \) is a surjective mapping, then the following statements hold:

1. \( \text{SPo}(g \circ f) \wedge \text{SPI}(g) \leq \text{SPo}(f) \).
2. \( \text{SPcl}(g \circ f) \wedge \text{SPI}(g) \leq \text{SPcl}(f) \).

**Theorem 3.10.** Let \( f : (X, \mathcal{I}_1) \rightarrow (Y, \mathcal{I}_2) \) be a bijective mapping between \((L, M)\)-fpts's on \( X \) and \( Y \), then

1. \( \text{SPI}(f) = \bigwedge_{G \in L^X} \left\{ (\mathcal{I}_{sp}^2(f_L^-(G)) \Rightarrow (\mathcal{I}_{sp}^1(G))) \right\} \).
2. \( \text{SPo}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{I}_{sp}^1(f_L^-(H)) \Rightarrow (\mathcal{I}_{sp}^2(H))) \right\} \).
3. \( \text{SPI}(f^{-1}) = \text{SPo}(f) = \text{SPcl}(f) \).

**Proof.** Proving (1) and (3) is enough since condition (2) is proved similarly to (1).

(1) Since \( f : X \rightarrow Y \) is a bijective mapping, we have \( f_L^{-1}(f_L^-(G)) = G \), for all \( G \in L^X \), and \( f_L^{-1}(f_L^-(H)) = H \), for all \( H \in L^Y \). The following inequalities are obtained

\[
\bigwedge_{G \in L^X} \left\{ (\mathcal{I}_{sp}^2(f_L^{-1}(G)) \Rightarrow (\mathcal{I}_{sp}^1(G))) \right\} \leq \bigwedge_{H \in L^Y} \left\{ (\mathcal{I}_{sp}^2(H) \Rightarrow (\mathcal{I}_{sp}^1(f_L^{-1}(G)))) \right\} \geq \bigwedge_{H \in L^Y} \left\{ (\mathcal{I}_{sp}^2(f_L^{-1}(H)) \Rightarrow (\mathcal{I}_{sp}^1(f_L^{-1}(H)))) \right\}
\]
Let \( \bar{G} \).

The following corollaries and theorems characterize the degree of semi-preirresolutness, semi-preopenness.

**Corollary 3.12.**

(1) **Semipre-Hom**

\[
\begin{align*}
\text{Semipre-Hom} & = \bigwedge_{G \in L^X} \left\{ (T_{sp})_2(f^{-1}_L(G)) \Rightarrow (T_{sp})_1(G) \right\}.
\end{align*}
\]

Hence

\[
\begin{align*}
\text{SPi}(f) & = \bigwedge_{H \in L^X} \left\{ (T_{sp})_2(f_L^{-1}(H)) \Rightarrow (T_{sp})_1(H) \right\} = \bigwedge_{G \in L^X} \left\{ (T_{sp})_2(f_L^{-1}(G)) \Rightarrow (T_{sp})_1(G) \right\}.
\end{align*}
\]

(3) Since \( f : X \rightarrow Y \) is a bijective mapping, we have \( (f^{-1})_L^{-1}(G) = f_L^{-1}(G) \) and \( f_L^{-1}(G') = f_L^{-1}(G) \) for any \( G \in L^X \).

Then

\[
\begin{align*}
\text{SPi}(f^{-1}) & = \bigwedge_{G \in L^X} \left\{ (T_{sp})_1(G) \Rightarrow (T_{sp})_2(f^{-1}_L(G)) \right\} \\
& = \bigwedge_{G \in L^X} \left\{ (T_{sp})_1(G) \Rightarrow (T_{sp})_2(f_L^{-1}(G)) \right\} \\
& = \text{SPo}(f).
\end{align*}
\]

and

\[
\begin{align*}
\text{SPo}(f^{-1}) & = \bigwedge_{G \in L^X} \left\{ (T_{sp})_1(G) \Rightarrow (T_{sp})_2(f^{-1}_L(G)) \right\} \\
& = \bigwedge_{G \in L^X} \left\{ (T_{sp})_1(G) \Rightarrow (T_{sp})_2(f_L^{-1}(G)) \right\} \\
& = \text{SPcl}(f).
\end{align*}
\]

