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Research Article

Yevgen Gorash* and Donald MacKenzie

On cyclic yield strength in definition of limits for characterisation of fatigue and creep behaviour

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Abstract: This study proposes cyclic yield strength as a potential characteristic of safe design for structures operating under fatigue and creep conditions. Cyclic yield strength is defined on a cyclic stress-strain curve, while monotonic yield strength is defined on a monotonic curve. Both values of strengths are identified using a two-step procedure of the experimental stress-strain curves fitting with application of Ramberg-Osgood and Chaboche material models. A typical S-N curve in stress-life approach for fatigue analysis has a distinctive minimum stress lower bound, the fatigue endurance limit. Comparison of cyclic strength and fatigue limit reveals that they are approximately equal. Thus, safe fatigue design is guaranteed in the purely elastic domain defined by the cyclic yielding. A typical long-term strength curve in time-to-failure approach for creep analysis has two inflections corresponding to the cyclic and monotonic strengths. These inflections separate three domains on the long-term strength curve, which are characterised by different creep fracture modes and creep deformation mechanisms. Therefore, safe creep design is guaranteed in the linear creep domain with brittle failure mode defined by the cyclic yielding. These assumptions are confirmed using three structural steels for normal and high-temperature applications. The advantage of using cyclic yield strength for characterisation of fatigue and creep strengths is a relatively quick experimental identification. The total duration of cyclic tests for a cyclic stress-strain curve identification is much less than the typical durations of fatigue and creep rupture tests at the stress levels around the cyclic yield strength.

Keywords: creep, cyclic plasticity, fatigue life, fatigue limit, failure, plasticity, steel, yield strength.

1 Introduction

Characterisation of long-term strength of structural materials is an important engineering task for prevention of potential catastrophic failures of critical equipment. However, studies of this type are usually very long-lasting, technically challenging and involve expensive experimental work. Thus, the main scope of this study is the formulation of a simple way to predict characteristics of the long-term material behaviour under creep and fatigue conditions using basic material properties. Based upon the extensive availability of experimental material data, a significant progress toward this challenge has been achieved so far and may be observed in the literature as indicated below.

One should start with the fundamental work of Bäumel and Seeger [1], who discussed a few methods for estimating fatigue behaviour of metals using strain-life approach on the basis of monotonic test results. They included the Method of Universal Slopes (MUS) proposed by Manson [2] and modification of MUS developed by Muralidharan & Manson [3], which are applicable to all metals. However, this approach contained a critical limitation regardless of a good accuracy, which is the requirement of reduction in area availability.

The flaw of MUS formulations induced Bäumel and Seeger [1] to develop the Uniform Material Law (UML) using as its basis the extensive collection of fatigue data [4] with over 1500 experimental results. Unalloyed and low-alloy steels were analysed separately from aluminium and titanium alloys resulting into two sets of equations both being based on only elasticity modulus $E$ and ultimate tensile strength (UTS) $\sigma_u$, which can be easily correlated with Vickers hardness $HV$. Later, the applicability of UML concept was extended to high-strength steels by Korkmaz [5].

Comparative study by Kim et al. [6] evaluated seven basic methods for estimating uniaxial fatigue properties (including fatigue limit $\sigma_{lim}^f$) from tensile properties or hardness. This study was based upon the fatigue test data for eight ductile steels under axial and torsional loading. Three of the evaluated methods were able to predict over 93% of the test cases within a factor of 3 compared to the
observed lives. The formulas for $\sigma_{\text{lim}}^f$ prediction included mechanical properties such as $E$, $\sigma_0$ and true fracture ductility $\epsilon_t$. Among the variety of empirical formulations for $\sigma_{\text{lim}}^f$ prediction with different combinations of aforementioned mechanical properties, the simplest are based on $\sigma_0$: $\sigma_{\text{lim}}^f = 1.9018 \sigma_0$ (MUS); $\sigma_{\text{lim}}^f = 1.5 \sigma_0$ (UML); and $\sigma_{\text{lim}}^f = \sigma_0 + 345$ MPa (Mitchell’s method), which showed an accuracy of $R^2 = 0.88$. Another simple method in this comparison, proposed by Roessle & Fatemi [7], used a Brinnell hardness $HB$ for prediction as $\sigma_{\text{lim}}^f = 4.25 HB + 225$ MPa. This approach showed a reasonable accuracy of $R^2 = 0.86$ for the experimental data fit.

Casagrande et al. [8] investigated a relationship between $\sigma_{\text{lim}}^f$ and Vickers hardness $HV$ in steels and developed a method to predict $\sigma_{\text{lim}}^f$. A good correlation was observed between $HV$ and $\sigma_{\text{lim}}^f$ for four kinds of steels in different metallurgical states. However, the proposed empirical method is not straightforward and involves a number of parameters and equations to achieve a reasonable accuracy of $\sigma_{\text{lim}}^f$ predictions. Recently, Bandara et al. [9] proposed a formula for predicting $\sigma_{\text{lim}}^f$ of steels in the Giga-Cycle Fatigue (GCF) regime. It uses a combination of $\sigma_0$ and $HV$ as material parameters and was verified using the experimental results for 45 steels.

An alternative approach was suggested by Li et al. [10], who estimated theoretically the cyclic yield strength $\sigma_y^c$ and $\sigma_{\text{lim}}^f$ using the test data for 27 alloy steels. One formula expresses $\sigma_y^c$ by two conventional mechanical performance parameters $\sigma_0$ and the reduction in area $\psi$. The other formula expresses the fatigue endurance limit through the cyclic yield strength with a reasonable accuracy of $R^2 = 0.883$ as $\sigma_{\text{lim}}^f = 1.13 (\sigma_y^c)^{0.9}$. Despite the relative simplicity, the proposed relation can’t be considered as mathematically elegant, most probably because of the conventional assumption of 0.2% plastic strain offset for yield strength. Nevertheless, this formula by Li et al. [10] demonstrated the tendency that $\sigma_{\text{lim}}^f$ is not very different from $\sigma_y^c$.

