



# Fractional Calculus & Applied Analysis

An International Journal for Theory and Applications

VOLUME 22, NUMBER 6 (2019)

(Print) ISSN 1311-0454  
(Electronic) ISSN 1314-2224

## EDITORIAL

### FCAA SPECIAL ISSUE – IN MEMORY OF LATE PROFESSOR WEN CHEN (FCAA–VOLUME 22–6–2019)

Guest-Editors: YangQuan Chen, Changpin Li,  
Igor Podlubny, Hongguang Sun

#### 1. Editorial Note

Prof. Wen Chen was born on February 21, 1967, in Fuzhou, Fujian, China, and passed away in Nanjing at 3:58 am on November 22, 2018, at the shockingly young age of 51. Among many contributions to the community of fractional calculus research, Wen Chen is best known for organizing successfully the ICFDA 2012 (as 5th event of the traditional “International Conferences on Fractional Differentiation and Applications”) in Nanjing, China in May 2012.

Many who worked and interacted with Wen in one way or the other expressed strong emotions. For example, Prof. Changpin Li, Shanghai University, China, said “*Wen is one of fractional pioneers in China. In the past summer holidays, he called me with very weak breath for discussing some possible co-operations... His death is a big loss in fractional community.*” In the same emails thread, Prof. YangQuan Chen of University of California Merced, USA said highly about Wen: “... *His engineering insights to fractional order operators are excellent. His book in Chinese is widely read. He has a wide caliber of interests driven by his curiosity. I wish to say, his work on prime number distribution (ML fitting), as well as his work on LRD toolbox are both very fascinating to me among many others. Of course, his outstanding organization of ICFDA 2012 Nanjing is one of the many of his contributions. We will remember Wen Chen for his contribution to our community in various ways.*” Prof. Dumitru Baleanu of Cankaya University, Turkey shared: “*It is not easy to describe in words the lost of Wen Chen ... My lecture during the conference organized last week by Professor Luis Vazquez in Madrid was dedicated to him. I met Wen in 2006 in Porto and in several many occasions in Europe and China. In December 2017 I discussed last time with him about the nonlocality and fractional calculus .... I am sure that the group Wen established in his faculty will continue to work very hard in the future and the ideas of Wen will continue ...*”

© 2019 Diogenes Co., Sofia  
pp. 1437–1448, DOI: 10.1515/fca-2019-0075

DE GRUYTER

In response to the initial proposal of Prof. Changpin Li, we formed a team of four Guest Editors led by Prof. YangQuan Chen to prepare a formal Special Issue proposal to FCAA Editor-in-Chief Virginia Kiryakova to commemorate the late Professor Wen Chen by invited contributions. We received 11 contributions and each paper went through a regular peer review process with at least three returned reviews with constructive comments. We wish to thank Virginia for supporting this Special Issue and the reviewers for professional services.

Prof. Wen Chen was a member of Editorial Board of “*Fract. Calc. Appl. Anal.*” and worked wholeheartedly for its progress. On behalf of journal’s board we share our sadness about the lost of the colleague and friend and pass condolences to family and close circle of collaborators and followers.

## 2. Prof. Wen Chen’s Biography

Prof. Wen Chen was born on February 21, 1967, in Fuzhou, Fujian, China. He was a distinguished professor and Ph.D. supervisor at Hohai University. He was a recipient of China National Funds for Distinguished Young Scientists, a member of the China Association for Promoting Democracy, and a member of the 11th and 12th Chinese People’s Political Consultative Conference (CPPCC) Jiangsu Provincial Committee. Unfortunately, he passed away in Nanjing at 3:58 am on November 22, 2018, at the age of 51.

Prof. Wen Chen graduated from Huazhong University of Science and Technology in July 1988 with a bachelor degree majored in engineering mechanics. Then, he received a master’s degree in February 1994 and a doctorate in February 1997, from Shanghai Jiao Tong University.