**Corollary 3.11.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a bijective mapping between \((L, M)\)-fpts’s \( X \) and \( Y \), then

1. **Semipre-Hom**

\[
\text{Semipre-Hom}(f) = \text{SPi}(f) \land \text{SPi}(f^{-1}) = \text{SPi}(f) \land \text{SPo}(f).
\]

2. **Semipre-Hom**

\[
\begin{align*}
\text{Semipre-Hom} & = \bigwedge_{G \in L^X} \left\{ (T_{sp})_2(f_L^{-1}(G)) \Rightarrow (T_{sp})_1(G) \right\}.
\end{align*}
\]

3. **Semipre-Hom**

\[
\begin{align*}
\text{Semipre-Hom} & = \bigwedge_{H \in L^Y} \left\{ (T_{sp})_1(f_L^{-1}(H)) \Rightarrow (T_{sp})_2(H) \right\}.
\end{align*}
\]

The following corollaries and theorems characterize the degree of semi-preirresolutness, semi-preopenness by \((L, M)\)-fuzzy quasi semi-preneighborhood systems, \((L, M)\)-fuzzy semi-preclosure operators, and \((L, M)\)-fuzzy semi-preinterior operators.

**Corollary 3.12.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a mapping between \((L, M)\)-fpts’s \( X \) and \( Y \), then

1. **SPi(f)**

\[
\begin{align*}
\text{SPi}(f) & = \bigwedge_{H \in L^Y} \bigwedge_{x \in P(L^X)} \left\{ \text{SPo}^{(T_{sp})_1}(f_{H}(x)) \Rightarrow \text{SPo}^{(T_{sp})_2}(f^{-1}_L(H)) \right\}.
\end{align*}
\]

2. **SPi(f)**

\[
\begin{align*}
\text{SPi}(f) & = \bigwedge_{H \in L^Y} \bigwedge_{x \in P(L^X)} \left\{ \text{SPN}^{(T_{sp})_1}(f_{H}(x)) \Rightarrow \text{SPN}^{(T_{sp})_2}(f^{-1}_L(H)) \right\}.
\end{align*}
\]

3. **SPi(f)**

\[
\begin{align*}
\text{SPi}(f) & = \bigwedge_{H \in L^Y} \bigwedge_{x \in P(L^X)} \left\{ \text{SPC}^{(T_{sp})_1}(f_{H}(x)) \Rightarrow \text{SPC}^{(T_{sp})_2}(f^{-1}_L(H)) \right\}.
\end{align*}
\]

4. **SPi(f)**

\[
\begin{align*}
\text{SPi}(f) & = \bigwedge_{H \in L^Y} \bigwedge_{x \in P(L^X)} \left\{ \text{SPC}^{(T_{sp})_1}(f_{H}(x)) \Rightarrow \text{SPC}^{(T_{sp})_2}(f^{-1}_L(H)) \right\}.
\end{align*}
\]
Proof. (1) Since $spQ_{x_a}^{(T_{sp})_1}(G) = \bigvee_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(f_L^-(G))$ for all $G \in L^X$, and for any $x_a \in P(L^X)$ and $H_1, H \in L^Y$, we have $f(x)_a \not\preceq H_1', \preceq H' \Rightarrow x_a \not\preceq f_L^-(H_1)' \preceq f_L^-(H)'$. Then
\[
\bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(T_{sp})_1}(f_L^-)(H) \implies spQ_{x_a}^{(T_{sp})_1}(f_L^-(H)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{x \in H_1', \preceq H'} (T_{sp})_2(H_1) \implies (T_{sp})_1(H_1) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{x \in H_1', \preceq H'} (T_{sp})_2(H_1) \implies (T_{sp})_1(f_L^-(H_2)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X) \setminus f(x)_a \cup H_1', \preceq H'} \left\{ (T_{sp})_2(H_1) \implies (T_{sp})_1(f_L^-(H_1)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \left\{ (T_{sp})_2(H) \implies (T_{sp})_1(f_L^-(H)) \right\} = SPi(f).
\]

