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Modeling the Supply Process Using the Application of Selected Methods of Operational Analysis

DOI 10.1515/eng-2017-0009
Received June 24, 2016; accepted August 1, 2016

Abstract: Supply process is one of the most important enterprise activities. All raw materials, intermediate products and products, which are moved within enterprise, are the subject of inventory management and by their effective management significant improvement of enterprise position on the market can be achieved. For that reason, the inventory needs to be managed, monitored, evaluated and affected. The paper deals with utilizing the methods of the operational analysis in the field of inventory management in terms of achieving the economic efficiency and ensuring the particular customer’s service level as well.

Keywords: inventory theory, stochastic models, static models, stationary models, fixed order quantity, fixed order period

1 Introduction

Modeling techniques with the application of operations research methods are often utilized within the inventory management. There is a separate discipline within operational analysis which is applicable in solving various economic and technical issues related to inventory management - inventory theory. Inventory theory deals with the rationalization of marketing materials, semi-products and the overall effectiveness of their management.

The differences among the models of inventory theory are based mainly on the facts of how to express the expected consumption and the level of risk, and test the consumption and delivery cycle. Within the operational analysis, there are two kinds of mathematical models - analytic and simulation models. This paper is focused on the practical application of the analytical models in the field of inventory management.

2 Analytical models of the inventory theory

In enterprise, the stocks level and the stocks movement are determined by the way of their procurement, the demand for stocks and the interval of stocks order. Each particular case requires an individual approach. This contributes to create a number of mathematical models in the field of inventory theory [1–3]. The basic division of the mathematical models is illustrated in Figure 1.

![Figure 1: Mathematical models in the field of inventory theory [1].](image-url)

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mostly. In such a case, the input data is expressed as probabilities. Unlike the deterministic models, the stochastic models also take into consideration the possibility of creation of the stocks deficit and the resulting need to maintain the safety stocks [4–6].

According to the way that the stocks are bound in time, the stochastic models can be divided into the static and dynamic. The issue of the quantity of one-time orders is dealt with in the static model. Static models place an emphasis on minimizing the losses resulting from redundant stocks, and vice versa, the deficient stocks. In the case that the individual decisions are fixed in time, it is suitable to apply the dynamic model. According to the changes recorded in the consumption, the dynamic models are further divided into stationary and non-stationary models [7–9].

If the random variable does not show the fluctuations, it is the stationary model. These are classified according to the nature of the order periods into [1, 8, 10]:
- models with the fixed order periods,
- models with the variable order periods.

The fluctuations in consumption caused by seasonality or development trends are solved by the non-stationary models. These are divided into [1, 8]:
- stochastic - deterministic dynamic models,
- adaptive models.

3 The stationary models of the inventory theory

The basic approaches in inventory management, used to absorb the probabilistic fluctuations in consumption, are called the P-System and the Q-System. These systems belong to the dynamic models, i.e., the individual decisions are fixed in time. The aim of these models is to find a suitable algorithm of orders, and thus, effectively manage the inventory. These models take into consideration all types of costs associated with the stocks - the costs related to procurement of stocks, the costs of stocks maintaining, the costs resulting from the stocks deficiency and the costs resulting from the stocks redundancy [1, 3].

The dynamic models take into account the extent of storage period without limit. On that basis, the stocks redundancy, generated within one period, can be disposed by reducing the amount of the orders in the next period. After all, the costs resulting from the stocks redundancy are occurred in the form of stocks maintaining costs. And thus, the costs resulting from the stocks redundancy and the stocks maintaining costs are compared in these models. From the aforementioned, the determination of appropriate replenishment intervals represents a significant part of the inventory management [5, 11].

Inventory management can be performed by two ways:
- by determining the order period,
- by determining the order quantity.

Determining the optimal order period is processed within the Q-System (Fixed Order Quantity System) and determining the optimal order quantity is processed within the P-System (Fixed Order Period System). The way to capture the fluctuations in consumption represents the main difference between the Q-System and P-System [1, 12].

In practice, application of the Q-System means a determination of constant order quantity and the signal stocks level. The order is realized in such a time when the stocks level decreases to a predetermined signal level. The size of the signal stocks is determined according to the need of stocks for covering the consumption in the time period between the particular stocks item order and the given stock delivery to the warehouse. In the case of fluctuation in demand during the interval of order execution, it is necessary to maintain a certain level of the safety stocks as well [1, 12]. Q-System is illustrated in Figure 2.

**Figure 2:** Q-System for the inventory management [11]. Where: L - delivery time, Q - order quantity, R - signal stocks level.

For application of the P-Systems in practice, it is necessary to determine fixed order terms. The size of individual orders is determined in order that the size of the actual stock in the warehouse, in the order time period, was equal to a predetermined variable which takes into consideration the fluctuations in consumption. Using the model, it is possible to find the optimal order cycle extent and optimal order level for replenishing the actual stocks level in order terms. The size of the specific order is determined by the difference between the order level of the stocks and the
actual stocks level in the order time \([3, 5, 13, 14]\). Illustration of the P-System is shown in Figure 3.

4 Modeling the supply process using the application of stationary models of inventory theory

If there is an appropriate model for the type of stocks consumption and if it is possible to ensure the input data for the given model in the enterprise, it is possible to apply the optimization models within inventory theory. For using and applying the inventory theory, the input data can be divided into the following groups \([1, 2]\):

- the estimated consumption and its characteristics,
- the delivery cycle and its characteristics,
- the costs associated with stocks,
- the purchase price, rebates, surcharges, etc.

For the proposal of the optimization model, the input data is summarized in Table 1.

