Research Article

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Wear-dependent specific coefficients in a mechanistic model for turning of nickel-based superalloy with ceramic tools

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Abstract: Difficult to cut materials such as nickel and titanium alloys are used in the aeronautical industry, the former alloys due to its heat-resistant behavior and the latter for the low weight - high strength ratio. Ceramic tools made out alumina with reinforce SiC whiskers are a choice in turning for roughing and semifinishing workpiece stages. Wear rate is high in the machining of these alloys, and consequently cutting forces tends to increase along one operation.

This paper establishes the cutting force relation between work-piece and tool in the turning of such difficult-to-cut alloys by means of a mechanistic cutting force model that considers the tool wear effect. The cutting force model demonstrates the force sensitivity to the cutting engagement parameters ($a_p$, $f$) when using ceramic inserts and wear is considered. Wear is introduced through a cutting time factor, being useful in real conditions taking into account that wear quickly appears in alloys machining. A good accuracy in the cutting force model coefficients is the key issue for an accurate prediction of turning forces, which could be used as criteria for tool replacement or as input for chatter or other models.

Keywords: Inconel 718, ceramics, cutting coefficients, time-dependent wear

1 Introduction

Gas and aero turbines manufacturing is a top notch application, in which machining plays an important role, being responsible for the final part precision and surface integrity. The use of new advanced materials has played a major role on this improvement; hence, up to 50% of the increase obtained in turbines efficiency is attributed to super alloys (nickel base) and titanium alloys, both kinds included in the group S (orange) in the ISO (513:2012) materials classification. The development of nickel-based superalloys led into an increase in the turbine entrance temperature up to 350°C over the last decades, and thus during the next years an increase of 200°C in the entrance temperature is estimated, based on Clean Sky European initiative [1, 2]. In manufacturing of turbines, turning is the most important operation for cases and disks for all kind of engine segments. Alloy Inconel 718 (UNS N07718) is by far the most widely used nickel-based superalloy and the most studied one. The high ramp up of new deliveries required by aeroengine market implies an increase in cutting speed, that can be provided by ceramic tools.

Alloy 718 includes nickel as main element, being more than half of its mass by weight. It also has important quantities of chromium and iron, around 19% wt. and other non-insignificant quantities of alloying elements such as niobium, molybdenum, titanium or aluminum. Turbine vases and discs represent the principal component made of Alloy 718 as high-strength is required at high temperatures given the high centrifugal force reached by engine disks running at full load, but it is also used in cases, parts of combustor, blades, etc. This alloy maintains high strength and good ductility up to about 650°C. For many workshops, it is the reference of the group S material regarding machinability.

However, these alloys are also difficult-to-cut materials (i.e., the present high specific cutting force, rapid tool wear) because of their tendency to suffer work hardening by the tooltip rubbing effect on just machine surface on
one hand, and their high shear strength and ductility on
the other. In addition, the highly abrasive nature of the al-
loy carbide particles reduces tool life and produces poor
surface finish. Because of that, the thermo-mechanical
and metallurgical phenomena between tool insert and ma-
chined surface need to be explored. That fact explains the
efforts done in the last years on this issue. A time ago,
Narutaki et al. [3] studied the wear resistance of several
ceramic tools during severe turning conditions up to 500
m/min. A complete state of the art in the field of ceram-
ics and Inconel 718 machining can be found in Dudzin-
ski et al. [4], in which wear evolution was also considered.
In order to define appropriate tool geometry and cutting
conditions for such difficult to cut materials, intense re-
search work under the concept of high speed machining
(HSM) was performed, employing carbide tools in the turn-
ing process. Hence, Fang and Wu [5] explained a detailed
comparative machining study between Inconel 718 and Ti-
6Al-4V, finding empirical relationships to determine cut-
ing forces depending on cutting speed and feedrate. Ad-
ditionally, Thakur et al. [6] studied tool wear effect and
showed the trends in the surface finish under different cut-
ing conditions.