Conversely, since
\[
(T_{sp})_1(G) = \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(G)
\]
for each $G \in L^X$, and for each $x_a \in P(L^X)$, $H \in L^Y$, and $x_a \not\preceq f_L^-(H)' \Rightarrow f(x)_a \not\preceq f_L^-(H)'$. The following is valid
\[
SPi(f) = \bigwedge_{H \in L^Y} \left\{ (T_{sp})_2(H) \implies (T_{sp})_1(f_L^-(H)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \left\{ \bigwedge_{x \in H_1', \preceq H'} spQ_{f(x)_a}^{(T_{sp})_1}(H) \implies \bigwedge_{x \in H_1', \preceq H'} spQ_{x_a}^{(T_{sp})_1}(f_L^-(H)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \left\{ \bigwedge_{x \in H_1', \preceq H'} spQ_{f(x)_a}^{(T_{sp})_1}(H) \implies spQ_{f(x)_a}^{(T_{sp})_1}(f_L^-(H)) \right\}
\]
\[
\geq \bigwedge_{H \in L^Y} \left\{ \bigwedge_{x \in H_1', \preceq H'} spQ_{f(x)_a}^{(T_{sp})_1}(H) \implies spQ_{x_a}^{(T_{sp})_1}(f_L^-(H)) \right\}
\]

Thus we complete the proof of (1). By Theorem 2.16 and Theorem 2.19, we can prove (2), (3) and (4). \qed

Theorem 3.13. Let $f : (X, T_1) \rightarrow (Y, T_2)$ be a mapping between $(L, M)$-fpts $X$ and $Y$, then

1. $SPo(f) = \bigwedge_{G \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(T_{sp})_1}(G) \implies spQ_{f(x)_a}^{(T_{sp})_1}(f_L^-(G)) \right\}$.

2. $SPo(f) = \bigwedge_{G \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spN_{f(x)_a}^{(T_{sp})_1}(G(x)_a) \implies spN_{f(x)_a}^{(T_{sp})_1}(f_L^-(G)) \right\}$.

3. $SPo(f) = \bigwedge_{G \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spf^{(T_{sp})_1}(G(x)_a) \implies spf^{(T_{sp})_1}(f_L^-(G))(x)_a \right\}$.

4. $SPo(f) = \bigwedge_{G \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spC^{(T_{sp})_1}(G'(x)_a) \implies spC^{(T_{sp})_1}(f_L^-(G'))(f(x)_a)' \right\}$.

5. $SPo(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(T_{sp})_1}(f_L^-)(H) \implies spQ_{f(x)_a}^{(T_{sp})_1}(H) \right\}$.

6. $SPo(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spN_{f(x)_a}^{(T_{sp})_1}(f_L^-)(H)(x)_a \implies spN_{f(x)_a}^{(T_{sp})_1}(H) \right\}$.

7. $SPo(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spf^{(T_{sp})_1}(f_L^-)(H)(x)_a \implies spf^{(T_{sp})_1}(H)(x)_a \right\}$.
\[ (8) \, \text{SPo}(f) = \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \text{spCi}^{(T \circ p \circ)}(f_L^{-}(H))(x_a) \Rightarrow \text{spCi}^{(T \circ p \circ)}((H')(f(x)_a)) \right\}. \]

Proof. Proving (5) is enough since the other conditions are similarly proved.

\[ \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \text{spQ}_{x_a}^{(T \circ p \circ)}((f_L^{-}(H))(x_a)) \Rightarrow \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((H)) \right\} \]

\[ = \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \bigvee_{x \in X} \left( \text{sp}_1((H)) \Rightarrow \bigvee_{x \in X} \left( \text{sp}_2((H')) \right) \right) \right\} \]

\[ \geq \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \bigvee_{x \in X} \left( \text{sp}_1((H)) \Rightarrow \bigvee_{x \in X} \left( \text{sp}_2((H')) \right) \right) \right\} \]

\[ \geq \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \bigvee_{x \in X} \left( \text{sp}_1((H)) \Rightarrow \bigvee_{x \in X} \left( \text{sp}_2((H')) \right) \right) \right\} \]

\[ \geq \bigwedge_{H \in L^x} \left\{ \bigvee_{x \in X} \left( \text{sp}_1((H)) \Rightarrow \bigvee_{x \in X} \left( \text{sp}_2((H')) \right) \right) \right\} \]

\[ \geq \bigwedge_{H \in L^x} \left\{ \text{sp}_1((H)) \Rightarrow \bigvee_{x \in X} \left( \text{sp}_2((H')) \right) \right\} = \text{SPo}(f). \]

