Research Article

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To the theory of modeling of electric power and electric contact systems

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Abstract: This study evaluated questions of optimum control solution of ordinary differential equations’ nonlinear system which in particular, describes control processes of electric power systems. Conducted numerical experiments have shown sufficient efficiency of the implemented algorithms. It has been shown that essential feature of heating conditions in contact space occur in the moment of contact closure or contact breaking, which has $t^{-1/2}$ order.

Keywords: optimal control, electric power systems, electric contact systems, mathematical model

1 Introduction

This article describes the issues of mathematical modeling of electric power and electric contact systems. As known, electric power and electric contact systems are among the most important elements of energy supply of consumers. Ensuring reliable and trouble-free functioning of these systems is an important problem in applied and theoretical aspects.

The consistently improving algorithms optimal stabilization with an illustration of the obtained results by numerical examples are applied for models of electric power systems (see subclauses 1, 3–7).

The peculiarities of emerging thermal conditions in the contact space were studied based on the theory of parabolic equations in non-cylindrical areas for the electric contact systems (see subclauses 2, 8–10).

Mathematical model of modern electric power complex, consisting of turbo generators and complex multiply connected power units, is a system of nonlinear ordinary differential expressions. It is known [1–3] that this model serves as the basis of a broad and relevant class of control problem.

It should be noted that simulation analysis of various processes, including electric power systems, is closely related with problem of making the best decisions that is actual.

Methods of model identification by measurement data can be used to determine the parameters of the network computational model in the main and supply systems with sufficient telemetry. The network analyzer for the calculation mode is provided in the section of the network with lumped loads. According to the structure, the model is identical to a real network with average hourly loads, with values of power flows or currents as in a real electric network.

The equivalence of the model and the real network is based on the criterion of equality of power or currents unbalance in the model and in the network.

As a rule, the original nonlinear differential and algebraic equations of electric power systems for the current mode are set to a non-linear differential equations in the Cauchy form.

2 About the mathematical model and control problems of electric power system

As known, the electric power system is a dynamic system that is defined in the cylindrical phase space $Z^{n+2}(\delta, S, x)$, described by the differential equations $(n + 2)$-th order for $i$-th isolated subsystem

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = w_i - K_i S_i - f_i(\delta_i), \quad w_i = C_i^* x_i,$$

(1)

$$\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i + R_i(S_i, x_i),$$

(2)

where $\delta_i$ – angular coordinate; $S_i$ – angular velocity; $x_i$ – $n_i$-is the state vector of the regulator; $w_i$-the control action of the regulator; $K_i > 0$ – damping coefficient; $c_i$, $q_i$, $b_i$ – constants $n_i$-dimensional vectors; $A_i$ – constant $(n_i \times n_i)$-matrix; $u_i$ – feedback control. The symbol (*) denotes the
transpose operation. Differential equation of second order (1) describe the process control object, and the vector differential equation (2) defines the state regulator of the $i$-th isolated subsystem. $n$-dimensional vector-function form $f_i(\delta_i)$ is specified in each particular model, for example, in the form of Figure 1. Figure 2 shows the position of the eigenvalues for a statically stable multi-machine electric power system.

![Figure 1: Periodic curve $f(\delta_i)$](image)

![Figure 2: Example of eigenvalues position for a statically stable multi-machine electric power system](image)

Issues of global asymptotic stability of a coupled system with many angular coordinates [4]; observability and control of electric power systems using the model in the control loop that adjusts current process parameters; constructing of analytical methods using periodic Lyapunov functions and establish the stability of multidimensional phase systems; controllability of complex electric power systems and construction software control; development of model identification methods on observed parameters of the system: as statistical estimation of system status on its current operating parameters, monitoring stability factor:

(a) identification of critical sections in current scheme of core network,

(b) determination of maximum and emergency permissible flows in the critical sections of static stability,

(c) forecast of transmission capacity of dangerous sections in different scheme and mode situations,

(d) calculations and definition of bottlenecks in isometric diagrams,

(e) ensuring reliability of management mode of electric power systems,

(f) the economic effect is due to the lifting of restrictions power flows through dangerous sections in real time,

(g) maximum use of economic, competitive power stations,

(h) reduction of restrictions for consumers.