On the other hand, and referring to ceramic tools, the
so-called hard turning technique of hardened steel was
studied. Hard turning in automotive applications must be
stable, so statistical analysis of previous results is a valu-
able approach; so, Khotamasu et al. [7] presented a model
to predict cutting tool flank wear and forces in hard turning
based on experimental data; in addition, Kountanya [8]
developed a general 3D model for corner-radiuses, cham-
fered, edge-honed cutting worn tool. Some industrial ap-
plications of hard turning are found in [8–10], in which
wear evolution dramatically affects both part quality and
process economy; tool wear causes higher forces and tem-
peratures, affecting quality. Hard turning presents two
choices, the expensive PCBN tools on one hand and the
cheaper ceramic tool approach on the other. The best op-
tion in ceramic tool involves the use of hard ceramic matrix
(Al2O3) reinforced with extremely resistant silicon-carbide
whiskers. This type of ceramic inserts is able to withstand
2000°C, providing the whiskers enough toughness for in-
terrupted turning. The growing requirement to increase
productivity, along with the interest for reducing coolant
consumption during machining of nickel-base alloys, has
turned attention to ceramic tools, which are able to with-
stand the high temperatures reached during the machin-
ing of Fe-Ni based alloys, and making the dry cutting ap-
proach feasible. In this issue, several authors investigated,
such as Arunachalam et al. [11] who analyzed the CBN and
mixed ceramic (Al2O3 and TiC) tools behaviour us-
ing optimal cutting parameters [12], observing the resid-
ual stresses and surface integrity of workpieces. In [13] it is
studied the ceramic tool failure in intermittent hard turn-
ing, proposing a method to stablish stresses along cutting
length. Cermet were also under evaluation by Xu et al. [14]
at cutting speed of 50 m/min, speed lower than ceramic
values usually in the range of 300-400 m/min.

Regarding carbide tools, a practical view is shown in
[15], focusing on the effect of excessively worn tools in
nickel-based superalloys. Also, in [16], the effect of carbide
size and spacing on the fretting wear resistance of Inconel
690 in experiments is presented, in the same way that Mi
et al. [17] did. Fernández-Valdivielso et al. [18] identified
as well the best combination of carbide grades, chipbreak-
ers shapes, and other features for having the best tool per-
formance. Additionally, they checked surface integrity ef-
fects.

High pressure cooling strategies help in turning pro-
cess due to its ability to form an hydraulic wedge on tool
rake face, lifting the chip and gain access to the cutting
zone, which leads to a reduction of the tool-workpiece con-
tact region lowering the friction zone, which in turn re-
sults in reduction in cutting temperature and component
forces. This phenomenon was observed by Ezugwu and
Bonney [19], who machined Alloy 718 with carbide tools
at speeds up to 50 m/min using conventional and high
coolant pressures, up to 203 bar. Vagnorius and Sørby [20]
claimed that SiAlON ceramic inserts with improved resis-
tance to notching in machining of Alloy 718 under high-
pressure cooling comparing it with conventional one. They
observed that notch wear increased but it was not critical
to tool live, and even flank wear was reduced. Wei et al. [21]
studied flank wear and the influence of hydrogen contents
on the rake crater wear in Ti-alloys. Other aeronautic alloys
have been tested in terms of surface finish and residual
stresses, such as Jomaa et al. [22] in dry machining con-
ditions. Finally, other emerging approaches such as plasma
or laser assisted processes [23], or ultrasonic assisted pro-
cesses [2] are in discussion, being surface integrity always a
concern.

In this study, straight cylindrical turning experiments
were carried out in Inconel 718 using ceramic inserts to
obtain specific cutting force coefficients. Face turning
could be also an option, but operational performing is
longer [24]. Other works focused on polycrystalline dia-
mond (PCD) tools instead of ceramics [25] are also an in-
teresting alternative, but temperature is always a serious
drawback for PCD. Testing is necessary to study the com-
plex tool wear in turning of such a low machinability ma-
terial. Starting from this experimental base, the effects of
depth of cut and cutting speed on tool flank wear were also investigated.

The model can be used to determine forces on turning of thin-walled cases or other turbine components, or being part of a chatter model.