For any \( G \in L^x \), we have to prove the following

\[ \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)) \geq \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

It is obvious that

\[ \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)) \leq \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

We will show that

\[ \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)) \leq \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

For each \( y_b \in P(L^x) \) with \( y_b \not\in f^{-}(G) \), we obtain \( \mu \not\in (f_L^{-}(G)) \mu' = \bigwedge_{(f(x)_a \in (f^{-}(G))} G(x)'. \) Then there exists \( x \in X \) such that \( f(x) = y \) and \( b \not\in G(x)'. \) This implies \( \mu \not\in \bigwedge_{(f(x)_a \in (f^{-}(G))} G(x)'. \) Hence \( f(x)_a \mu \leq f_L^{-}(G)'. \) On the other hand, since

\[ \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)) \leq \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

then

\[ \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)) \leq \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

Therefore

\[ \bigwedge_{y \in (f^{-}(G))} \text{spQ}_{y_a}^{(T \circ p \circ)}((f_L^{-}(G)) \leq \bigwedge_{f(x)_a \in (f^{-}(G))} \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(G)). \]

(2)

\[ \bigwedge_{x \in X} \left\{ \bigwedge_{x \in X} \left( \text{spQ}_{x_a}^{(T \circ p \circ)}((f_L^{-}(H))(x_a)) \Rightarrow \bigwedge_{x \in X} \left( \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((f_L^{-}(H))' \right) \right) \right\} \]

\[ \geq \bigwedge_{H \in L^x} \bigwedge_{x \in P(L^x)} \left\{ \text{spQ}_{x_a}^{(T \circ p \circ)}((f_L^{-}(H))(x_a)) \Rightarrow \text{spQ}_{f(x)_a}^{(T \circ p \circ)}((H)) \right\} \]
for each $c \in M$, such that
\[
c \in \bigwedge_{H \in L^Y, x_a \in P(L^X)} \left\{ spQ_{x_a}^{(T_{sp})_1}(f^{-}(H)) \rightarrow spQ_{f(x_a)}^{(T_{sp})_1}(H) \right\}.
\]

Then $c \leq spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(H)) \rightarrow spQ_{f(x_a)}^{(T_{sp})_1}(H)$ for any $H \in L^Y$ and $x_a \in P(L^X)$. By Lemma 2.1(1), we have $c \wedge spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(H)) \leq spQ_{f(x_a)}^{(T_{sp})_1}(H)$. For all $G \in L^X$ and $f(x)_a \leq f_L^{--}(G)$, we obtain $a \leq f_L^{--}(G)(f(x))' = \bigwedge_{f(x)_a \leq f_L^{--}(G)} G(z)'$. Then there exists $z \in X$ such that $f(z) = f(x)$ and $a \not\in G(z)'$. This implies $a \not\in G(z)'$. On the other hand, since
\[
c \wedge \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(G)) \leq c \wedge spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(G)) \leq spQ_{f(x_a)}^{(T_{sp})_1}(f_L^{--}(G)) = spQ_{f(x_a)}^{(T_{sp})_1}(f_L^{--}(G))
\]

The following is valid
\[
c \wedge \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(G)) \leq \bigwedge_{f(x)_a \leq f_L^{--}(G)'} spQ_{f(x)_a}^{(T_{sp})_1}(f_L^{--}(G)).
\]

By Lemma 2.1(1), we obtain
\[
c \leq \bigwedge_{x_a \in G'} \left\{ \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(G)) \leq \bigwedge_{f(x)_a \leq f_L^{--}(G)'} spQ_{f(x)_a}^{(T_{sp})_1}(f_L^{--}(G)) \right\}.
\]

By the arbitrariness of $c$, we have
\[
\bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \in G'} \left\{ \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(f_L^{--}(G)) \leq \bigwedge_{f(x)_a \leq f_L^{--}(G)'} spQ_{f(x)_a}^{(T_{sp})_1}(f_L^{--}(G)) \right\} \right\}
\]