Source of information for technological algorithms is the current steady state achieved on the basis of state estimate of a power system, which is the basic task of the real-time operations control complex.

As a result of its decision, the information model of the current or historical steady-state mode is formed. Subsequently, on the basis of this model other tasks were solved, in particular, simulation, stability testing, reliability and optimization.

### 3 About modeling of thermal processes in electric contact devices

In many applications, in particular, when examining heating and diffusion processes in mobile (closed or open) electric contact devices of multiple purpose, there is the necessity to solve modeling problems for equations of non-stationary transfer in the areas of moving boundaries (in tapered areas). When the area size depends on time or the area degenerates in some points, it is not always possible to agree with the solution of the heat-conduction equation with the movement of heat transfer area boundary.

We note that the consideration of a wide range of problems of mathematical physics [15, 16]. Particularly, the solution of boundary value problems of heat conduction equation in the degenerate regions leads to the need for necessary research of special integral equations of Volterra of the second kind, when the integral operator norm equal to one.

It is the mathematical explanation of special effects in the initial moment of time of contacts breaking or in the final moment of time of contacts closure.

Theoretical studies show that in these times there are features of the functions describing the processes of heat transfer in the contact gap.

Thus, theoretically, there is a need to study special integral equations of Volterra of the second kind, which, as it turned out, they have a direct connection with the objec-
tives of spectrally loaded heat conduction equations [22, 23].

Previously we determined the results (see Clause 8) for the case of closed or open contact on condition that one part of the contact does not move and the other part of the contact moves with the single constant speed [18–21].

In this paper, we applied the results of papers [18–21] for the cases when either only one part of the contact moves with an arbitrary constant, but not necessarily with unit speed or both contact parts are moving with respective constant speeds. The investigation showed that a significant feature of the thermal conditions in the contact space appears at moments of either contact closing or opening, which is of the order \( t^{-1/2} \). These results are shown in sections 9 and 10.

4 Formulation of the optimal control problem for the power systems

As it is known, the basic theory of optimal control is the maximum principle of L.S. Pontryagin [5] and method of dynamic programming of P. Bellman [6–9]. These methods of optimal control theory are connected with methods of sufficient optimality conditions of V.F. Krotov and principle of expansion [10–12], characterized by considerable diversity of approaches and results. They found wide and effective application in solving a number of problems of optimum control of large dimension and various complexities [12].

When solving the control problems in this study, the principle of the expansion of extreme problems was used based on sufficient optimality conditions, and construction of Bellman–Krotov function method for the considered power system [10–12].

It is required to minimize the functional

\[
J(u) = 0.5 \sum_{i=1}^{l} \int_0^T \left( k_i y_i^2 + r_i u_i^2 \right) dt + A(x(T), y(T)),
\]

under the condition:

\[
\frac{dx_i}{dt} = y_i, \quad \frac{dy_i}{dt} = -\lambda_i y_i + f_i(x) + b_i u_i,
\]

\[
x_i(0) = x_{i0}, \quad y_i(0) = y_{i0}, \quad i = 1, \ldots, l, \quad t \in (0, T),
\]

where \((x_i, y_i)_{i=1}^l\) is system condition \((u_i)_{i=1}^l\) – control; \((f_i(x))_{i=1}^l, A(x, y)\) – the given continuously differentiable functions and functions \(f_i(x)\) satisfy the integrability conditions:

\[
\frac{\partial f_i(x)}{\partial x_k} = \frac{\partial f_j(x)}{\partial x_l}, \quad \forall i \neq j; \quad (5)
\]

we consider point in time \(T\) and initial states \((x_{i0}, y_{i0})\) be given; \(r_i, \lambda, k_i, b_i\) – positive constants; terminal values \(x(T), y(T)\) are unknown earlier.

We must note that if we appropriately set the \(f_i(x), i = 1, \ldots, l\), function, non-linear problem of Cauchy (3)–(4) images the electric power system, for which the problem of synthesis is an important practical problem of optimal control.