2 Force Modelling

Similar in essence to other predictive models in turning and milling [26, 27], a mechanistic model is proposed for calculating turning force. But, here an additional component is considered. So, the total force during the cutting process is divided into two main components. Firstly, cutting force components due to the chip removal process itself, which it is supposed to remain steady along the process. Secondly, a component related with the tool wear growing along the cutting process, which the novelty of the model. The latter component depends on wear and consequently on machining time, so it must time-dependent, as it is shown in the Equation (1) for modelling the total force during the chip removal process.

\[ F(t) = F_{cutting} + F_{wear}(t) \] (1)

At the same time, the so-called cutting force is subdivided into two effects: the force due to the shear stress, \( F_c \), which is the dominant component of the removal process and responsible of the great fraction of the energy consumption. And secondly, the edge force \( F_e \) at the rake and relief surfaces, due to the friction contact between the chip and the tool insert produced during chip removal and some from the rubbing of relief face on part surface. If those three effects are decomposed, the equation results as follows:

\[ F(t) = F_c + F_e + F_c(t) \] (2)

The wear component allows adapting this model to the full process time, up to flank wear reaches the maximum threshold, usually \( V_B \) 0.3 mm is the ISO recommendation is followed. The models for each of the three sub-components are as follows.

2.1 Shear force component

The shear force \( F_c \) comes from the metal cutting process itself, involving the shear deformation process inherent to chip formation in ductile alloys. Using a linear model, and introducing the effects of the side cutting edge angle and the depth of cut \( (a_p) \) inside the cutting coefficient, cutting force can be expressed as a function of the feed. The approach includes the effect of hardening in the coefficient dependency on \( a_p \). Equation (3) shows the model for the coefficient and force component [28]:

\[ F_{cx}(ap) = K_{cx}(ap) \cdot f \] (3)

where \( K_{cx} \) is the cutting coefficient for the \( X \) direction in [N/m] considering the depth of cut inside the cutting coefficient, \( a_p \) the depth of cut and \( f \) the feed per revolution. Similar relationships are established for the \( Y \) and \( Z \) directions.

2.2 Edge force component

The interaction between the workpiece surface and the tool flank face is still not well understood. From the works of Kobayashi and Thomsen [29], along with Thomsen et al. [30] and Zorev [31], it can be said that the deformation located in the primary shear zone and the friction phenomena on the rake face are not mainly affected by flank wear, that is, the basic cutting quantities remain unaffected by the tool flank wear land size.

The edge force is considered as the friction, adhesion and diffusion phenomena over the chip-rake face, and it can be written as in [28]:

\[ F_{ex}(ap) = K_{ex}(ap) \] (4)

where \( K_{ex} \) is the corrected specific edge force in \( X \) direction in [N], one that depth of cut is introduced. Similar relationships are found in \( Y \) and \( Z \) directions.

2.3 Wear force component

As it was explained in the previous subsection, flank wear has several consequences on process. For instance, it tends to stabilize the system mainly at low spindle speeds: this is the so-called process damping effect, very known in study of the chatter vibration problem. On the other hand, cutting becomes more aggressive with cutting edge dullness and forces increase.

Two main different research lines were used to model the effect of the tool flank wear. The first approach based on the contact force model [32, 33] establishes a proportionality relationship between the contact force and the displaced volume of the workpiece under the tool. This model is suitable for mild steels turning, but not for hard materials because of the difficulty in estimating the displaced volume in short tool life spans. Besides, the mea-
surement of the flank wear length \( (V_B) \) becomes complicated for ceramic inserts, where cutting is so aggressive that tool life is short and chipping frequent.

The other proposed approach is based on the slip-line model [34]. In this concept, the frictional force originated by the sliding of a hard relatively smooth surface over a softer one can be assumed as the force needed to push layers of plastically deformed material along the soft surface ahead of asperities on the hard surface. So, Kobayashi and Thomsen considered the slip-line field under the tool flank wear. In the same way Waldorf et al. [35] introduced non-uniform load distribution along the flank wear area to estimate the ploughing force.

In this work, an alternative view of the edge wear is put into practice for the prediction of the total force. The method lies basically in the fact that it does not include any considerations onto the tool-workpiece material interaction, introducing them by means of coefficients. As the forces involved during machining of low-machinability materials are strongly time-dependent, even more if using ceramics tools (because of the high cutting speeds in excess of 300 m/min), it seems reasonable as a first approach an expression like Eq. (5):

\[
F_{wx} (t) = K_{wx}(ap) \frac{V_c}{V_f} t
\]  

(5)

where \( K_{wx} \) is the specific edge force in \( X \) direction as function of the depth of cut. Further on, the physical sense and composition of this coefficient will be explained. The formula (5) also shows a ploughing force proportional to the chip removal speed given by the cutting speed and an inversely proportional dependence on the linear feed rate \( V_f \) given in mm/min.