For each $G \in L^X$ and $G \leq f_L^{--}(f_L^{--}(G))$, it can be verified that
\[
\bigwedge_{G \in L^X} \left\{ (T_{sp})_1(G) \rightarrow (T_{sp})_2(f_L^{--}(G)) \right\} = \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \in G'} \left\{ \bigwedge_{x_a \in G} spQ_{x_a}^{(T_{sp})_1}(G) \rightarrow \bigwedge_{f(x)_a \leq f_L^{--}(G)'} spQ_{f(x)_a}^{(T_{sp})_1}(f_L^{--}(G)) \right\} \right\}
\]

The desired equality is obtained.

\[\square\]

4 Semi-precompactness, semi-preconnectedness, semipre-$T_1$, and semipre-$T_2$ degree in $(L, M)$-fuzzy pretopological spaces

**Theorem 4.1.** Let $(X, T_1)$ and $(Y, T_2)$ be two $(L, M)$-fpts's, $f : X \rightarrow Y$ be a mapping and $L = M$. Then $spcom_{(T_{sp})_1}(G) \wedge SPI(f) \leq spcom_{(T_{sp})_2}(f_L^{--}(G))$ for all $G \in L^X$. 
Proof. Take any \( c \in M \) such that \( c \preceq \text{spcom}_{(\mathcal{T}_{sp})_1}(G) \land \text{SPI}(f) \). Then

\[
 c \preceq \text{SPI}(f) = \bigwedge_{H \in L^Y} \left( (\mathcal{T}_{sp})_2(H) \rightarrow (\mathcal{T}_{sp})_1(f^-_{L}(H)) \right),
\]

and

\[
 c \preceq \text{spcom}_{(\mathcal{T}_{sp})_1}(G) = \left( \bigwedge_{G_1 \in \mathbb{G}} (\mathcal{T}_{sp})_1(G_1) \land \bigwedge_{x \in X} (G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1)(x) \right) \rightarrow \bigvee_{H \in \mathcal{H}} \left( G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1 \right)(x).
\]

Then for all \( H \in L^Y \) and \( \mathbb{G} \subseteq L^X \), by Lemma 2.2, we have \( c \leq (\mathcal{T}_{sp})_2(H) \rightarrow (\mathcal{T}_{sp})_1(f^-_{L}(H)) \), and

\[
 c \leq \left( \bigwedge_{G_1 \in \mathbb{G}} (\mathcal{T}_{sp})_1(G_1) \land \bigwedge_{x \in X} (G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1)(x) \right) \rightarrow \bigvee_{H \in \mathcal{H}} \left( G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1 \right)(x).
\]

By Lemma 2.1(1), we can obtain

\[
 c \land (\mathcal{T}_{sp})_2(H) \leq (\mathcal{T}_{sp})_1(f^-_{L}(H)), \forall H \in L^Y
\]

and

\[
 c \land \bigwedge_{G_1 \in \mathbb{G}} (\mathcal{T}_{sp})_1(G_1) \land \bigwedge_{x \in X} (G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1)(x) \leq \bigvee_{H \in \mathcal{H}} \left( G' \lor \bigvee_{G_1 \in \mathbb{G}} G_1 \right)(x).
\]

In order to prove that for all \( H \subseteq L^Y \),

\[
 c \preceq \text{spcom}_{(\mathcal{T}_{sp})_2} \left( f^-_{L}(G) \right) = \left( \bigwedge_{y \in Y} \left( \bigwedge_{H \in \mathbb{H}} (\mathcal{T}_{sp})_2(H) \land \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathbb{H}} H \right)(y) \right) \right)
\]

\[
 \rightarrow \bigvee_{\mathcal{D} \in \mathcal{D}^X} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathbb{H}} H \right)(y)
\]

let \( f^-_{L}(\mathbb{H}) = \{ f^-_{L}(H) \mid H \in \mathbb{H} \} \subseteq L^X \). Then we have

\[
 c \land \bigwedge_{H \in \mathbb{H}} (\mathcal{T}_{sp})_2(H) \land \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathbb{H}} H \right)(y) \leq c \land \bigwedge_{H \in \mathbb{H}} (\mathcal{T}_{sp})_2(f^-_{L}(H)) \land \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathbb{H}} H \right)(y)
\]