A computational approach was developed by Tomasella et al. [11], who applied the Artificial Neural Networks to estimation of the cyclic material properties used in strain-life fatigue approach from a set of monotonic material properties. This approach was implemented into the software called Artificial Neural Strain Life Curves (ANSLC), and has been tested on a large database of steels [1, 4]. In comparison with the largely used UML [1], the results of the estimation with ANSLC program, even without including the support of real experimental tests in the regression, showed a considerably higher accuracy in the life-time estimation.

Recently, Pang et al. [12] did a comprehensive review of the relations between $\sigma_{\text{lim}}^f$ and other mechanical properties of metallic materials. They include the qualitative / quantitative relations between $\sigma_{\text{lim}}^f$ and hardness (HV or HB), strength ($\sigma_u$ and $\sigma_0$) and toughness. Analysis of the numerous fatigue data resulted in the General Fatigue Formula (GFF): $\sigma_{\text{lim}}^f = \sigma_0 (C - P \cdot \sigma_u)$, where $C$ and $P$ are fitting parameters. GFF was found applicable to $\sigma_{\text{lim}}^f$ prediction with increasing $\sigma_u$ in a wide range for many materials such as conventional metals and newly developed alloys. Pang et al. [12] suggested that GFF can provide a new clue to predict $\sigma_{\text{lim}}^f$ and select the appropriate materials within engineering design by adjusting parameters $P$ and $C$ adequately.

The concept of the fatigue limit $\sigma_{\text{lim}}^f$ has been comprehensively studied on microstructural scale by Bathias [13]. This experimental and theoretical study was based upon a thorough and authoritative examination of the coupling between plasticity, crack initiation and heat dissipation for lifetimes that exceed the billion cycle. Moreover, the validity of GCF concept was proved, what questions the idea of infinite fatigue life, at least for practical applications.

Less progress has been achieved in the methods for creep rupture strength evaluation, but recently an important observation was discovered by Kimura [14]. The creep strength of ferritic and austenitic steels has been investigated in [14] through the correlation between creep rupture curve, presenting stress vs. creep rupture life, and 50% of 0.2% offset yield stress (half yield) at a wide range of temperatures. The inflection of the creep rupture curve at half yield was recognised for ferritic creep resistant steels with martensitic or bainitic microstructure, e.g. T91, T92 and T122. This was explained in terms of different mechanisms of microstructural evolution during creep at high- and low-stress regimes. The purpose of this study was to point out a significant risk of overestimation of long-term creep rupture strength by extrapolating the data for martensitic and bainitic steels (e.g. ASTM T91/P91) in high-stress regime to low-stress regime, which are separated by half yield.

A similar problem with particular application to ASTM P91 steel was investigated and discussed earlier by Gorash et al. [15–17] for the purpose of a creep constitutive model development. In these works, apart from inflection of the creep rupture curve, the simultaneous inflection of the minimum creep rate curve, presenting minimum creep rate vs. stress, was recognised. Alternation of the minimum creep rate slope was explained in terms of different creep deformation mechanism (linear creep for low stress and power-law for high stress), while alternation of the creep rupture life slope was explained in terms of different damage accumulation modes (brittle fracture for low
stress and ductile for high stress). The inflection of both curves was characterised by the same value $\sigma_{cy}^*$ called transition stress, which had the meaning of material parameter in the developed “double power law” creep model. However, $\sigma_{cy}^*$ was identified in [15–17] using the minimum creep rate data, and no relation of $\sigma_{cy}^*$ to basic mechanical properties of ASTM P91 steel was recognised.

The principal aim of the present study is to investigate a link in characterisation of long-term strength of structural steel by finding a similar quantitative feature in available experimental data. This work establishes relationships between characteristics of creep and fatigue behaviour on one hand and yield strength as a basic material property and characteristic of plasticity on other hand.

## 2 Concept of the safe structural design

### 2.1 Definition of the yield strength

Dowling [18] discusses several methods to characterise the yield strength $\sigma_y$. The first is the proportional limit $\sigma_p$, which is the stress where the first departure from linearity occurs. The second is the elastic limit $\sigma_{el}$, which is the highest stress that does not cause plastic deformation. The third is the offset yield strength $\sigma_y^{0.2\%}$, which is the stress in the point on stress-strain curve typically defined by the plastic strain offset of 0.2% from elastic line. This value is generally the most practical means of defining the yielding event for engineering metals. Therefore, $\sigma_y^{0.2\%}$ is usually meant to define the yield strength $\sigma_y$ in the literature. However, here the elastic limit $\sigma_{el}$, defined in the scope of unified Chaboche model [19, 20], is used as the yield strength $\sigma_y$.

This study proposes the cyclic yield strength $\sigma_{cy}^*$ as a key characteristic for the definition of safe design for engineering structures operating under fatigue and creep con-
stress range (MPa) for each cyclic test respectively; where 
\[ \Delta \varepsilon \]
is the total strain range and \[ \bar{\Delta} \varepsilon \] is the plastic strain rate, and \[ \bar{p} \] is its magnitude. The total backstress \( X \) in Eq. (3) is given by the superposition of a number \( N \) of kinematic backstresses \( X_i \) with a corresponding evolution equation initially proposed by Armstrong & Frederick [25] for \( \dot{X}_i \), where \( C_i \) and \( \gamma_i \) are kinematic material constants. Chaboche et al. [19] recommended \( N = 3 \) in order to provide a good fit of experimental SSCs, which include large strain areas. Therefore, three backstresses are considered in this study providing an excellent match of the R-O fit (1) for a whole range of strains.