Prof. Wen Chen had 20 plus years’ working experiences in a variety of national laboratories, industries, and universities, including 6 years at overseas research entities. After graduation, he had 3 years professional experiences in industries, worked as an assistant engineer in Mechanics Institute of Zhenjiang Huatong Machinery Group Co., China. Then he worked as a research engineer for a short time in Chinese Underwater Technology Institute, China. From November 1997 to October 1998, he worked as a research engineer in Department of Mechanical & Production Engineering, National University of Singapore, Singapore. Then he worked as a postdoctoral research fellow at Shinshu University in Japan and University of Oslo in Norway until September 2000. After that, he experienced four months visit in Department of Mathematics, City University of Hong Kong, Hong Kong. From March 2001 to December 2001, he worked as a postdoctoral research fellow in Department of Informatics, University of Oslo, Norway.

From January 2002 to December 2003, he worked as a research scientist and project manager in Simula Research Laboratory, Oslo, Norway. From January 2004 to January 2006, he worked as a research professor in the Institute of Applied Physics and Computational Mathematics in Beijing. Since March 2006, he began to work in the Department of Engineering Mechanics at Hohai University in China. He served as director of Mechanics and vice dean (January 2007-December 2012) of College of Civil Engineering and College of Mechanics and Materials. Later he served as the dean of College of Mechanics and Materials (January 2013-September 2016). Besides, he also served as deputy director of “Chinese Society of Environmental Mechanics”, associate general secretary of Jiangsu Society of Theoretical and Applied Mechanics, the former TC chair of computational mechanics software under China Mechanics Society, the IFAC (the International Federation of Automatic Control) TC member on Linear Control Systems. Moreover, he had served as associate editor or member of editorial board of 14 journals, among which 5 international journals are indexed by SCI.

Prof. Chen attended more than 50 academic conferences with more than 10 invited talks and 6 plenary talks. He was a member of scientific committee for more than 15 different conferences, chair of organization committee for 4 international academic conferences, organizer of many mini-symposiums and workshops, and had visited over 20 countries for various academic activities.

Prof. Wen Chen showed prodigious ability on academic research and most of his projects was of multidisciplinary undertaking. He had diverse professional experiences in mechanics, mathematics, acoustics, and informatics, as summarized in his 368 plus academic journal publications (318 indexed by the Science Citation index) with more than 3000 non-self SCI citations, 6 monographs, 9 patents, and 12 software copyrights. His research works mainly include the two major directions. The first one is about scientific computing and computational mechanics, involving radial basis function methods, differential quadrature method, inverse elastodynamics, heat conduction problems, and nonlinear dynamics. The major original contributions in computational mechanics are:

1. An introduction of the singular boundary method [9, 18] and the boundary knot method [3, 17], which are symmetric boundary-only discretization techniques free of mesh, integration and fictitious boundary;
2. A composite recursive multiple reciprocity [8, 20] is developed to solve a wide variety of inhomogeneous problems in truly boundary-only fashion;

3. An introduction of the kernel distance functions, space-time distance functions, pre-wavelet and orthogonal wavelet distance functions [7], which place the radial basis function on a new mathematical basis for multiscale, multivariate, scattered data processing and meshfree computations;
4. A discovery of the high-order fundamental solutions and general solutions of convection-diffusion, thin plate vibration, Winkler plate, and Burger plate [14, 21];
5. The first use of the special matrix product in the nonlinear matrix computations, and the introduction of the new special matrix product, and the discovery and proof of an explicit Jacobian matrix representation theorem of nonlinear polynomial discretization equations [15];
6. An improvements of efficiency and applicability of the differential quadrature methods [12] involving the study of numerical integrators for stiff and structural dynamic problems.

The second one is about mechanics and physics of soft matter (e.g., polymers, colloids, emulsions, foams, living organisms, rock layers, sediments, plastics, glass, rubber, oil, soil, DNA) and their mathematical modeling, involving modeling of medical ultrasonic imaging, fractional calculus, mesoscopic physics, Lévy statistics, fractional Brownian motion, and anomalous diffusion (frequency-dependent dissipation and damping). Below are his original contributions in the modeling of soft matter and complex fluids:

1. A new definition of the fractional Laplacian, which overcomes the hyper-singularity of the traditional definition and naturally includes boundary conditions in finite domains [11]; and then the development of the linear and nonlinear causal fractional Laplacian wave equations and the corresponding FEM numerical models for lossy media exhibiting arbitrary frequency power law attenuation, with the classical proportional Rayleigh damping being a special case [1]. The models have successfully been used in the modeling of ultrasonic medical imaging of breast cancer [2].
2. First mathematical physics explanation of [0,2] power dependency of attenuation coefficient on frequency in various lossy media via the Lévy stable distribution theory [4].
3. An introduction of the concept of the positive fractional time derivative and accordingly the presentation of the modified Szabo wave equations [10], where the hyper-singularity of the original Szabo wave equation models for anomalously attenuative media is significantly eased and the integer-order initial condition is naturally

included [19]. The modified Szabo's wave equation for arbitrarily frequency-dependent viscous dissipation in soft matter has successfully been used to 3D ultrasonic imaging [24].

4. A discovery of the fractal time-space transforms underlying "anomalous" physical behaviors of soft matter, and the presentation of the two hypotheses concerning the effect of fractal time-space fabric on physical behaviors, and the introduction of the novel mathematical concept of the Hausdorff derivative [5] for modeling "anomalous" diffusion and conduction of soft matter and complex fluids [16, 23].

5. An introduction of the new intermittent statistical equation of turbulence via fractional derivative and the fractional Laplacian representation of eddy viscosity in the Reynolds equation [6].

6. A definition of structural derivative [13], a novel approach to tackle the perplexing modeling problem of ultraslow diffusion, in which the structural function plays a central role in this strategy as a kernel transform of underlying time-space fabric of physical systems [22].

Based on his excellent research works, he served as the principle investigators of 34 academic research projects and participants of 9 research or industrial projects. Those projects include one "973 Project", one China National Fund for Distinguished Young Scientists, 9 China National Natural Science Funds, one R&D Special Fund for Public Welfare Industry (Hydrodynamics), and one Innovative Talent Project of New Century by the Ministry of Education of China (2006).

Prof. Wen Chen devoted his life to the teaching and research on computational mechanics, environmental mechanics, mechanical models of soft matter, statistical mechanics, industrial design, and other fields. He was capable to teach in English and the Level 2 proficiency of Japanese. During his teaching at Hohai University, he had hosted two brand courses in English for foreign students in China sponsored by the Ministry of Education and Jiangsu Province. His Ph.D. students are excellent as well, four of them are awarded excellent doctor degree dissertation in Jiangsu Province, two are awarded Baogang scholarship, and one has been nominated for the "Chinese college students annual figure selection". Prof. Wen Chen provided many overseas communication opportunities for students. More than 40 graduate students have long-term or short-term oversea academic visit or study experiences. His graduated students have played important roles in various jobs, five of them have become professors, another five are associate professors, one obtained a permanent position at an overseas research center, and two are employed by large enterprises as project manager.

Because of the excellent contributions of Prof. Wen Chen, he has been granted several academic awards including the JSPS (Japanese Society for Promotion of Science) Fellowship, Germany Humboldt Fellowship for Experienced Researchers, Australian Leadership Awards Fellowship, Du Qinghua Medal of Computational Method in Engineering, ICCES MM 2010 Award for Promising Research on Novel Computational Method, and ICCES Distinguished Fellowship, Contribution Award for Returned Overseas Chinese (Innovative Talent, 2014). From 2014 to 2018, he had been on the list of the most cited Chinese researchers for five years. In addition, he had been selected as Jiangsu Provincial Distinguished Professor (2012), Jiangsu provincial Young and Middle-aged Science and Technology Leading Talents in “333 Talents Project”, Top Ten Science and Technology Stars in Nanjing City (2013).

Prof. Wen Chen was hardworking during his whole life, diligent in thinking and willing to contribute. Unfortunately, he was diagnosed with lung cancer in April 2013. However, he still persisted in teaching, research, management and fought against disease with unimaginable perseverance and courage for a long time. He treated people sincerely and everyone who knows him is moved by his positive and optimistic attitude toward disease and work. He had devoted a lifetime of hard work and made several significant contributions to academia and earned extensive respect and praise.