\[
 = c \land \bigwedge_{H \in \mathbb{H}} (\mathcal{T}_{sp})_2(f^-_{L}(H)) \land \bigwedge_{x \in X} \left( G' \lor \bigvee_{H \in \mathbb{H}} f^-_{L}(H) \right)(x)
\]

\[
 = c \land \bigwedge_{G_1 \in f^-_{L}(\mathbb{H})} (\mathcal{T}_{sp})_1(G_1) \land \bigwedge_{x \in X} \left( G' \lor \bigvee_{G_1 \in f^-_{L}(\mathbb{H})} G_1 \right)(x)
\]

\[
 \leq \bigvee_{\mathcal{D} \in \mathcal{D}^X} \bigwedge_{x \in X} \left( G' \lor \bigvee_{G_1 \in \mathbb{V}} G_1 \right)(x)
\]

\[
 \leq \bigvee_{\mathcal{D} \in \mathcal{D}^X} \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathcal{D}} H \right)(y).
\]

By Lemma 2.1(1), we have

\[
 c \leq \left( \bigwedge_{H \in \mathbb{H}} (\mathcal{T}_{sp})_2(H) \land \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathbb{H}} H \right)(y) \right) \rightarrow \bigvee_{\mathcal{D} \in \mathcal{D}^X} \bigwedge_{y \in Y} \left( f^-_{L}(G) \lor \bigvee_{H \in \mathcal{D}} H \right)(y).
\]
Therefore

\[
\begin{align*}
  c & \leq \bigwedge_{\mathcal{H} \subseteq L^Y} \left\{ \left( \bigwedge_{H_1 \in \mathcal{H}} (\mathcal{I}_{sp})_2(H_1) \right) \wedge \bigvee_{y \in Y} \left( f_L^{-1}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) \right\} \\
  \implies \bigvee_{\mathcal{D} \subseteq 2^{(Y)} \setminus \{Y\}} \bigwedge_{y \in Y} \left( f_L^{-1}(G)' \vee \bigvee_{H_1 \in \mathcal{D}} H_1 \right)(y) & = \text{spcom}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G)).
\end{align*}
\]

By the arbitrariness of \( c \), we obtain that \( \text{spcom}_n(\mathcal{I}_{sp}_1)(G) \wedge \text{SPI}(f) \leq \text{spcom}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G)). \)

The following corollaries are direct results from Theorem 4.1.

**Corollary 4.2.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a surjective mapping between \((L, M)\)-fpts's \( X \) and \( Y \) and \( L = M \), then \( \text{spcom}_n(\mathcal{I}_{sp}_1)(1_{L^X}) \wedge \text{SPI}(f) \leq \text{spcom}_n(\mathcal{I}_{sp}_1)(1_{L^Y}). \)

In general topology, if \( G \) is connected and \( f \) is continuous, then \( f(G) \) is connected. Now we generalize it to the setting of \((L, M)\)-fpt as follows.

**Theorem 4.3.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a mapping between \((L, M)\)-fpts's \( X \) and \( Y \), then \( \text{spcom}_n(\mathcal{I}_{sp}_1)(G) \wedge \text{SPI}(f) \leq \text{spcom}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G)) \) for all \( G \in L^X \).

**Theorem 4.4.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a mapping between \((L, M)\)-fpts's \( X \) and \( Y \), then \( \text{SPc}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G))' \wedge \text{SPI}(f) \leq \text{SPc}_n(\mathcal{I}_{sp}_1)(G)' \) for all \( G \in L^X \).

**Proof.** Take any \( c \in M \) such that \( c \wedge \text{spCon}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G))' \wedge \text{SPI}(f) \). By Theorems 2.21 and 3.4 (2), we obtain

\[
c \wedge \text{SPc}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G))' = \bigvee_{V \in L^Y} \left\{ (\mathcal{I}_{sp})_2(V_1) \rightarrow (\mathcal{I}_{sp}_1)(f_L^{-1}(V_1)' \left( H_1 \right) \right\},
\]

and

\[
c \wedge \text{SPI}(f) = \bigwedge_{V \in L^Y} \left\{ (\mathcal{I}_{sp}_2)(V_3) \rightarrow (\mathcal{I}_{sp}_1)(f_L^{-1}(V_3)'). \right\}.
\]