The kinematic hardening constants \( (C_i, \gamma_i) \) and \( \sigma_y \), which define the size of the yield surface, are identified as recommended in [20]. The cyclic SSC is fitted by the following relation:

\[ \frac{\Delta \sigma}{2} = \sigma_y^c + \sum_{i=1}^{N} \frac{C_i}{\gamma_i} \tanh \left( \frac{\gamma_i \Delta \varepsilon}{2} \right), \]

which is obtained in [20] by integrating Eq. (3) and considering \( \Delta \varepsilon = \text{const} \) at the peak stresses for strain-controlled cyclic loading. Relation (4) is valid for the cyclic curve after stabilisation of the hardening or softening effects. Constants \( (C_i, \gamma_i) \) and cyclic \( \sigma_y^c \) are identified by automatic fitting Eq. (4) to the R-O extrapolation (2) with "cyclic" values of constants \( B \) and \( \beta \). The identification procedure is implemented in Microsoft Excel using an add-in Solver [26]. The Solver searches for an optimal (minimum in this case) value for a formula in one cell – called the objective cell – subject to constraints, or limits, on the values of other formula cells on a worksheet. The Solver works with a group of material constants sequentially to the both SSCs providing an excellent match of the R-O fit (1) for a whole range of strains.

The resultant R-O fits for monotonic and cyclic curves are then used to identify the parameters for the Chaboche material model. The range of applicability for the R-O fit is usually quite narrow not exceeding 1% of \( \Delta \varepsilon^{\text{tot}} \) depending on the grade of curvature grade for a SSC.

The basic variant of the rate-independent Chaboche model [19, 20] is presented as a combination of nonlinear kinematic hardening and nonlinear isotropic hardening models. The model allows the superposition of several independent backstress tensors and can be combined with any of the available isotropic hardening models. Since in this study monotonic and cyclic SSCs are fitted separately only for the identification of \( \sigma_y \), only the kinematic hardening component is considered:

\[ X = \sum_{i=1}^{N} X_i, \quad \text{with} \quad \dot{X}_i = C_i \dot{\varepsilon}^p - \gamma_i \varepsilon_i \dot{p}, \]

where \( \dot{\varepsilon}^p \) is the plastic strain rate, and \( \dot{p} \) is its magnitude. The total backstress \( X \) in Eq. (3) is given by the superposition of a number \( N \) of kinematic backstresses \( X_i \) with a corresponding evolution equation initially proposed by Armstrong & Frederick [25] for \( \dot{X}_i \), where \( C_i \) and \( \gamma_i \) are kinematic material constants. Chaboche et al. [19] recommended \( N = 3 \) in order to provide a good fit of experimental SSCs, which include large strain areas. Therefore, three backstresses are considered in this study providing an excellent match of the R-O fit (1) for a whole range of strains.

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of cells, called decision variables or simply variable cells, that participate in computing the formulas in the objective and constraint cells. In this case, the Solver adjusts the values in the decision variable cells containing material constants \((C_i, \gamma_i)\) in order to minimise the value in the objective cell. This cell contains an average value of the absolute difference between columns containing \(\frac{\Delta\sigma}{\tau}\) calculated by Eq. (2) and Eq. (4) correspondingly in a particular range of \(\Delta\varepsilon^p\). Applying this approach, an excellent match of Eqs (2) and (4) is achieved.

The monotonic SSC is fitted by the different relation in the following form \([20]\):

\[
\sigma = \sigma_y + \sum_{i=1}^{N} \frac{C_i}{\gamma_i} \left[1 - \exp(-\gamma_i \varepsilon^p)\right],
\]

which contains the monotonic \(\sigma_y\) and different values of kinematic hardening constants \((C_i, \gamma_i)\). These constants are identified by fitting Eq. (5) to the R-O extrapolation (2) with “monotonic” values of the R-O parameters \(B\) and \(\beta\). The identification procedure is implemented in Microsoft Excel using an add-in Solver in the same way as for cyclic SSC. An advanced step-by-step guideline for the estimation of the Chaboche viscoplasticity model parameters with their further optimisation was developed by Hyde et al. \([27]\).

### 2.3 Application to three structural steels

The above described fitting procedure is applied to SSCs of three structural steels for the purpose of \(\sigma_y\) and \(\sigma_y^p\)
Table 1: Fitting parameters of the Ramberg-Osgood model (1) and (6) for different steels and temperatures

<table>
<thead>
<tr>
<th>Type of plastic material response</th>
<th>Elasto-plastic constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E$ (MPa)</td>
</tr>
<tr>
<td>ASTM A36 RT cycl.</td>
<td>189606</td>
</tr>
<tr>
<td>AISI 4340 RT cycl.*</td>
<td>193053</td>
</tr>
<tr>
<td>ASTM P91 RT mono.</td>
<td>215000</td>
</tr>
<tr>
<td>ASTM P91 RT cycl.</td>
<td>1180</td>
</tr>
<tr>
<td>ASTM P91 500°C m.</td>
<td>180000</td>
</tr>
<tr>
<td>ASTM P91 500°C c.</td>
<td>763</td>
</tr>
<tr>
<td>ASTM P91 550°C m.</td>
<td>482</td>
</tr>
<tr>
<td>ASTM P91 550°C c.</td>
<td>613</td>
</tr>
<tr>
<td>ASTM P91 600°C m.</td>
<td>330</td>
</tr>
<tr>
<td>ASTM P91 600°C c.</td>
<td>446</td>
</tr>
<tr>
<td>ASTM P91 650°C m.</td>
<td>269</td>
</tr>
<tr>
<td>ASTM P91 650°C c.</td>
<td>343</td>
</tr>
</tbody>
</table>

* Extended version of the R-O model (6) is used for data fitting.