### References

- [1] A. Bounaim, W. Chen, Computations for a breast ultrasonic imaging technique and finite element approach for a fractional derivative modeling the breast tissue acoustic attenuation. *Int. J. Tomogr. Stat.* **10**, No 8 (2008), 31–43.
- [2] A. Bounaim, S. Holm, W. Chen, A. Odegard, Quantification of the CARI breast imaging sensitivity by 2D/3D numerical time-domain ultrasound wave propagation. *Math. Comput. Simulat.* **65**, No 4-5 (2004), 521–534.
- [3] W. Chen, High-order fundamental and general solutions of convection-diffusion equation and their applications with boundary particle method. *Eng. Anal. Bound. Elem.* **26**, No 7 (2002), 571–575.
- [4] W. Chen, Lévy stable distribution and  $[0,2]$  power law dependence of acoustic absorption on frequency in various lossy media. *Chinese Phys. Lett.* **22**, No 20 (2005), 2601–2603.
- [5] W. Chen, Time-space fabric underlying anomalous diffusion. *Chaos. Soliton. Fract.* **28**, No 4 (2006), 923–929.

- [6] W. Chen, A speculative study of 2/3-order fractional Laplacian modeling of turbulence: Some thoughts and conjectures, *Chaos* **16**, No 2 (2006), 023126.
- [7] W. Chen, Z.J. Fu, C.S. Chen, *Recent Advances on Radial Basis Function Collocation Methods*. SpringerBriefs in Applied Sciences and Technology (2013); <https://www.springer.com/gp/book/9783642395710>.
- [8] W. Chen, Z. Fu, B.T. Jin, A truly boundary-only meshfree method for inhomogeneous problems based on recursive composite multiple reciprocity technique. *Eng. Anal. Bound. Elem.* **34**, No 3 (2010), 196–205.
- [9] W. Chen, Z.J. Fu, X. Wei, Potential problems by singular boundary method satisfying moment condition. *CMES-Comp. Model. Eng.* **54**, No 1 (2009), 65–85.
- [10] W. Chen, S. Holm, Modified Szabo's wave equation models for lossy media obeying frequency power law. *J. Acoust. Soc. Am.* **114**, No 5 (2003), 2570–2574.
- [11] W. Chen, S. Holm, Fractional Laplacian time-space models for linear and nonlinear lossy media exhibiting arbitrary frequency power-law dependency. *J. Acoust. Soc. Am.* **115**, No 4 (2004), 1424–1430.
- [12] W. Chen, S. Liang, T. Zhong, On the DQ analysis of geometrically nonlinear vibration of immovably simply supported beams. *J. Sound. Vib.* **206**, No 5 (1997), 745–748.
- [13] W. Chen, Y. Liang, X. Hei, Structural derivative based on inverse Mittag-Leffler function for modeling ultraslow diffusion. *Fract. Calc. Appl. Anal.* **19**, No 5 (2016), 1250–1261; DOI: 10.1515/fca-2016-0064; <https://www.degruyter.com/view/j/fca.2016.19.issue-5/issue-files/fca.2016.19.issue-5.xml>.
- [14] W. Chen, Z.J. Shen, L.J. Shen, G.W Yuan, General solutions and fundamental solutions of varied orders to the vibrational thin, the Berger, and the Winkler plates. *Eng. Anal. Bound. Elem.* **29**, No 7 (2005), 699–702.
- [15] W. Chen, C. Shu, W. He, The DQ solution of geometrically nonlinear bending of orthotropic rectangular plates by using Hadamard and SJT product. *Comput. Struct.* **74**, No 1 (2000), 65–74.
- [16] W. Chen, H.G. Sun, X. Zhang, D. Korosak, Anomalous diffusion modeling by fractal and fractional derivatives. *Comput. Math. Appl.* **59**, No 5 (2010), 1754–1758.
- [17] W. Chen, M. Tanaka, A meshless, exponential convergence, integration-free, and boundary-only RBF technique. *Comput. Math. Appl.* **43** (2002), 379–391.
- [18] W. Chen, F. Wang, A method of fundamental solutions without fictitious boundary. *Eng. Anal. Bound. Elem.* **34**, No 5 (2010), 530–532.