3 Experimental setup and data acquisition

The straight turning experiments were conducted in a CMZ® TBI-450MC turning center, see Figure 1. The workpiece was a cylinder of Inconel 718 (\( \varnothing 100 \text{ mm} \times \text{L300 mm} \)) in annealed state, clamped in a power chuck, important at high speeds, and using a tailstock. Dry condition was used because of ceramics resistance to heat, although MQL and cryogenic gases also have lead to good results [36]. Figure 1 also shows the Cartesian coordinate system: the mean tangential cutting force (parallel to cutting speed), \( Z \) direction (workpiece axis) and \( Y \) axis, the radial direction.

![Figure 1: Experimental setup. turning workspace with dynamometer.](image)

Figure 2: ASTM grain size near 8.

Alloy 718 is a Nickel-Chromium alloy, this 718 batch containing 53.60% of Ni and 18.30% of Cr. There are other important constituents as 18.02% Fe or important alloying elements, e.g. Mo 3.06%, Al 0.49%, Ti 0.98% or Nb+Ta 5.02%.

Studying wear when machining Inconel with ceramic tools is difficult because of the extremely aggressive nature of this material. This leads to very different results even when machining theoretically the same material with the same tool. So, material from a single batch was used for all tests to prevent any additional effect from chemistry or metallurgical variations. Specimen grain size was ASTM 8-9, microstructure on Figure 2.
Rhombic ceramic inserts supplied by NTK® Cutting Tools with ISO reference CN120708 T00520 WA1. The insert grade was a whisker-reinforced composite ceramic material with silicon-carbide whisker added to alumina (Al2O3+SiC: Density: 3.7 g/cm³; Hardness: 94.5 HRA; Bending Strength: 1200 MPa; Young modulus: 400 GPa, Thermal expansion coefficient: 7.0×10⁻⁶/K; Thermal conductivity: 35 W/mK).

The recording system used was composed by a KISTLER® dynamometer, type 9257B and a signal amplifier 5017, using an OROS® multichannel analyzer by NVGATE software 6.2 and a PC for data processing and storage.

Cutting tests were done at two cutting speeds: Vc = 250 and 300 m/min, applying six different depths of cut: ap ranging from 0.4 to 0.9 mm, in steps of 0.1 mm. Finally, four different feed rates were tested for each of the depths of cut, thus f = 0.05, 0.1, 0.15, 0.2 mm/rev.

Each of the series was driven with new inserts, being the wear measured at the end of each cutting experiment. All tests were repeated twice, and values were just the average one of both. If divergence between same tests were in excess of 7%, a third one was repeated in order to disregard the previous biased one.

4 Cutting force analysis

A typical turning force measurement is shown in Figure 3. In order to capture only the effect of cutting process and eliminating the tool wear effect, the force magnitude was measured at seven points/times for each experiment, along with the beginning of the first feed rate, and in the transition point between two different feed rates.

In this way, the forces due to the cutting process under each feed rate can be defined as:

\[ F_{f=0.05} = F_1 \]
\[ F_{f=0.10} = F_{f=0.05} + \Delta_1 \]
\[ F_{f=0.15} = F_{f=0.10} + \Delta_2 \]
\[ F_{f=0.20} = F_{f=0.15} + \Delta_3 \]

Following the above criterion, the force components due to the cutting process show the trend represented in the left side of Figure 4. For the subsequent analysis of the measured forces, a linear regression was carried out at each depth of cut under the four different feed rates adopting the form:

\[ F = K_c \cdot f + K_e \]
Figure 5: Experimental linearized cutting coefficients: shear cutting coefficients (up) in [N/m] and edge coefficients (down) in [N].

(see Figure 4 the correspondence between simulated and experimental ones).

Table 1: Shear cutting coefficient functions. Dependence on the depth of cut.

<table>
<thead>
<tr>
<th>Shear cutting coefficient</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{cx}$</td>
<td>$2628.5 \cdot a_p - 296.5$</td>
</tr>
<tr>
<td>$K_{cy}$</td>
<td>$276.6 \cdot a_p + 241.5$</td>
</tr>
<tr>
<td>$K_{cz}$</td>
<td>$797.1 \cdot a_p - 391.2$</td>
</tr>
</tbody>
</table>

Table 2: Edge cutting coefficient functions. Dependence on the depth of cut.