This implies that there exist \( H_1, H_2 \in L^Y \) with \( f_L^{-1}(G) \wedge H_1 \neq \emptyset \), \( f_L^{-1}(G) \wedge H_2 \neq \emptyset \), \( f_L^{-1}(G) \wedge H_1 \wedge H_2 = \emptyset \), \( f_L^{-1}(G) \wedge H_1 \neq H_1 \vee H_2 \) such that \( c \leq (\mathcal{I}_{sp}_2)(H_1') \wedge (\mathcal{I}_{sp}_2)(H_2') \), and \( c \leq (\mathcal{I}_{sp}_2)(H_1') \rightarrow (\mathcal{I}_{sp}_1)(f_L^{-1}(H_1)' \left( H_2' \right) \right) \) for all \( H_3 \in L^Y \). This implies that there exist \( H_1, H_2 \in L^Y \) with \( G \wedge f_L^{-1}(H_1) \neq \emptyset \), \( G \wedge f_L^{-1}(H_2) \neq \emptyset \), \( G \wedge f_L^{-1}(H_1) \wedge f_L^{-1}(H_2) = \emptyset \), \( G \leq f_L^{-1}(H_1) \vee f_L^{-1}(H_2) \) where

\[
c \leq (\mathcal{I}_{sp}_2)(H_1') \wedge (\mathcal{I}_{sp}_2)(H_2'), \tag{4.1}
\]

and

\[
c \wedge (\mathcal{I}_{sp}_2)(H_3') \leq (\mathcal{I}_{sp}_2)(f_L^{-1}(H_3)'), \quad \text{for all } H_3 \in L^Y. \tag{4.2}
\]

From Eqs. (4.1) and (4.2), the following is obtained,

\[
c = c \wedge (\mathcal{I}_{sp}_2)(H_1') \wedge (\mathcal{I}_{sp}_2)(H_2') \leq (\mathcal{I}_{sp}_2)(f_L^{-1}(H_1)' \left( H_2' \right) \right) \leq (\mathcal{I}_{sp}_1)(f_L^{-1}(H_1)' \left( H_2' \right) \right) = \text{SPc}_n(\mathcal{I}_{sp}_1)(G)'.
\]

Hence, \( c \) is arbitrary, we have \( \text{SPc}_n(\mathcal{I}_{sp}_1)(f_L^{-1}(G))' \wedge \text{SPI}(f) \leq \text{SPc}_n(\mathcal{I}_{sp}_1)(G)' \).

**Lemma 4.5.** Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a bijective mapping between \((L, M)\)-fuzzy pretopological spaces \( X \) and \( Y \). Then
(1) \( \text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{SPo}(f) \leq \text{Semipre-} T_1(Y, \mathcal{T}_2) \).
(2) \( \text{Semipre-} T_2(X, \mathcal{T}_1) \land \text{SPo}(f) \leq \text{Semipre-} T_2(Y, \mathcal{T}_2) \).

Proof. (1) Take any \( c \in M \), such that

\[
\mathfrak{c} \leq \text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{SPo}(f) = \bigwedge_{g_1,g_2} \bigvee_{g \in L^X} (\mathcal{T}_{sp}(g \cdot f)(G')) \land \bigwedge_{g \in L^X} (\mathcal{T}_{sp}(f \cdot G_1) \rightarrow \mathcal{T}_{sp}(f \cdot G_2)).
\]

Then for any \( g_1, g_2 \in P(L^X) \) with \( g_1 \neq g_2 \), there exists \( G \in L^X \) such that \( g_1 \neq G \geq g_2 \) and \( c \leq \mathcal{T}_{sp}(G) \). For any \( G_1 \in L^X \), \( c \leq \mathcal{T}_{sp}(G_1) \rightarrow \mathcal{T}_{sp}(f \cdot G_1) \). By Lemma 2.1 (1), we have \( c \land \mathcal{T}_{sp}(G_1) \leq \mathcal{T}_{sp}(f \cdot G_1) \). In order to prove

\[
\mathfrak{c} \leq \text{Semipre-} T_1(Y, \mathcal{T}_2) = \bigwedge_{h_1,h_2} \bigvee_{h \in L^Y} (\mathcal{T}_{sp}(f \cdot G'))
\]