5 About the algorithm of sequential improvement in function pairs state-control

Following previous works [10–12], let us describe an iterative method for the problem of optimal control:

\[
J(x, u) = \int_0^T f^0(x, u, t) dt + F(x(T)) \to \min,
\]

\[
x'(t) = g(x, u, t), \quad x(0) = x_0.
\]

Let us converse the problem (6) by using Krotov function \(\varphi(x, t)\), which satisfies the formula

\[
\int_0^T \left( \frac{\partial \varphi(x(t), t)}{\partial t} + \sum_{i=1}^n \frac{\partial \varphi(x(t), t)}{\partial x^i} g^i(x(t), u(t), t) \right) dt = \varphi(x(T), T) - \varphi(x_0, 0), \quad (7)
\]

\[
L(\varphi; x, u) = -\int_0^T R(\varphi; x(t), u(t), t) dt + G(\varphi; x(T)), \quad (8)
\]

where

\[
R(\varphi; x, u, t) = -f^0(x, u, t) + \frac{\partial \varphi(x, t)}{\partial t} + \sum_{i=1}^n \frac{\partial \varphi(x, t)}{\partial x^i} g^i(x, u, t),
\]

\[
G(\varphi; x) = F(x) + \varphi(x, T) - \varphi(x_0, 0), \quad (10)
\]

It is necessary to maximize the functional (8) on the required function \(\varphi(x, t)\) in the transformed problem.

Consistent improvement algorithm consists of the following steps:

**Step 1.** We found out that the s-ordered approximation of pair of functions: control and state \((x_i(t), u_i(t))\), satisfying the Cauchy problem (6).
Step 2. We found the function \( \varphi_s(x, t) \) from the condition
\[
R(\varphi_s; x_s(t), u_s(t), t) = \min_x R(\varphi_s; x, u_s(t), t),
\]
for \( t \in (0, T) \),
\[
G(\varphi_s; x_s(T)) = \max_x G(\varphi_s; x).
\]
Let note that the value (11-12) provides the following inequality:
\[
L(\varphi_s; x_s(t), u_s(t)) \leq \max L(\varphi; x_s(t), u_s(t)) \leq \min \int R(x, u),
\]
for \( \forall \{x(t), u(t)\} \), satisfying the problem (6), and for \( \forall \varphi(x, t) \), satisfying the equality (7). Necessary conditions of magnitude relation (11–12) will be the following:
\[
\frac{\partial R(\varphi_s; x_s(t), u_s(t), t)}{\partial x} = 0,  \\
\frac{\partial G(\varphi_s; x_s(T))}{\partial x} = 0.
\]
Step 3. We find function \( \tilde{u}(x, t) \)
\[
\tilde{u}(x, t) \in \text{Arg} \max_u R(\varphi_s; x, u, t)
\]
solving Cauchy problem (6) when \( u = \tilde{u}(x, t) \), we define the state \( x_{s+1}(t) \) function and control function \( u_{s+1}(t) = \tilde{u}(x_{s+1}(t), t) \).
And then, transition to a step 2 and so on.
Note, expression
\[
J(x_s(t), u_s(t)) - J(x_{s+1}(t), u_{s+1}(t)) =
\]
\[
= \int_0^T [R(\varphi_s; x_{s+1}(t), u_{s+1}(t), t) - R(\varphi_s; x_s(t), u_s(t), t)]dt + \\
\int_0^T [R(\varphi_s; x_s(t), t) - R(\varphi_s; x_s(t), u_s(t), t)]dt + \\
+ G(\varphi_s; x_s(T)) - G(\varphi_s; x_{s+1}(T))
\]
is positive in force of condition of (11)–(12) and proportion
\[
u_{s+1}(t) = \tilde{u}(x_{s+1}(t), t) \in \text{Arg} \max_u R(\varphi_s; x_{s+1}(t), u, t)
\]
if the function pair \( \{x_s(t), u_s(t)\} \) does not satisfy the Pontryagin maximum principle. Thus, the described algorithm results in the improvement of the current s-order approximation.