<table>
<thead>
<tr>
<th>Edge cutting coefficient</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{ex}$</td>
<td>$60.4 \cdot a_p + 26.8$</td>
</tr>
<tr>
<td>$K_{ey}$</td>
<td>$21.2 \cdot a_p + 48.7$</td>
</tr>
<tr>
<td>$K_{ez}$</td>
<td>$152.0 \cdot a_p - 5.3$</td>
</tr>
</tbody>
</table>

An example of prediction for the cutting force, using the expressions described above, is illustrated in Figure 6. In this case, a straight cylindrical turning operation is carried out with $a_p = 0.6$ mm and the selected tool geometry (CNGN, rhombic geometry with $\kappa_r = 93^\circ$) leads to a more pronounced increment of the cutting force in the tangential direction (cutting speed direction) and in the radial direction (Y direction) than in Z direction. This is the main reason for the small effect on the force in axial direction, if wear is not considered (Figure 6, down). It is evident that the consideration of the wear effect over the force would improve the accuracy between experimental data and predictions. This is a key issue when cutting low-machinability materials with ceramic inserts resulting in an accumulative effect over the forces. The proposed treatment for the computation of wear effects will be discussed in the next section.

5 Wear force analysis

In the scientific literature, cutting forces are often decomposed into pure (shear) cutting forces and friction or edge forces if wear is not studied. It is reasonable to consider that wear and edge forces are both due to friction mechanisms between the tool and workpiece. However, especially when machining Inconel, wear mechanisms will affect edge force components (also shear cutting compo-
Wear-dependent specific coefficients in a mechanistic model because the tool continuously loses material and so, the edge geometry continuously changes with respect to its theoretical profile. Under this approach, these effects were absorbed by the wear component, i.e., the one that changes against time. In other words, the linearization of the cutting and friction components was done prior to wear analysis, because wear is time varying and needs a different treatment. So, cutting and edge components are considered stationary (constant) phenomena while evolving phenomena will be packaged inside the wear coefficient.

Figure 6 shows a relative distortion between the real cutting forces and the predicted ones without considering tool wear. This slope due to the wear effect is constant along each feed rate but slightly varies from one to another. Let $K_w$ be the ratio between increasing force and time and $\Delta t_1$ the length for the first feed rate. From Equation (7), it can be said that the force at the beginning of the first feed rate could be expressed as:

$$F_{1,st} = k_c(ap) \cdot f + k_e(ap)$$  \hspace{1cm} (8)

Then, considering the effect of wear, the force at the end of the first feed rate is defined by:

$$F_{1,end}(\Delta t_1) = F_{1,st} + k_w(V_c/V_f)\Delta t_1$$  \hspace{1cm} (9)

where, $k_w$ is the wear coefficient to determine the effect in the force in N/s. Thus, under the consideration that there is not significant wear in the transitions between feed rates, the force at the beginning of the second feed rate is:

$$F_{2,st} = F_{1,end} + \Delta 1$$  \hspace{1cm} (10)

The same approach can be extended to predict the total force as dependent on time, because wear effect causes an increasing effect on the cutting force.

The analysis to obtain the wear coefficients for the time dependent term in equation (7) uses a linear approach as well, as it was explained for $k_c$ and $k_e$ coefficients. The analysis finds the relationship between the slope force-time as dependent on the depth of cut. The expressions to describe this wear coefficient are in Table 3.

The force thus obtained, considering the effects of the cutting process as well as the effect introduced by wear as tool life decreases, is shown in Figure 7. Here, it is clearly denoted the matching between experimental measurements and force predictions.

At the view of Figure 7, a machinist could establish a threshold value for considering the tool replacement, allowing programmers to foretell turning force components at different wear stages. The tool life is very short when ceramics are used at 300-400 m/min, so estimation is useful for work preparation. For instance, Figure 8 shows the state of two tools after experiments in Figure 7, for 250 and 300 m/min respectively. Notch wear is disregarded when flank wear is determined.

