Some properties of asymmetric Hopfield neural networks with finite time of transition between states

Ibragim Suleimenov*, Grigoriy Mun, Sergey Panchenko, and Ivan Pak

Abstract: There were implemented samples of asymmetric Hopfield neural networks which have finite time of transition from one state to another. It was shown that in such systems, various oscillation modes could occur. It was revealed that the oscillation of the output signal of certain neuron could be treated as extra logical variable, which describes the state of the neuron. Asymmetric Hopfield neural networks are described in terms of ternary logic. Such logic may be employed in image recognition procedure.

Keywords: artificial neural networks; autooscillations; flip flop; image recognition; bureaucracy; management

1 Introduction

At present neural networks are principally used as a computational means [1, 2]. There are a number of software tools, for example [3, 4], which are based on algorithms of artificial neural networks.

At the same time more and more attention is attracted by physically existing objects which behavior has similarities with neural networks.

In [5, 6] was shown that each partially dissociated macromolecule is a direct analog of the Hopfield neural network. A macromolecule fragment having dissociable functional group could be considered as a neuron. In this case the presence or absence of electrical charge in functional group is possible to treat as logical variable describing the state of neuron output.

Systems having behavior, which is similar to neural networks, also could be found in social sphere. (Early approaches of neural network methods in this way were described in [7, 8]). The proof of legality of such way was presented in [9]. It is shown in [9] that every set of individuals who take place in voting procedure (for example Scientific Council or Board of Directors) could be considered as a Hopfield neural network. Here analogue of neuron is a member of Council of Board and analogue of synaptic link is a mutual influence of members on each other.

Employing analogues with neural networks for the analysis of collective decisions mechanism becomes more and more actual at present stage because exactly at present stage we observe all attributes of loss of control crisis in complex systems [10]. According to conclusions of [11] as complexity of a system grows the response of a system becomes more and more unpredictable due to noises, which arise in channels of control information and due to appearance of parasitic information flows. One of examples is the horizontal links between officials but these links are not covered in job description. Such bureaucratic apparatus may be considered as information system.

Moreover, there is a reason to believe that when the density of parasitic relationships in administrative system exceeds the certain critical value, there is a transpersonal structure [10, 13], so-called information “golem” in terms of [12]. Such structure has its own behavior and instinctive reactions for self-preservation. Results of [9] in this context can be considered as mathematical foundation for point of view in [11, 12].

Such point of view is confirmed by well-known facts: in particular, top managers of Russian Federation according to media reports are forced to use a “manual” control mode. Senior managers are increasingly forced to solve local problems personally but it assumed that those problems could be solved automatically during normal operation of control mechanisms.
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<thead>
<tr>
<th>Table 1: Asymmetric weight coefficients of neural network (case 1)</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>-0.22921</td>
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<th>Table 2: Asymmetric weight coefficients of neural network (case 2)</th>
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<th>Table 3: Symmetric weight coefficients of neural network (case 3)</th>
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<td>-1.41462</td>
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We can assume that the loss of control crisis actually has significantly more complex nature. Each bureaucratic apparatus generates its own information system which structure remains unknown to end-user. (This implies the logical intransparency of neural networks in terms of [1]).

Analogues of Hopfield neural network formed by society and social institutions are different from models studied in [14] due to its weight matrix is not symmetric. (The direct influence of one individual to another may be significantly greater that reverse influence.)

Hopfield neural networks having asymmetric matrix of weight coefficients exhibit quite certain features rather than networks with symmetric matrix. Primarily, there are nonstationary oscillations in networks with asymmetric matrix. This situation has significant consequences, particularly it confirms the conclusions made in [15, 16]. Socio-economic systems have not definitions based on analogues with thermodynamics. Particularly, it is not possible to specify a certain function whose extremum describes certain stationary state of society.

Hopfield neural network can be described by following system of equations

\[ Y_i = f \left( \sum_{j=1}^{n} w_{ij} Y_j + X_i \right), \]

where \( Y_i \) is the output state of \( i \)-th neuron having discrete values -1 and 1; \( X_i \) is the input signal of \( i \)-th neuron; \( f(x) = \text{sign}(x) \) is the activation function; \( w_{ij} \) is weight coefficient between \( i \)-th and \( j \)-th neurons.

2 Experimental

In present paper we simulate Hopfield neural network consisting of six neurons. Weight coefficients are generated randomly and their values are presented in tables 1 – 3. As we see from (1) an image recognized by neural network is the solution of system (1) with the given set of variables \( \{X_i\} \). Simulation results show that not for any value of \( \{X_i\} \) the system (1) has a certain solution.
To find solutions we employ the following iteration procedure

\[ Y_{k+1}^{i} = f \left( \sum_{j} w_{ij} Y_{k}^{j} + X_{i} \right), \]  

where \( k \) is the iteration number; network output values were initiated randomly.

The obtained results show that if the matrix of weight coefficients is asymmetric it is not for all values of \( w_{ij} \) and \( X_{i} \) iterations converge to a certain fixed values of unknown variable \( Y_{i} \).

The most commonly found situation when outputs of neural network shape quasi-periodical oscillations (Fig. 1, Fig. 2). Opposite, iteration procedure converges, as it expected, when we use symmetric weight matrix. The appropriate example is shown on Fig. 3. Weight coefficients used in this simulation are given in Table 3. Here we see that neural network described by (1) has outputs which converge during relatively small number of iterations.

The obtained results can be interpreted as follows. The behavior of real systems that has neural network properties is different from artificial neural networks due to finite time of transition between states. This fact lead us to the occurrence of auto-oscillations when there is no formal solution of the system (1). The simplest case of such situation gives us a network consisting of one neuron. In this case system (1) has one equation, which has not solution. Example of such equation is follows.

\[ X = \theta (-X), \]  

where

\[ \theta (X) = \begin{cases} 
-1, & X < 0 \\ 1, & X \geq 0 
\end{cases} \]

From radio engineering point of view the function \( \theta (-X) \) describes an inverter and equation (3) can be graphically shown in Fig. 4.

From formal point of view equation (3) has no solution but scheme on Fig. 4 works as generator of periodical oscillations. Oscillations in such scheme occur due to finite time of transition between states. Exactly this time parameter specifies the frequency of oscillations.
This, in particular, suggests that for the description of the system (taking into account the finiteness of the switching time) is actually permissible to use an iterative procedure. This is acceptable to assume that each of the iterations corresponds approximately to the beat, the duration of which is comparable with the switching time.

**Conclusion**

Thus, the behavior of the Hopfield neural network, characterized by asymmetric matrix of weight coefficients, indeed significantly different from the case when matrix is symmetric. This fact, in combination with the analogies between neural networks and some social and economic systems, allows us to say that for some systems does not always exist certain “equilibrium” state. So this conclusion is an additional argument in favor of the view previously expressed by V.M. Polterovich [15] who emphasizes the attributes of crisis of economic theory including issues in social choice and general equilibrium theory. Also results actualize the further research in the field of application of neural network theory in mathematical sociology.

**References**