6 Implementation of iterative algorithm for the problem (3)–(4)

Let us describe the procedure of improving a given s-order approximation
\[
\psi_s(t) = \{x_1, s(t), ..., x_i, s(t), y_1, s(t), ..., y_i, s(t), u_1, s(t), ..., u_i, s(t)\}.
\]
Step 1. Let us find a solution the next adjoint problem
\[
\begin{align*}
\psi_i, s(t) &= -\frac{\partial j_1((x_s(t), y_s(t), \nabla_{x,y} \varphi(x_s(t), y_s(t), t), t)}{\partial x_i}, \\
\psi_{s+1, s}(t) &= -\frac{\partial j_2((x_s(t), y_s(t), \nabla_{x,y} \varphi(x_s(t), y_s(t), t), t)}{\partial y_j},
\end{align*}
\]
where
\[
\begin{align*}
\partial j_1((x_s(t), y_s(t), \nabla_{x,y} \varphi(x_s(t), y_s(t), t), t) = \\
= \max_u H(x_s(t), y_s(t), \nabla_{x,y} \varphi(x_s(t), y_s(t), t), u, t), \\
H(x, y, \nabla_{x,y} \varphi, u, t) = -0.5[k_1 y_1^2 + r_i u_i^2] + \\
\sum_{i=1}^l \left[ \frac{\partial \varphi(x, y, t)}{\partial y^i} y^i + \frac{\partial \varphi(x, y, t)}{\partial x^i} x^i \right] [-\lambda_i y_i + f_i(x) + b_i u_i],
\end{align*}
\]
And then, transition to a step 2 and so on.
Note, expression
\[
J(x_s(t), u_s(t)) - J(x_{s+1}(t), u_{s+1}(t)) =
\]
\[
= \int_0^T [R(\varphi_s; x_{s+1}(t), u_{s+1}(t), t) - R(\varphi_s; x_s(t), u_s(t), t)]dt + \\
\int_0^T [R(\varphi_s; x_s(t), t) - R(\varphi_s; x_s(t), u_s(t), t)]dt + \\
+ G(\varphi_s; x_s(T)) - G(\varphi_s; x_{s+1}(T))
\]
is positive in force of condition of (11)–(12) and proportion
\[
u_{s+1}(t) = \tilde{u}(x_{s+1}(t), t) \in \text{Arg} \max_u R(\varphi_s; x_{s+1}(t), u, t)
\]
if the function pair \( \{x_s(t), u_s(t)\} \) does not satisfy the Pontryagin maximum principle. Thus, the described algorithm results in the improvement of the current s-order approximation.
7 Application of iterative algorithm to solve the problem of optimal control of steam turbines’ capacity

One of the models describing the transient processes in electrical power system is the following system of differential equations [1, 2]:

\[
\frac{d\delta_i}{dt} = S_i, \quad H_i \frac{dS_i}{dt} = -D_iS_i - E_i^2 Y_{ii} \sin a_{ij} - P_i \sin (\delta_i - \alpha_i) - \sum_{j=1,j\neq i}^l P_{ij} \sin (\delta_{ij} - a_{ij}) + u_i, \quad i \in \overline{1,l}, \quad t \in (0, T),
\]

where \(\delta_i\) is an angle of rotor deflection of \(i\)-alternator towards some synchronous roll axis (roll axis of constant voltage bus, which makes rotation at a speed of 50 rpm/sec.; \(S_i\) – slip of \(i\)-alternator; \(H_i\) – an inertia constant of \(i\)-alternator; \(a_{ij}\), \(\alpha_i\) – mechanical outputs, which feed to alternator; \(E_i\) – EMF of \(i\)-alternator; \(Y_{ij}\) – mutual conductance of system branches \(i\) and \(j\); \(U\) = const – is tension in constant voltage bus; \(Y_{i,n+1}\) – characterizes connection (conductivity) of \(i\)-alternator with constant voltage bus; \(D_i\) = const ≥ 0 – mechanical damping; \(a_{ij}\), \(\alpha_i\) – constant values with active resistance influence in armature alternator circuits.

The complexity of the model’s analysis (22) is in taking account \(a_{ij}\), \(\alpha_i\) = \(a_{ij}\). Because \(\delta_{ij} = -\delta_{ji}\), then the model (22) is not a conservative; you cannot build a Lyapunov function for it in the form of the first integral. The system is called positional model.