6 Wear effects on stability

Unfortunately, tool wear cannot be neglected when machining difficult-to-cut alloys. The quality of machined parts is affected by changing conditions at the chip-tool interface. Changes in tool geometry will have drastic con-

<table>
<thead>
<tr>
<th>Wear cutting coefficient</th>
<th>$V_c = 250$ m/min</th>
<th>$V_c = 300$ m/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{wx}$</td>
<td>$0.1474 \cdot a_p + 0.4338$</td>
<td>$K_{wx} = 0.1987 \cdot a_p + 0.5845$</td>
</tr>
<tr>
<td>$K_{wy}$</td>
<td>$0.3850 \cdot a_p + 0.7763$</td>
<td>$K_{wy} = 0.6186 \cdot a_p + 1.2475$</td>
</tr>
<tr>
<td>$K_{wz}$</td>
<td>$0.6226 \cdot a_p + 0.5504$</td>
<td>$K_{wz} = 0.9714 \cdot a_p + 0.8589$</td>
</tr>
</tbody>
</table>

Figure 7: Force prediction considering the wear effect.
sequences on the dynamics of the cutting process. First, cutting coefficients rapidly increase leading to lower admissible depths of cut. However, tool wear has also positive effects at low cutting speeds, where process damping tends to increase the available free chatter zones [37].

On the side of monitoring, many authors in the literature pointed towards the importance of recording the vibration (acceleration) signal [38, 39]. These approaches need further improvements in order to be reliable solutions.

On the side of modelling, we present some stability lobes considering tool wear from the corresponding dynamic turning model. The above expressions for the specific cutting energy were introduced in the chatter model presented in [40–42]. The Chebyshev collocation method (CCM) was used with meshes of 200x150 and 250 discretization points and the tested modal parameters were: $f_n = 711.75 \text{ Hz}$, $k = 1 \times 10^7 \text{N/m}$, $m = 0.5 \text{ kg}$, $\xi = 0.01$. A tool with $\kappa r = 45^\circ$ angle, similar to round inserts, is assumed. The cutting coefficients considering tool degradation over time were calculated at times $t = 0$ (new), 15 and 30 s (Table 4) using $ap = 0.5 \text{ mm}$ and $f = 0.1 \text{ mm/rev}$.

Figure 9 shows how important is keeping tool wear under control to avoid chatter. It significantly decreases the chatter free zones in a few seconds what is particularly risky in aerospace industry when turning expensive parts. Additionally, when combining worn tools and low spindle speeds (for instance, in turning or boring) it is important considering that process damping effect could appear. This phenomenon tends to increase the stability zone and is relatively easily to consider within the model. However, it depends on a number of parameters (tool geometry, preparation, cutting parameters and modal parameters) and it is difficult to obtain experimentally.

### 7 Conclusions

The cutting force prediction has been always a subject of concern in the planning of turning operations in Inconel 718 with ceramic inserts. The estimation could help machinist to program tool replace or reduce the risks in cases of thin-wall parts machining.

This study presents a predictive model to obtain the three force components in the straight turning process, based on a linearization of the tool wear progression. This linearization lies in the slope observed during the experiments performed at several cutting speeds. Because of the aggressive cutting of ceramics on superalloys causing high temperatures and stresses, flank wear effect need to be included in the force prediction. In fact, the weight of the wear component into the total force seems to be so variable that estimation cannot be achieved without considering the ploughing effects caused by tool corner chamfering due to wear.

This work proposes a model that considers the wear coefficients deviation with the depth of cut, offering a set of equations for different cutting speeds. Similar relationships are found for cutting and edge coefficients. The effects of feed rate are included for the cutting and wear terms in the general expressions (for equation (8) and for equation (9)).

Therefore, model makes possible for given cutting conditions to predict turning force components at a certain time. The model achieved has been adjusted in the width
range from $V_c = 250$ to 300 m/min, typical for the application of $\text{Al}_2\text{O}_3 + \text{SiC}_w$ ceramic tools in Inconel 718.

The so proposed model can help to obtain forces or being part of a complex dynamic model.

**Acknowledgement:** We are grateful to research projects TURBO from MINECO, project INNPACTO DESAFIO II by MINECO as well, initiative ZABALDUZ, and UPV/EHU number UFI 11/29.

**Nomenclature**

- $\kappa$: cutting edge approach angle in $^\circ$
- $\xi$: damping ratio
- $a_p$: depth of cut in mm
- $f$: feed per revolution in mm/rev
- $k$: modal stiffness in N/m
- $K_{sx}$: shear cutting coefficient in N/mm$^2$
- $K_{ex}$: specific edge force in N/mm
- $K_{wx}$: specific edge wear force in N/s
- $m$: mass
- $V_B$: flank wear in mm
- $V_C$: cutting speed in m/min
- $V_f$: linear feed rate in mm/min

**References**