Identification. The first is ASTM A36 steel, with mechanical properties available in [28] and [21], which is a standard low carbon steel, without advanced alloying and is a principal carbon steel employed for bridges, buildings, and many other structural uses. The monotonic SSC for this steel at room temperature (RT) shown in Fig. 2a exhibits perfectly plastic behaviour when reaching the stress of 36 ksi = 248.211 MPa in average, which is considered as $\sigma_\text{ym}$. The perfectly plastic yielding lasts for approximately of $\varepsilon_\text{p} = 1$ (%) of strain plateau, which is followed by the strain hardening area, then gradually approaching failure at $\varepsilon\text{tot} = 30$ (%). The cyclic SSC for this steel shown in Fig. 2a from [21] is fitted by the 2-step procedure, and the obtained material parameters for the R-O (1) and Chaboche (3)-(5) models are listed in Tables 1 and 2 correspondingly.

The second material is AISI 4340 steel [22], a high-strength alloy steel, which has good machinability features and used for a wide range of applications including aircraft landing gears, shafts or axels for power transmission, gears, high pressure pump housings, etc. Both monotonic and cyclic SSCs shown in Fig. 2b and mechanical properties are taken from [22]. Since it is available explicitly, the monotonic SSC is fitted by the Chaboche model (5) directly, and the material parameters are listed in Table 2. The cyclic SSC for this steel shown in Fig. 2b from [22] is available at ten times wider strain range than for the ASTM A36 steel. Therefore, the R-O model (1) is not able to provide an accurate fit of the cyclic SSC. In this case, the following modification of the R-O equation (1) proposed by Lemaitre & Chaboche [29] is used for fitting analysis:

$$
\varepsilon\text{tot} = \frac{\sigma}{E} + \left(\frac{\sigma - \sigma_\text{y}}{B}\right)^{1/\beta}
$$

and

$$
\frac{\Delta\varepsilon\text{tot}}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma - \sigma_\text{y}}{2B}\right)^{1/\beta},
$$

Compared to Eq. (1), this notation contains an additional parameter of the yield strength $\sigma_\text{y}$ in the meaning of $\sigma_\text{ym}$, and can be applied for an accurate fitting of much wider strain range than Eq. (1). Thus, the cyclic SSC is fitted by the 2-step procedure. The obtained material parameters for the modified R-O (6) and Chaboche (3)-(5) models are listed in Tables 1 and 2 correspondingly.

The third material is ASTM P91 (modified 9Cr-1Mo) steel [23, 30], an advanced ferritic steel with martensitic microstructure, which has already been widely used over the last 2 decades as tubes/pipes for heat exchangers, plates for pressure vessels, and other forged, rolled and cast components for high temperature services. Both monotonic and cyclic SSCs shown in Figs 2c and 2d and mechanical properties at room temperature (RT), 500°C, 550°C, 600°C and 650°C are taken from [23]. Firstly, the monotonic SSCs are presented in [23] by the material parameters for the R-O model (1) listed in Table 1. The cyclic SSCs are presented in [23] by raw data, which is fitted by the R-O model (1) with material parameters listed in Table 1. Secondly, both monotonic and cyclic R-O extrapolations are fitted by the Chaboche model (3)-(5) with material parameters listed in Table 2.
Table 2: Fitting parameters of the Chaboche model (3)-(5) for different steels and temperatures

<table>
<thead>
<tr>
<th>Type of plastic material response</th>
<th>Three kinematic hardening backstresses</th>
<th>Yield ( \sigma_c ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A36 RT cyc.</td>
<td>( C_1 ) (MPa) ( \gamma_1 ) ( C_2 ) (MPa) ( \gamma_2 ) ( C_3 ) (MPa) ( \gamma_3 )</td>
<td>( \sigma_y ) (MPa)</td>
</tr>
<tr>
<td>AISI 4340 RT mono.</td>
<td>87345.7 984.7 14013.4 111.78 3918.32 13.477 115.792</td>
<td></td>
</tr>
<tr>
<td>AISI 4340 RT cyc.</td>
<td>205524.6 535.8 8966.94 92.268 782.893 1.0739 341.153</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 RT mono.</td>
<td>35912.1 650.7 6972.29 53.297 4221.72 5.7356 330.727</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 RT cyc.</td>
<td>1120466 23911 125301.9 2539.9 17295.23 227.86 406.098</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 500°C m.</td>
<td>1030320 11608 136282.4 1245.6 29535.03 148.08 197.493</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 500°C c.</td>
<td>659430 11229 87028.5 1248.7 19146.80 149.22 134.541</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 550°C m.</td>
<td>1059420 23359 122317.7 2469.7 17631.89 219.49 270.687</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 550°C c.</td>
<td>659430 11229 87028.5 1248.7 19146.80 149.22 134.541</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 600°C m.</td>
<td>511703 24975 56536.0 2630.3 7588.97 232.90 199.970</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 600°C c.</td>
<td>444752 12216 11344.6 160.13 56238.9 1347.6 107.731</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 650°C m.</td>
<td>498277 23543 56252.6 2433.8 8263.19 217.10 115.346</td>
<td></td>
</tr>
<tr>
<td>ASTM P91 650°C c.</td>
<td>353928 12801 44816.6 1396.6 8916.41 162.14 80.6307</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Material parameters of the S-N curves for ASTM A36, AISI 4340 and ASTM P91 steels.