Let the state variable and control variable in the established post-emergency mode be equal to:

\[
S_i = 0, \quad \delta_i = \delta_i^F, \quad u_i = u_i^F, \quad i \in \overline{1,l}.
\]

To obtain the system of perturbed motion let us pass on to equations in fluctuations, supposing that:

\[
S_i = \Delta S_i, \quad \delta_i = \delta_i^F + \Delta \delta_i, \quad u_i = u_i^F + \Delta u_i, \quad i \in \overline{1,l}.
\]

Next, for the convenience of the variables \(\Delta u_i\), \(\Delta \delta_i\), \(\Delta S_i\) again symbolizing \(u_i\), \(\delta_i\), \(S_i\), from (21) we get:

\[
\frac{d\delta_i}{dt} = S_i,
\]

\[
\frac{dS_i}{dt} = \frac{1}{H_i} [-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i], \quad i \in \overline{1,l}, \quad t \in (0, T),
\]

where

\[
f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F) - \sin(\delta_i^F - \alpha_i)],
\]

\[
N_i(\delta) = \sum_{j=1,j\neq i}^l N_{ij}(\delta_1, ..., \delta_i) = \sum_{j=1,j\neq i}^l \Gamma_{ij} |\sin(\delta_{ij} + \delta_i^F) - \sin \delta_{ij}^F|,
\]

\[
M_i(\delta) = \sum_{j=1,j\neq i}^l M_{ij}(\delta_1, ..., \delta_i) = \sum_{j=1,j\neq i}^l \Gamma_{ij} |\cos(\delta_{ij} + \delta_i^F) - \cos \delta_{ij}^F|,
\]

\[
\Gamma_{ij} = P_i \cos \alpha_i, \quad \Gamma_{ij}^2 = P_i \sin \alpha_i, \quad \Gamma_{ij} = I_{ij}, \quad k = 1, 2.
\]

The control will be searched in the form of:

\[
u_i = v_i - M_i(\delta), \quad i \in \overline{1,l},
\]

where functions \(v_i\) to be determined.

It is required to minimize the functional.

\[
J(u) = J(v_1, ..., v_l) = 0.5 \sum_{i=1}^l \int_0^T \left[ w_{si} S_i^2 + w_{vi} v_i^2 \right] dt + \Lambda(\delta(T), S(T)),
\]

under the conditions (25)–(26), where \(w_{si}\), \(w_{vi}\) – positive constants of weight coefficients; \(f_i(\delta_i)\) – 2\(\pi\)-continuously differentiable periodic function; \(N_i(\delta)\) – \(2\pi\)-continuously differentiable periodic function towards \(\delta_{ij}\); for \(N_i(\delta)\) the condition of the integrability of the type (5) is accomplished; \(T\) – the duration of the transition process is considered as given. In addition, the initial conditions have been given:

\[
\delta_i(0) = \delta_{i0}, \quad S_i(0) = S_{i0}, \quad i \in \overline{1,l},
\]

and values \(\delta(T), S(T)\) are unknown.

8 Numerical example. The optimal motion control of two-unit electric power system

In the system (22) we take \(i = 1, 2\), and assume that the mechanical damping is not available, i.e. the coefficients \(D_1\), \(D_2\) are equal to zero. According to the values (22)–(28) the optimal control problem takes the form of [3, 13, 14]:

\[
J(u) = J(u_1, u_2) =
\]
where 
\[ f_i(\delta_i) = P_i[\sin(\delta_i + \delta_i^f - \alpha_i) - \sin(\delta_i^f - \alpha_i)], \quad i = 1, 2, \]
\[ N_i(\delta) = \Gamma_i[\sin(\delta_{12} + \delta_i^f - \delta_{12}) - \cos \delta_{12}^f - \cos \delta_{12}], \quad \Gamma_i = P_{12} \cos \alpha_{12}, \quad \Gamma_2 = P_{12} \sin \alpha_{12}, \] 
\[ \delta_{12} = \delta_1 - \delta_2, \quad \delta_{21} = -\delta_{12}. \]

Numerics of the system (30):
\[ \alpha_1 = -0.052; \quad \alpha_2 = -0.104; \quad H_1 = 2135; \quad H_2 = 1256; \]
\[ P_1 = 0.85; \quad P_2 = 0.69; \quad P_{12} = 0.9; \quad \delta_{1f} = 0.827; \]
\[ \delta_{2f} = 0.828; \quad \alpha_{12} = -0.078; \]

and initial data:
\[ \delta_1(0) = 0.18; \quad \delta_2(0) = 0.1; \quad S_1(0) = 0.001; \quad S_2(0) = 0.002. \]