- [19] W. Chen, X. Zhang, X. Cai, A study on modified Szabo's wave equation modeling of frequency-dependent dissipation in ultrasonic medical imaging. *Phys. Scripta* **T136** (2009), # 014014.
- [20] Z. Fu, W. Chen, H.T. Yang, Boundary particle method for Laplace transformed time fractional diffusion equations. *J. Comput. Phys.* **235**, No 15 (2013), 52–66.
- [21] Z. Fu, W. Chen, W. Yang, Winkler plate bending problems by a truly boundary-only boundary particle method. *Comput. Mech.* **44**, No 6 (2009), 757–763.
- [22] Y. Liang, W. Chen, A non-local structural derivative model for characterization of ultraslow diffusion in dense colloids. *Commun. Nonlinear. Sci.* **56** (2018), 131–137.
- [23] H.G. Sun, M.M. Meerschaert, Y. Zhang, J.T. Zhu, W. Chen, A fractal Richards' equation to capture the non-Boltzmann scaling of water transport in unsaturated media. *Adv. Water. Resour.* **52** (2013), 292–295.
- [24] X. Zhang, W. Chen, C. Zhang, Modified Szabo's wave equation for arbitrarily frequency-dependent viscous dissipation in soft matter with applications to 3D ultrasonic imaging. *Acta. Mech. Solida. Sin.* **25**, No 5 (2012), 510–519.

Prepared by: *Hongguang Sun*

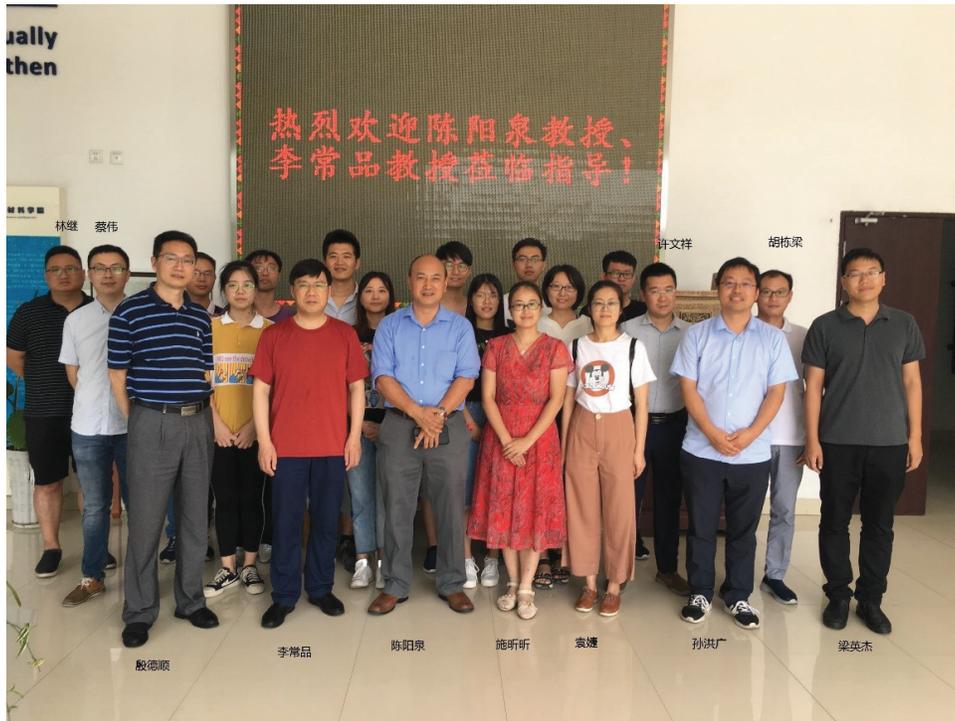
### **3. Report on a 2019 FC Related Meeting:**

**A half day workshop dedicated to the late Prof. Wen Chen  
at Nanjing, China, August 8, 2019**

**<http://mechatronics.ucmerced.edu/rememberwenchen>**

A half day workshop was organized and hosted by Prof. Hongguang Sun of Hohai University to remember the late Professor Wen Chen was held on August 8, 2019 at Hohai University's Jiangning Campus, Nanjing, China. Professor Changpin Li of Shanghai University and Professor YangQuan Chen of University of California Merced, both close friends of the late Prof. Wen Chen, attended this event. Profs. C. Li and Y. Q. Chen respectively shared some personal recollections and memories of Wen and then each gave a technical talk. Prof. Li's talk is entitled "Caputo-Hadamard fractional differential equation: analysis and computation" and Prof. Y.Q. Chen's talk title is "Optimal way to optimize using optimized randomness and its connection to fractional calculus". Details and slides can be found in the above web page dedicated to remembering Prof. Wen Chen where readers can find detailed biography of the late Prof. Wen Chen, list of papers, as well as a poem collection written by Wen in Chinese. This event was well

attended and the attendees are posed for a group photo after the workshop as seen below.