<table>
<thead>
<tr>
<th>Steel</th>
<th>( \sigma_d )</th>
<th>( \sigma_f^p/\sigma_d^p )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A36 RT</td>
<td>413.7</td>
<td>115.8</td>
<td>0.23405</td>
<td>4778.8</td>
<td>1.0</td>
</tr>
<tr>
<td>AISI 4340 RT</td>
<td>827.4</td>
<td>330.7</td>
<td>33.187</td>
<td>2955.7</td>
<td>0.3795</td>
</tr>
<tr>
<td>ASTM P91 RT</td>
<td>658.0</td>
<td>406.1</td>
<td>29.037</td>
<td>10258</td>
<td>0.5154</td>
</tr>
<tr>
<td>ASTM P91 400°C</td>
<td>534.0</td>
<td>[350]*</td>
<td>28.827</td>
<td>10303</td>
<td>0.4738</td>
</tr>
<tr>
<td>ASTM P91 550°C</td>
<td>380.0</td>
<td>[0.0]**</td>
<td>0.0375</td>
<td>7806.7</td>
<td>1.1472</td>
</tr>
</tbody>
</table>

* \( \sigma_d^p \) is not available, hence \( \sigma_f^p \) from [35] is used.
** \( \sigma_f^p \) is observed at high temperatures because of creep.

3 Relation in mechanical characteristics

The next step is a search for possible correlations between the experimentally obtained yield strength values (\( \sigma_d^p \) and \( \sigma_f^p \)) for ASTM A36, AISI 4340 and ASTM P91 steels and their fatigue and creep behaviour characteristics. This identifies a clear similarity for characteristic transition stresses in S-N fatigue, minimum creep strain rate and creep rupture curves, as explained below.

3.1 Fatigue behaviour at normal temperature

Engineering structures operating under cyclic loading conditions at normal temperature are usually designed against the fatigue failure using the conventional stress-life approach. This approach involves experimental fatigue S-N curves with a number of cycles to failure \( N \) vs. stress. A typical S-N curve is a straight line in double logarithmic coordinates, which conventionally described by the Basquin model [36]. Referring to [18, 27], a distinctive minimum stress lower bound, which is called a fatigue endurance limit \( \sigma_{lim} \), is observed on S-N curves for a number of structural steels with polished surface of a specimen in benign (non-corrosive and RT) environment. Referring to [13], fatigue limit, endurance limit and fatigue strength are all expressions used to describe a property of materials under cyclic loading: the amplitude (or range) of cyclic stress that can be applied to the material without causing fatigue failure. In these cases, a number of cycles (usually \( 10^7 \)) are chosen to represent the fatigue life of the material.

Comparison of \( \sigma_f \) defined as a material constant and experimentally observed \( \sigma_{lim} \) reveals that they are close. This assumption is confirmed by the high-cycle fatigue (HCF) experimental data for ASTM A36 [31] and AISI 4340 [32–34] steels shown in Fig. 3. Comparison of \( \sigma_{lim} \) with \( \sigma_f \) summarised in Table 4 for ASTM A36 steel gives 27.6% accuracy and 5.5% accuracy for AISI 4340 steel. These observations indicate that the safe fatigue design is guaranteed in the purely elastic domain defined by \( \sigma_f \).
In general, each S-N curve exhibits two limits: one, when stress tends towards the static fracture $\sigma_u$ (fracture in a quarter of the cycle), and the other, when stress tends towards the fatigue limit $\sigma_{\text{lim}}$. The most known concepts able to describe a reverse sigmoidal shape of a generic S-N curve are presented by two models. The conventional one is the Bastenaire model [37, 38]:

$$N^* = \frac{A}{\sigma_a - \sigma_{\text{lim}}} \exp \left( - \left( \frac{\sigma_a - \sigma_{\text{lim}}}{B} \right)^C \right),$$

where $\sigma_a$ is an alternating stress. Three material parameters $A$, $B$, $C$ and fatigue limit $\sigma_{\text{lim}}$ are derived from fitting the experimental raw data.

A more advanced formulation was developed by Lemaitre & Chaboche [29] using the damage mechanics approach:

$$N^* = \frac{\sigma_u - \sigma_{\text{max}}}{a \left[ (\sigma_{\text{max}} - \bar{\sigma}) - \sigma_{\text{lim}} (1 - b \bar{\sigma}) \right]^{-\alpha}} \left[ \frac{\sigma_{\text{max}} - \bar{\sigma}}{c (1 - b \bar{\sigma})} \right]^{-\alpha},$$

where $\bar{\sigma}$ is the mean stress of the cycle; $\sigma_{\text{max}}$ is the maximum stress in the cycle; and the other variables are material parameters defined in the material property set: $a$ – non-linear damage sensitivity, $b$ – mean stress correction factor, $c$ – the Chaboche equation coefficient, and $\alpha$ – the Chaboche equation exponent. Because the Chaboche concept incorporates its own mean stress correction [29] resembling the Goodman method, the Eq. (8) is fitted to a family of S-N curves with different stress ratios $R$.

For the purpose of this study, the following equation for a S-N curve proposed in [39] was used as a basis:

$$\sigma_a(N^*) = \sigma_{\text{lim}} f_1 \left( \frac{\sigma_u - \sigma_{\text{lim}}}{\sigma_a - \sigma_{\text{lim}}} \right) \left[ 1 - \frac{N^* f_2}{N_{f1}} \right].$$

$$\Rightarrow$$

$$N^* (\sigma_a) = \left( \frac{\sigma_u - \sigma_{\text{lim}}}{\sigma_a - \sigma_{\text{lim}}} \right)^{\frac{1}{f_1}} \left( 1 + f_2 \right) \left( 1 - \frac{1}{N_{f1}} \right)^{\frac{1}{f_1}}.$$

The proposed modification (10) is more convenient than the original formulation (9), and it is a reasonable alternative to previously available equations (7) and (8) because:
it produces a fully adjustable reverse sigmoidal shape with a mathematical minimum of fitting parameters;
- it contains an upper and lower bounds as \( \sigma_u \) and \( \sigma_{u0} \);
- it is fully reversible.