The results are shown in Figures 3 and 4. Herewith, the value of a functional (31) has been reduced to the value 
\[ = 0.006865. \]

**9 About the task of modeling heating conditions of contact space of high voltage and heavy current switches**

The issue of investigating the boundary value problems concerning the degeneration in the initial moment of time is topical so far. The research in this direction dynamically continues. Let us consider the boundary problem in 
\[ G = \{ x, t : t > 0, \quad 0 < x < t \} \]
(Figure 5):
\[ u_t(x, t) = a^2 u_{xx}(x, t), \quad \text{(34)} \]
\[ u(x, t) |_{x=0} = 0, \quad u(x, t) |_{x=t} = 0, \quad \text{(35)} \]

\[ u(x, t) = \max \left\{ \sqrt{t-x} \exp \left\{ - \left( \frac{2t-x}{2a} \right)^2 \cdot \frac{1}{t} \right\}, \right. \]
\[ 1 + \exp \left\{ - \frac{t-x}{a^2} \right\}, \quad \{ x, t \} \in G, \quad \text{(37)} \]

i.e.
\[ \exp \left\{ - \frac{t-x}{a^2} \right\} \cdot [\gamma(x, t)]^{-1} \cdot u(x, t) \in L^\infty(G). \]
Below when the separation of the area into subareas is built, the following condition (36) will be useful. In straight lines \( x = b t, \ 0 < b < 1, \ t > 0, \) from (36) we have

\[
\begin{align*}
\frac{u(b, t)}{\sqrt{\pi}} & = G \cdot \exp \left\{ -b^2 \cdot t \right\} \\
\times \max \left[ \frac{1}{(1 - b)^{\sqrt{t}}} \exp \left\{ -\left( \frac{2 - b}{2a} \right)^2 \cdot t \right\} \right] + 1 + \exp \left\{ -\left( \frac{1}{2a} \right)^2 \cdot t \right\}, \ t > 0.
\end{align*}
\]

Next, we note that functions:

\[ u_1(x, t) = x^2 + 2a^2 t, \ (x, t) \in G, \ u_2(x, t) = t^{-1/2} \exp \left\{ -\left( \frac{x^2}{4a^2} \right) \right\}, \ (x, t) \in G, \]

that meet equation (34), agree with the requirements from (36)–(37) to solve \( u(x, t) \). However, the traces of those functions with \( x = 0 \) and \( x = t \) do not meet the boundary conditions (35).

The results of the problem (34)–(35) are described in detail and published in papers [18–20]. Here we first briefly present the results of these works, and then consider their application to other tasks, the relevant cases: (a) one contact is stationary and the other moves with an arbitrary constant given speed, not necessarily equal to one; (b) both contacts move at the same velocity in the process of breaking.

It will be shown that the characteristics that were inherent to that considered in [18–20] are maintained. This is the mathematical explanation of special effects in the very beginning moment of time of contacts breaking or in the final moment of time of contacts closure. Our studies show that in these times there are features of the functions describing the processes of heat transfer in the contact gap.

We are searching the solution of problems (34)–(35) in the sum of heat potentials of double layer [17]:

\[
\begin{align*}
\phi(t) & - \int_0^t K(t, \tau) d\tau = 0, \ K(t, \tau) = k(t, \tau) \exp \left\{ -\left( \frac{t - \tau}{4a^2} \right) \right\}, \ (40) \\
k(t, \tau) & = \frac{1}{2a \sqrt{\pi}} \left[ \frac{2t}{(t - \tau)^{3/2}} \exp \left\{ -\frac{t \tau}{a^2(t - \tau)} \right\} \right. \\
& \quad + \left. \frac{1}{(t - \tau)^{3/2}} \left( 1 - \exp \left\{ -\frac{t \tau}{a^2(t - \tau)} \right\} \right) \right].
\end{align*}
\]

According to the results of the work [18], we are receiving the solution of the integral equation (39):

\[
\phi(t) = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{t}{4a^2} \right\} + \frac{\sqrt{\pi}}{2a} \left[ 1 + \text{erf} \left( \frac{\sqrt{t}}{2a} \right) \right]. \ (41)
\]

Thus, we have the solution of the source boundary problems (34)–(35), that is determined by the formula (39) and the function \( \varphi(t) \) has been found as per (41), and the function \( \nu(t) \) is determined through the function \( \varphi(t) \), which is shown above.