From left to right of the front row:  
 Prof. Deshun Yin, Prof. Changpin Li, Prof. YangQuan Chen,  
 Prof. Xinxin Shi, Ms. Jie Yuan, Prof. HongGuang Sun, and Prof. Yingjie  
 Liang; Two more faculty on the left (back row): Prof. Ji Lin,  
 Prof. Wei Cai; Two faculty on the right (back row):  
 Prof. Wenxiang Xu, Prof. Dongliang Hu.

Prepared by: *YangQuan Chen*

#### 4. Book Review

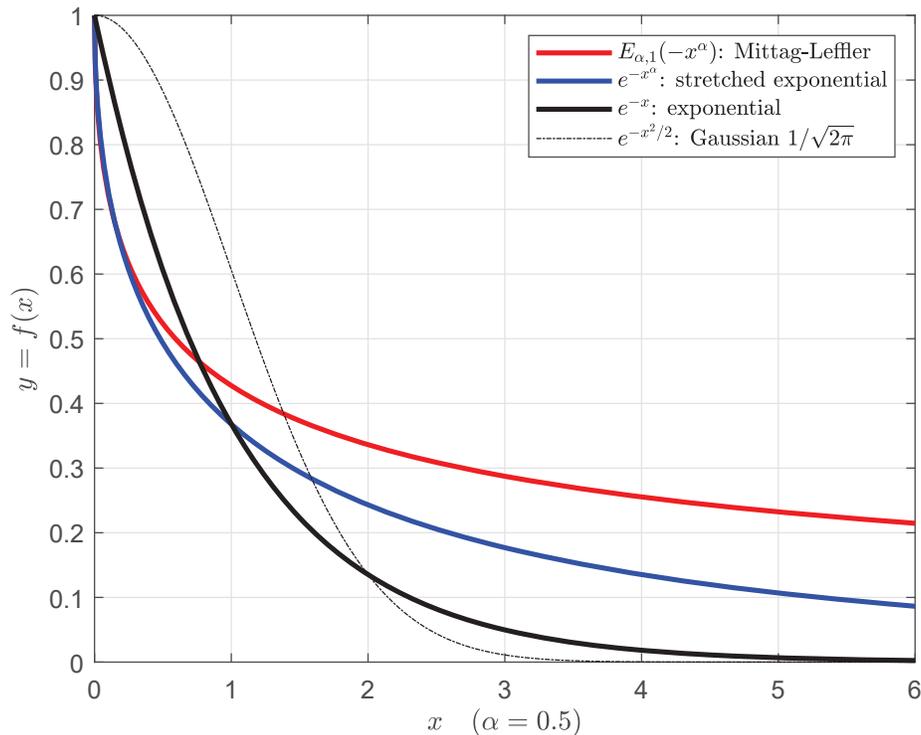
**Yingjie Liang, Wen Chen, Wei Cai, *Hausdorff Calculus: Applications to Fractal Systems***, Vol. 6 of “Fractional Calculus in Applied Sciences and Engineering” Book Series. ISBN 978-3-11-060852-6, March 2019, De Gruyter GmbH. 24.0 x 17.0 cm, 138 pages, 97 Figures, 30 Tables, <https://www.degruyter.com/view/product/506187>.

**Mini Book Review:****Why?**

It is a perfect occasion to present this mini-review for the last book of Wen Chen. Another reason is, we have some fresh views to share on this book that we believe is useful to the community.