Using a previously suggested assumption, a S-N curve lower bound \( \sigma_{u0} \) in Eqs (7, 8, 10) can be replaced with \( \sigma_{u0}^f \). This would let to identify two material parameters (\( \sigma_u \) and \( \sigma_{u0} \)) from tensile and cyclic experiments correspondingly rather than from fatigue tests. The efficiency of Eq. (10) for description of S-N curves is demonstrated in Fig. 3 in application to ASTM A36 steel with \( \sigma_u \) from [21] and AISI 4340 steel with \( \sigma_u \) from [22]. For both steels \( \sigma_u^f \) is taken from Table 2. Identification of the fatigue fitting parameters \( (f_1, f_2 \) and \( f_3) \) is implemented in Microsoft Excel using an add-in Solver in the same way as described previously for fitting of cyclic and monotonic SSCs by the Chaboche model. All parameters for the S-N curves are reported in Table 3. Finally, the whole range of mathematical formulations for S-N curves is discussed by Castillo & Fernández-Canteli [40].

3.2 Creep behaviour at elevated temperature

Engineering structures operating under constant loading conditions at high temperature are usually designed against the creep failure using the conventional time-to-failure approach. This approach involves experimental creep rupture curves with stress vs. time to failure \( t_r \). A typical creep rupture curve is a trilinear smoothed curve in double logarithmic coordinates, with two inflections corresponding to \( \sigma_u^f \) and \( \sigma_{u0}^f \). These inflections separate three sections on the creep rupture curve, which are characterised by three different creep damage accumulation modes – brittle, ductile and mixed. Three sections with different creep deformations mechanisms can be typically observed on the minimum creep rate curve, presenting minimum creep strain rate vs. stress, which is also a trilinear smoothed curve in double logarithmic coordinates. The deformations mechanism (linear creep, power-law creep and power-law breakdown) are separated by the same two inflections. This assumption is confirmed by the experiments for ASTM P91 steel at elevated temperatures and corresponding theoretical developments.

Previously, the creep modelling using different creep exponent values in different stress ranges was studied by Gorash et al. [15–17]. These studies were devoted to the formulation of constitutive creep model, called double power law and applied to ASTM P91 steel at 600°C:

\[
\dot{\varepsilon}_c^*(\sigma) = C \frac{\sigma}{\sigma_{u0}^f} + \frac{C}{1 + \left(\frac{\sigma}{\sigma_{u0}^f}\right)^{-n-1}}
\]

(11)

where \( \sigma_{u0}^f = 100 \text{ MPa} \) is a material parameter called “transition stress”, which characterises a transition from linear creep to power creep. Other material parameters were identified as \( n = 12 \) and \( C = 2.5 \times 10^{-7} \text{ MPa}^{-1}/\text{h} \).

To implement two transitions into the constitutive model, Eq. (11) is modified by adding the third “power-law breakdown” component to become the triple power law:

\[
\dot{\varepsilon}_c^*(\sigma) = C_1 \sigma^{\alpha_1} + C_2 \sigma^{\alpha_2} + C_3 \sigma^{\alpha_3}
\]

(12)

where \( \alpha = 100 \text{ MPa} \) is a stress exponent, \( C \) and \( n \) are secondary creep parameters, which have the temperature dependence expressed by Arrhenius-type functions as follows [43]:

\[
\sigma_{u0}^f(T) = \sigma_{u0}^f \exp \left( \frac{Q_s}{RT} \right) \text{[MPa]}
\]

(13)

with \( \sigma_s = 0.0587 \text{ [MPa]} \) and \( Q_s = 54100 \text{ [J/mol]} \),

\[
C(T) = 10^{-C_0} \exp \left( \frac{Q_c}{RT} \right) \left[ \frac{1}{T} \right] \text{[1/h]}
\]

(14)

with \( C_0 = 1.9916 \text{ [1/h]} \) and \( Q_c = 8753.7 \text{ [J/mol]} \),

\[
n(T) = n_0 \exp \left( \frac{Q_n}{RT} \right)
\]

(15)

with \( n_0 = 0.2479 \) and \( Q_n = 28648.4 \text{ [J/mol]} \).

In notations (13)-(15): \( T \) is a temperature in K; \( Q_s, Q_c \) and \( Q_n \) are creep activation energies and \( R = 8.314 \text{ [J mol}^{-1} \cdot \text{K}^{-1}] \) is the universal gas constant. The transition stress \( \sigma_{u0}^f(T) \) and creep parameters \( C(T) \) and \( n(T) \) were obtained by fitting the data for minimum creep strain rate from studies by Sklenička et al. [42], Kloc & Fiala [41] and Kimura [30] and shown in Fig. 4. The inflections of corresponding curves are observed at 550, 600 and 650°C and explained in terms of transition between different creep deformation mechanisms. Comparison of \( \sigma_{u0}^f \) defined by Eq. (13) and reported in Table 4 with \( \sigma_{u0} \) from Table 2 reveals that they are close. As summarised in Table 4, the accuracies are 26.7, 6.4 and 19.2% corresponding to 550, 600 and 600°C.