let \( h_1, h_2 \in P(L^Y) \) with \( h_1 \neq h_2 \). Since \( f \) is a bijective mapping, there exist \( g_1, g_2 \in J(L^X) \) with \( g_1 \neq g_2 \), such that \( h_1 = f \cdot g_1 \) and \( h_2 = f \cdot g_2 \). From \( g_1 \neq g_2 \), there exists \( G \in L^X \), with \( g_1 \neq G \geq g_2 \) such that \( c \leq \mathcal{T}_{sp}(G) \). Then \( h_1 = f \cdot g_1 \neq f \cdot g_2 = h_2 \). Since \( f \) is a bijective mapping, we have

\[
\mathfrak{c} = \mathfrak{c} \land (\mathcal{T}_{sp}(G') \leq (\mathcal{T}_{sp}(f \cdot G')) = (\mathcal{T}_{sp}(f \cdot G')).
\]

This implies

\[
\mathfrak{c} \leq \bigwedge_{h_1,h_2} \bigvee_{h \in L^Y} (\mathcal{T}_{sp}(f \cdot G')) = \text{Semipre-} T_1(Y, \mathcal{T}_2).
\]

By the arbitrariness of \( c \), we obtain

\[
\text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{SPo}(f) \leq T_1(Y, \mathcal{T}_2).
\]

Other case is similarly proved.

(2) Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a bijective mapping between \((L, M)\)-fuzzy pretopological spaces \( X \) and \( Y \). Then

(1) \( \text{Semipre-} T_1(Y, \mathcal{T}_2) \land \text{SPI}(f) \leq \text{Semipre-} T_1(X, \mathcal{T}_1) \).
(2) \( \text{Semipre-} T_2(Y, \mathcal{T}_2) \land \text{SPI}(f) \leq \text{Semipre-} T_2(X, \mathcal{T}_1) \).

Proof. It is proved in the same way as Lemma 4.5.

By combining Lemmas 4.5, 4.6 and Definition 3.2(1), we can state the following theorem.

Theorem 4.7. Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a bijective mapping between \((L, M)\)-fpts’s \( X \) and \( Y \). Then

(1) \( \text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{Semipre-} \text{Hom}(f) \leq \text{Semipre-} T_1(Y, \mathcal{T}_2) \), \( \text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{Semipre-} \text{Hom}(f) \leq \text{Semipre-} T_1(X, \mathcal{T}_1) \).
(2) \( \text{Semipre-} T_2(X, \mathcal{T}_1) \land \text{Semipre-} \text{Hom}(f) \leq \text{Semipre-} T_2(Y, \mathcal{T}_2) \), \( \text{Semipre-} T_2(Y, \mathcal{T}_2) \land \text{Semipre-} \text{Hom}(f) \leq \text{Semipre-} T_2(X, \mathcal{T}_1) \).

The following corollary is obtained from Theorem 4.8.

Corollary 4.8. Let \( f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2) \) be a bijective mapping between \((L, M)\)-fpts’s \( X \) and \( Y \). Then

(1) \( \text{Semipre-} T_1(X, \mathcal{T}_1) \land \text{Semipre-} \text{Hom}(f) = \text{Semipre-} T_1(Y, \mathcal{T}_2) \land \text{Semipre-} \text{Hom}(f) \).
(2) \( \text{Semipre-} T_2(X, \mathcal{T}_1) \land \text{Semipre-} \text{Hom}(f) = \text{Semipre-} T_2(Y, \mathcal{T}_2) \land \text{Semipre-} \text{Hom}(f) \)
5 Conclusion

In this study, we introduced and showed new degrees of weak forms of functions in \((L, M)\)-fuzzy pretopological spaces by using the implication operations and Ghareeb’s operators. We also investigated some properties of semi-preopen, semi-precontinuous, and semi-preirresolute degree of functions in \((L, M)\)-fuzzy pretopology. We also proved that the function can be regarded as semi-preopenness, semi-precontinuity, and semi-preirresoluteness to some degree. Furthermore, various relationships with semi-precompactness, semi-preconnectedness, Semipre-\(T_1\), and Semipre-\(T_2\) have been constructed and analyzed.

References