**Importance:**

During the summer 2019, we were discussing that “Tails matter” in complex worlds and the inverse power law (IPL) and exponential law (EL) seem to be two isolated worlds. We were both impressed by the new book “Hausdorff Calculus: Applications to Fractal Systems” by Liang, Chen and Cai in the sense that it bridges the IPL and EL by using a stretched exponential law (SEL) based on the introduced Hausdorff Calculus as seen in the following figure we created:



We comment that the need of richer methods to characterize the tail behavior is for sure going to continue as we face more and more complex

worlds. We mention here that, the so-called “tempered” fractional calculus is among such methods for tail behavior characterization and regulation.

### **What?**

Unlike the fractional derivative, the Hausdorff fractal derivative, one kind of fractal derivatives introduced by Wen Chen in 2006, is a local derivative instead of a global operation. Note that here “local” does not really imply integer order calculus and we should add “tunable degree of locality” in the context of understanding. Thus, the computing costs of the Hausdorff fractal derivative are far less than the global fractional derivative, while performing almost equally well in modeling a variety of complex problems. In particular, the Hausdorff fractal derivative diffusion equation characterizes the stretched Gaussian process in space and fractal exponential decay, also known as the stretched relaxation, in time. In contrast, the fractional derivative diffusion equation underlies the Levy statistics in space and the ML function power law decay in time. Hausdorff calculus operator in space also underlies the non-Euclidean distance and has potential applications in a variety of complex problems, and the corresponding fundamental solutions of the Hausdorff partial differential equations have been analytically derived and numerically verified. The methodology consists of self-contained approaches and theory of calculus, statistics, geometry, metric, and computation. The Hausdorff calculus method significantly extends the application scope of the classical calculus modeling approach under the framework of continuum mechanics to fractal materials. It is noted that nowadays there exist quite a few different definitions of fractal derivative, among which, to the best understanding and knowledge, the Hausdorff fractal derivative is mathematically simplest and numerically easiest to implement with clear physical significance and the most real-world applications.

### **Conclusions:**

This is the first book of fractal derivative and its comprehensive applications, which are related to but evidently different from the fractional calculus. It is also physics informed and application oriented and we strongly recommend this book to the readers who are still being puzzled by the two separate worlds of IPL and EL.

Prepared by: *YangQuan Chen and Igor Podlubny*

---

**P.S. Editor-in-Chief's Last Minute note:  
Calendar of FC related events**

---

**ICFDA 2020: International Conference on Fractional  
Differentiation and Applications '2020**

Organizers's announce: We hereby inform that ICFDA 2020 will take place on **23-25 September 2020 in Lodz, Poland**. At the moment we are determining the conference organization details. Please visit our website for some details, it will be updated soon: <https://icfda2020.p.lodz.pl/>

---

**NSFDE&A'20, International Workshop  
"Numerical Solution of Fractional Differential Equations  
and Applications"  
June 8-13, 2020, Sozopol, Bulgaria**

**1st Announce at: [http://parallel.bas.bg/Conferences/  
NSFDE&A\\_2020-Sozopol.pdf](http://parallel.bas.bg/Conferences/NSFDE&A_2020-Sozopol.pdf)**

See details in the previous Editorial Note of FCAA issue No 5, at [https://www.degruyter.com/view/j/fca.2019.22.issue-5/  
fca-2019-0061/fca-2019-0061.xml?format=INT](https://www.degruyter.com/view/j/fca.2019.22.issue-5/fca-2019-0061/fca-2019-0061.xml?format=INT).

This is a reminder, since some deadlines are coming soon: – Intention to participate: Jan. 31, 2020; – Submission of abstracts: Feb. 15, 2020, etc. E-mail for contacts: [nsfdea20@parallel.bas.bg](mailto:nsfdea20@parallel.bas.bg).

---

Guest-Editors of this Special Issue:

*YangQuan Chen* (University of California Merced, USA)

*Changpin Li* (Shanghai University, CHINA)

*Igor Podlubny* (Technical University of Kosice, SLOVAKIA)

*Hongguang Sun* (Hohai University, CHINA)

---

Please cite to this paper as "Editorial, FCAA–Volume 22–6–2019", publ. in: *Fract. Calc. Appl. Anal.*, Vol. **22**, No 6 (2019), pp. 1437–1448, DOI: 10.1515/fca-2019-0075; at <https://www.degruyter.com/view/j/fca>.