Another important observation was done in [15–17] for creep rupture behaviour of this steel. The creep-rupture
On cyclic yield strength for characterisation of fatigue and creep

Figure 4: Min. creep rate vs. stress of ASTM P91 steel based on several sets of data [30, 41, 42]

The fatigue performance of ASTM P91 steel is analysed using the HCF experimental data by Matsumori et al. [35] at three different temperatures (RT, 400 and 550°C) illustrated in Fig. 6. From these data, it can be concluded that at elevated temperatures the heat-resistant steels don’t exhibit $\sigma_{lim}^f$ on S-N fatigue curves, which is usually observed at normal temperature. The reason for this is the elimination of purely elastic behaviour at high temperature, since...
Figure 5: Stress vs. creep rupture life of ASTM P91 steel based on the data by [14]

there is always some amount of inelastic strain, which is caused by creep. Therefore, there is always a permanent accumulation of creep damage, even at low stress levels and high-strain rate, which leads to inevitable failure. This fact is confirmed by the experimental observations [35], which demonstrated the extinction of $\sigma_{lim}^f$ at 550°C for over $10^8$ loading cycles. However, a good match of $\sigma_{lim}^f$ with $\sigma_m$ with accuracy of 2.8% is observed at RT for this steel, which makes advanced martensitic steels different from simple ferritic steels for $\sigma_{lim}$ prediction. This effect can be explained by the assumption of Terent’ev [45], who recognised two types of the fatigue endurance limit $\sigma_{lim}^f$ – standard in HCF range ($N = 10^2$-$10^7$ cycles) and ultrahigh in giga-cycle fatigue (GCF) range ($N = 10^7$-$10^{11}$ cycles). The existence of ultrahigh $\sigma_{lim}^f$ was proved by the experimental data for high-strength steels (50CrV4, 54SiCrV6 and 54SiCr6), which demonstrated two inflections of the fatigue curves followed by horizontal plateaus – first in HCF area ($N = 10^5$-$10^6$), second in GCF area ($N = 10^8$-$10^9$). The correspondence of $\sigma_m$ with ultrahigh $\sigma_{lim}^f$ for ASTM P91 steel is expected to be found at $N > 10^8$ cycles, but no experimental data is available for this range.

Following these assumptions, experimental S-N curves for ASTM P91 steel by Matsumori et al. [35] are described by the Eq. (10), where $\sigma_m$ is replaced by $\sigma_{lim}^f$ as shown in Fig. 6. The experimental values of $\sigma_m$ for all temperatures are taken from [35]. The value of $\sigma_m^c$ for RT is taken from Table 2, while $\sigma_{lim}^f$ from [35] is taken instead of $\sigma_m^c$ for 400°C, and $\sigma_{lim}^f = 0$ is assumed for 550°C because of creep. All parameters for the S-N curves are reported in Table 3.

Finally, the values of $\sigma_m^c$ and $\sigma_m^c$ for ASTM P91 steel defined by Chaboche model as shown in Figs 2c and 2d are listed in Table 4. They are plotted versus temperature in Kelvins (K) in Fig. 7. The temperature dependence of a yield strength defined as $\sigma_m^c$ is extrapolated by a simple elliptic equation, which can be assumed as an extension of the von Mises yield criterion by temperature consideration, in the following form:

$$\left( \frac{T}{T_{eut}} \right)^2 + \left( \frac{\sigma_m}{\sigma_{y0}} \right)^2 = 1 \quad \Rightarrow \quad \sigma_m(T) = \sigma_{y0} \sqrt{1 - \left( \frac{T}{T_{eut}} \right)^2},$$

where $T_{eut} = 1000$ K is a typical eutectic temperature for steel alloys; $\sigma_{y0} = 210$ MPa and $\sigma_{y0}^c = 2 \cdot \sigma_{y0}^c = 420$ MPa are theoretical yield strengths at absolute zero temperature for monotonic and cyclic responses correspondingly. Since it is not possible to measure the values of $\sigma_{y0}^c$ and $\sigma_{y0}^c$ experimentally, they can be estimated using the results of only one experimental measurement (for instance, at RT) in Eq. (17). However, the value of $T_{eut} = 1000$ K is proved experimentally to be an eutectic temperature in the iron-carbon system, which characterises the co-
existence of solid and liquid phases on iron-carbon phase diagram.

### 4 Conclusions

This study explains the existence of the fatigue limit \( \sigma_{\text{f,lim}} \) and creep transition transition stress \( \sigma_{\text{cr}^*} \) by the cyclic yield strength \( \sigma_{\text{cy}} \) using the fatigue and creep experimental data for a few structural steels at normal and elevated temperatures. The comparison of yield strengths (\( \sigma_{\text{my}}, \sigma_{\text{cy}}, \sigma_{\text{f,lim}}, \sigma_{\text{cr}^*} \)) for ASTM A36, AISI 4340 and ASTM P91 steels is summarised in Table 4. Monotonic and cyclic yield strengths (\( \sigma_{\text{cy}}, \sigma_{\text{f,lim}}^* \)) are defined as elastic limit specified in the scope of the Chaboche model [19, 20]. Fatigue limit \( \sigma_{\text{f,lim}}^* \) of ASTM A36 and AISI 4340 steels complies with \( \sigma_{\text{cy}}^* \). Equality of \( \sigma_{\text{cy}}^* \) and \( \sigma_{\text{f,lim}} \) at RT for ASTM P91 steel can be explained by the GCF concept [45] introducing two fatigue limits.