Separation of the area \( G \) into subareas \( G_1 \) and \( G_2 \) is shown in Figure 6.
10 Problem (34)–(35) in area
\[ G_c = \{ x, t' : 0 < x < c t', \ t' > 0, \ 0 < c < +\infty \} \]

The replacement of independent variable \( t \) with \( c t' \) and coefficient \( a^2 \) with \( a^2/c \) (34)–(35) results (3) in the boundary problem (Figure 7):
\[
\frac{\partial u_c}{\partial t'} = a^2 \frac{\partial^2 u_c}{\partial x^2}, \ \{ x, t' \} \in G_c, \tag{34}
\]
\[
u_c(x, t')|_{x=0} = 0, \quad u_c(x, t')|_{x=c t'} = 0, \tag{35}
\]
\[
u_c(x, t') = u(x, c t').
\]

The estimation of the non-trivial solution:
\[
[\gamma_c(x, t')]^{-1} \cdot u_c(x, t') \leq C \cdot \exp \left\{ \frac{c t' - x}{a^2} \right\},
\]
\[
\gamma_c(x, t') = \max \left[ \frac{\sqrt{ct'}}{ct' - x} \exp \left\{ -\left( \frac{2ct' - x}{2a} \right)^2 \frac{1}{ct'} \right\}, 1 + \exp \left\{ -\frac{c t' - x}{a^2} \right\} \right], \ \{ x, t' \} \in G_c.
\]

11 Boundary value problem in area
\[ G_{sc} = \{ x, t : |x| < c t, \ t > 0, \ 0 < c < +\infty \} \]

Let us get over the problem in area \( G'_{sc} = \{ x' : t > 0, \ 0 < x' < 2c t, \ 0 < c < +\infty \} \) (Figure 8).

For it, we will make the replacement of independent variables \( x' = x + c t \) and \( t = t' \), which is leading the problem (34)–(35) in \( G'_{sc} = \{ x', t : t > 0, \ 0 < x' < 2c t, \ 0 < c < +\infty \} \) area to the equivalent type:
\[
u_c(x', t) = a^2 u_{c x}(x', t) - c u_{c x}(x', t), \tag{42}
\]
\[
u_c(x', t)|_{x'=0} = 0, \quad u_c(x', t)|_{x'=2c t} = 0, \tag{43}
\]
and \( u_c(x', t) = u(x - c t, t) \).

Let us transform problem (42)–(43) by entering new unknown variable \( \tilde{u}(x', t) \) as per the formula:
\[
u_c(x', t) = \exp \left\{ -\frac{a^2}{4a^2} t + \frac{c}{2a^2} x' \right\} \cdot \tilde{u}(x', t), \tag{44}
\]
from (42)–(43) we get:
\[
\tilde{u}_t(x', t) = a^2 \tilde{u}_{x x}(x', t), \tag{45}
\]
\[
\tilde{u}(x', t)|_{x'=0} = 0, \quad \tilde{u}(x', t)|_{x'=2c t} = 0. \tag{46}
\]

This procedure enables to obtain the estimation of the non-trivial solution:
\[
[\gamma(x, t)]^{-1} \cdot u(x, t) \leq C \cdot \exp \left\{ -\frac{c^2 + 4c t - c + 2}{2a^2} \right\},
\]
\[
\gamma(x, t) = \max \left[ \frac{\sqrt{2ct}}{ct - x} \exp \left\{ -\left( \frac{3ct - x}{2a} \right)^2 \frac{1}{2ct} \right\}, 1 + \exp \left\{ -\frac{ct - x}{a^2} \right\} \right], \ \{ x, t \} \in G_{sc}.
\]

Figure 9 shows the separation of \( G_{sc} \) area.
12 Conclusion

The questions of the solution of problems of optimal control of nonlinear system of ordinary differential expressions have been observed in this work. In particular, the study’s model describes the control processes in electric power systems. The conducted numerical experiments showed sufficient efficacy of the implemented algorithms.

It has been shown that the essential feature of heating conditions in contact space occur in the moment of contact closure or contact breaking, which has the order $t^{-1/2}$.

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References