### 4.1 Modeling the supply process using the application of the Q-System

The Q-System works with a fixed order quantity which is predetermined based on the calculation of the optimal order quantity using the Harris - Wilson’s equation \((1)\) \([1]\):

\[
Q_{opt} = \sqrt{\frac{2 \cdot C_p D}{C_t \cdot P}} \text{[units]}
\]

where: \(Q_{opt}\) - order quantity [units]; \(C_p\) - procurement costs/order [€]; \(D\) - averaged demand/year [units]; \(C_t\) - cost rate/day [%]; \(P\) - unit price [€].

\[
Q_{opt} = \sqrt{\frac{2 \cdot 20 \cdot 8450}{0.2 \cdot 4}} = 650 \text{ units}
\]

Buffer stocks are equal to the average demand during the delivery period \((2)\) \([1]\):

\[
BS = \frac{D}{\text{weeks per year}} \cdot LT \text{ [units]}
\]

where: \(BS\) - buffer stocks [units]; \(LT\) - average lead time [weeks].

\[
BS = \frac{8450}{52} \cdot 5 = 812.5 \text{ units}
\]

Reserve is determined through the standard deviation of the demand during the delivery period which is multiplied by a certain coefficient \((3)\) \([1]\):

\[
R = k \cdot (\sqrt{LT}) \text{ [units]}
\]

where: \(R\) - reserve [units]; \(k\) - constant [\(\cdot\)]; \(s\) - standard deviation of the demand [units].

\[
R = 1.64(\sqrt{5} \cdot 50) = 183.36 \text{ units}
\]

Safety stocks are equal to the average demand for a maximum delay time which is multiplied by the probability of delay \((4)\) \([1]\):

\[
SS = \left(\frac{D \cdot T_{d_{max}}}{\text{weeks per year}}\right) \cdot P_d \text{ [units]}
\]

where: \(SS\) - safety stocks [units]; \(T_{d_{max}}\) - max time of delay [weeks]; \(P_d\) - probability of the delay [\(\cdot\)].

\[
SS = \left(\frac{8450 \cdot 2}{52}\right) \cdot 0.1 = 32.5 \text{ units}
\]

The sum of the buffer stocks, the reserves and the safety stocks represents the size of the required stocks level. Simultaneously, the required stocks level represents the order point:

\[
Q = 812.5 + 183.36 + 32.5 = 1028.36 \text{ units}
\]
4.2 Modeling the supply process using the application of the P-System

Monitored time period depends on the optimal order quantity and the average annual demand (5) [1]:

\[ T = \frac{Q_{opt}}{D} \cdot \text{week per year [weeks]}, \]

where: \( T \) - monitored time period [weeks].

\[ T = \frac{650}{8450} \cdot 52 = 4 \text{ weeks} \]

Amount of the buffer stocks is equal to:

\[ BS = \frac{8450}{52} \cdot 10 = 1625 \text{ units} \]

Level of the reserve level is equal to:

\[ R = 1.64(\sqrt{10} \cdot 50) = 259.31 \text{ units} \]

Safety stocks level is equal to:

\[ SS = \left( \frac{8450 \cdot 2}{52} \right) \cdot 0.1 = 32.5 \text{ units} \]

Required stocks level (order point) is equal to:

\[ Q = 1625 + 259.31 + 32.5 = 1916.81 \text{ units} \]

For a better comparison of systems, Table 2 provides the summary of data regarding the types of stocks and the required stocks level (order point) determined based on those systems.

Table 2: Summary of data of types of stocks and the required stocks level.

<table>
<thead>
<tr>
<th>Type of stocks</th>
<th>Amount of the stock</th>
<th>Q-System</th>
<th>P-System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buffer stocks</td>
<td>812.5</td>
<td>1625</td>
<td></td>
</tr>
<tr>
<td>Reserve</td>
<td>183.3576</td>
<td>259.31</td>
<td></td>
</tr>
<tr>
<td>Safety stocks</td>
<td>32.5</td>
<td>32.5</td>
<td></td>
</tr>
<tr>
<td>Required stocks level</td>
<td>1028.358</td>
<td>1564.431</td>
<td></td>
</tr>
</tbody>
</table>

Resulting from Table 2, using the P-system, the level of all the types of stocks is higher compared to using the Q-system. Therefore, the required stocks level is higher when applying the P-system than the required stocks level when applying the Q-system, which means, that the costs related to the stocks and bound capital in stocks are higher. On the other hand, the risk of stocks deficiency is reduced when using the P-system. For better illustration, the results are presented in Figure 4.

5 Conclusion

Considering the fact, that there are fixed points in time of orders within the P-System, even if the order quantity is variable, stocks level can be temporarily high which has an adverse impact on the financial side of the business. On the other hand, when using the P-system, a lower risk of stocks deficiency is ensured and it does not require the permanent control of stocks as within the Q-System [2, 7, 15].

In contradiction with the previous, as for the Q-system, capital is not fixed in stocks and costs are decreased. In case of an unexpected increase in the consumption due to the variability of the order time period, interval of orders is decreased and the need for stocks during the delivery time period is covered by the safety stocks [4, 16–19].

Acknowledgement: The paper is supported by the VEGA Agency by the Project 1/0095/16 "Assessment of the quality of connections on the transport network as a tool to enhance the competitiveness of public passenger transport system", that is solved at the Faculty of Operations and Economics of Transport and Communication, University of Žilina.

References


[16] Stopka O., Kampf R., Kolar J., Kubasakova I., Savage Ch., Draft guidelines for the allocation of public logistics centres of international importance, Communications, 2014, 16(2), 14–19