Creep transition stress \( \sigma_{\text{cr}^*} \) of ASTM P91 steel complies with \( \sigma_{\text{cy}}^* \). Moreover, Kimura’s assumption [14] of half monotonic yield (\( \sigma_{\text{cy}}^{0.25}\sqrt{2} \)) agrees very well with the outcomes of the current study. According to Table 4, the relation \( \sigma_{\text{cy}}^* = \sigma_{\text{cy}}^0 / 2 \) is valid for all temperatures except the highest 650°C regarding ASTM P91 steel. This assumption is not relevant to AISI 4340 steel, which exhibits \( \sigma_{\text{cy}}^* = \sigma_{\text{cy}}^0 \).

An important finding is that the temperature dependence of yield strengths (\( \sigma_{\text{cy}}^0, \sigma_{\text{f,lim}}^* \)) resembles the von Mises yield criterion, which is elliptic in terms of the principle stresses. In the proposed formulation in form of Eq. (17), the yield surface is also presented by ellipse in coordinates of yield strength and temperature as shown in Fig. 7.

The principal advantage of the \( \sigma_{\text{cy}}^* \) application to the characterisation of fatigue and creep strength is the relatively fast experimental identification. The total duration of all cyclic tests, which are required to reach the stabilised stress response for the construction of cyclic SSC is much less than the typical durations of fatigue and creep rupture tests at stress levels around \( \sigma_{\text{cy}}^* \).

The critical point in the work presented here is an application of the advanced material model [19, 20] to the estimation of a single value of elastic limit \( \sigma_{\text{y,el}} \), which may seem to be complicated. However, this approach is effective in typical cases when experimental SSCs are unavailable in explicit form, but available in the form of R-O [24] fittings using Eq. (1). In other cases, when all necessary experimental SSCs are available in form of raw data, the modified form (6) of the R-O model proposed by Lemaitre & Chaboche [29] may reduce the fitting procedure just to one step. Since Eq. (6) contains \( \sigma_{\text{y}} \) as a material parameter, the application of Chaboche model equations (3)-(5) may no longer be needed.
Yield strength (MPa) vs. Temperature (K) for different steels.

Figure 7: Elliptic yield surfaces of ASTM P91 steel using temperature-dependent $\sigma_{y}^{m}$ and $\sigma_{c}^{m}$ from Table 4.

Table 4: Comparison of $\sigma_{y}^{m}$, $\sigma_{c}^{m}$, $\sigma_{y}^{\lim}$ and $\sigma_{c}^{\lim}$ for ASTM A36, AISI 4340 and ASTM P91 steels.

<table>
<thead>
<tr>
<th>Steel</th>
<th>ASTM A36</th>
<th>AISI 4340</th>
<th>ASTM P91</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp., °C</td>
<td>RT</td>
<td>RT</td>
<td>RT</td>
</tr>
<tr>
<td>$\sigma_{y}^{m}$, MPa</td>
<td>248.2</td>
<td>341.2</td>
<td>406.1</td>
</tr>
<tr>
<td>$\sigma_{c}^{m}$, MPa</td>
<td>115.8</td>
<td>330.7</td>
<td>197.5</td>
</tr>
<tr>
<td>$\sigma_{y}^{\lim}$ / $\sigma_{c}^{\lim}$</td>
<td>2.1</td>
<td>1.0</td>
<td>2.1</td>
</tr>
<tr>
<td>$\sigma_{y}^{\lim}$, MPa</td>
<td>160.0</td>
<td>350.0</td>
<td>418.0</td>
</tr>
<tr>
<td>$\sigma_{c}^{\lim}$, MPa</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\Delta\sigma$, %</td>
<td>27.6</td>
<td>5.5</td>
<td>28.7</td>
</tr>
</tbody>
</table>

References


On cyclic yield strength for characterisation of fatigue and creep

Nomenclature

Abbreviations

GCF  Giga-Cycle Fatigue
HCF  High-Cycle Fatigue
MUS  Method of Universal Slopes
R-O  Ramberg-Osgood
RT  room temperature
SSC  Stress-Strain Curve
UML  Uniform Material Law
UTS  Ultimate Tensile Strength

Variables, Constants

\( \sigma \)  stress
\( \Delta \sigma \)  stress range
\( \sigma_a \)  alternating stress
\( \bar{\sigma} \)  mean stress of the cycle
\( \sigma_{\text{max}} \)  maximum stress in the cycle
\( \sigma_y \)  yield strength
\( \sigma_{y0} \)  yield strength at absolute zero
\( \sigma_{\text{cy}} \)  cyclic yield strength
\( \sigma_{\text{my}} \)  monotonic yield strength
\( \sigma_{\text{pl}} \)  proportional limit
\( \sigma_{\text{el}} \)  elastic limit
\( \sigma_{y0.2\%} \)  offset yield strength
σ_{f, lim} fatique endurance limit
σ_u ultimate tensile strength
σ_{cr}^t creep transition stress
ε strain
ε strain rate
Δε strain range
ε_f true fracture ductility
ε_{tot} total strain
ε_p plastic strain
ψ reduction in area
HV Vickers hardness
HB Brinnell hardness
R^2 coefficient of determination
E Young’s (elasticity) modulus
B, β R-O model constants
X_i kinematic backstresses
C_i, γ_i kinematic material constants
N number of cycles to fatigue failure
t* time to creep failure
A, B, C fatigue parameters for Bastenaire model
a, b, c, a fatigue parameters for Chaboche model
f_1, f_2, f_3 fatigue parameters for Chaboche model
C, n secondary creep parameters
Q_o, Q_C, Q_n creep activation energies
k_1, k_2 secondary creep parameters
T temperature
T_{eut} eutectic temperature

Subscripts, Superscripts
y yield
c cyclic
a alternating
m monotonic
cr creep
f fatigue
el elastic
p plastic
* failure
tot total
lim limit
u UTS
eut eutectic