

# Connectedness and Continuous Sequences in Finite Topological Spaces

Yatsuka Nakamura  
Shinshu University  
Nagano, Japan

**Summary.** First, equivalence conditions for connectedness are examined for a finite topological space (originated in [9]). Secondly, definitions of subspace, and components of the subspace of a finite topological space are given. Lastly, concepts of continuous finite sequence and minimum path of finite topological space are proposed.

MML identifier: FINTOP06, version: 7.8.01 4.70.946

The articles [16], [5], [18], [13], [1], [19], [14], [3], [4], [2], [6], [12], [10], [15], [7], [11], [8], and [17] provide the terminology and notation for this paper.

## 1. CONNECTEDNESS AND SUBSPACES

In this paper  $F_1$  denotes a non empty finite topology space and  $A, B, C$  denote subsets of  $F_1$ .

Let us consider  $F_1$ . One can check that  $\emptyset_{(F_1)}$  is connected.

We now state two propositions:

- (1) For all subsets  $A, B$  of  $F_1$  holds  $(A \cup B)^b = A^b \cup B^b$ .
- (2)  $(\emptyset_{(F_1)})^b = \emptyset$ .

Let us consider  $F_1$ . Observe that  $(\emptyset_{(F_1)})^b$  is empty.

Next we state the proposition

- (3) Let  $A$  be a subset of  $F_1$ . Suppose that for all subsets  $B, C$  of  $F_1$  such that  $A = B \cup C$  and  $B \neq \emptyset$  and  $C \neq \emptyset$  and  $B$  misses  $C$  holds  $B^b$  meets  $C$  and  $B$  meets  $C^b$ . Then  $A$  is connected.

Let  $F_1$  be a non empty finite topology space. We say that  $F_1$  is connected if and only if:

(Def. 1)  $\Omega_{(F_1)}$  is connected.

We now state four propositions:

- (4) Let  $A$  be a subset of  $F_1$ . Suppose  $A$  is connected. Let  $A_2, B_2$  be subsets of  $F_1$ . Suppose  $A = A_2 \cup B_2$  and  $A_2$  misses  $B_2$  and  $A_2$  and  $B_2$  are separated. Then  $A_2 = \emptyset_{(F_1)}$  or  $B_2 = \emptyset_{(F_1)}$ .
- (5) Suppose  $F_1$  is connected. Let  $A, B$  be subsets of  $F_1$ . Suppose  $\Omega_{(F_1)} = A \cup B$  and  $A$  misses  $B$  and  $A$  and  $B$  are separated. Then  $A = \emptyset_{(F_1)}$  or  $B = \emptyset_{(F_1)}$ .
- (6) For all subsets  $A, B$  of  $F_1$  such that  $F_1$  is symmetric and  $A^b$  misses  $B$  holds  $A$  misses  $B^b$ .
- (7) Let  $A$  be a subset of  $F_1$ . Suppose that
  - (i)  $F_1$  is symmetric, and
  - (ii) for all subsets  $A_2, B_2$  of  $F_1$  such that  $A = A_2 \cup B_2$  and  $A_2$  misses  $B_2$  and  $A_2$  and  $B_2$  are separated holds  $A_2 = \emptyset_{(F_1)}$  or  $B_2 = \emptyset_{(F_1)}$ .
 Then  $A$  is connected.

Let  $T$  be a finite topology space. A finite topology space is said to be a subspace of  $T$  if it satisfies the conditions (Def. 2).

- (Def. 2)(i) The carrier of it  $\subseteq$  the carrier of  $T$ ,
- (ii)  $\text{dom}(\text{the neighbour-map of it}) = \text{the carrier of it}$ , and
  - (iii) for every element  $x$  of it such that  $x \in \text{the carrier of it}$  holds (the neighbour-map of it)( $x$ ) = (the neighbour-map of  $T$ )( $x$ )  $\cap$  the carrier of it.

Let  $T$  be a finite topology space. Note that there exists a subspace of  $T$  which is strict.

Let  $T$  be a non empty finite topology space. Note that there exists a subspace of  $T$  which is strict and non empty.

Let  $T$  be a non empty finite topology space and let  $P$  be a non empty subset of  $T$ . The functor  $T \upharpoonright P$  yields a strict non empty subspace of  $T$  and is defined as follows:

(Def. 3)  $\Omega_{T \upharpoonright P} = P$ .

We now state the proposition

- (8) For every non empty subspace  $X$  of  $F_1$  such that  $F_1$  is filled holds  $X$  is filled.

Let  $F_1$  be a filled non empty finite topology space. Note that every non empty subspace of  $F_1$  is filled.

Next we state a number of propositions:

- (9) For every non empty subspace  $X$  of  $F_1$  such that  $F_1$  is symmetric holds  $X$  is symmetric.
- (10) For every subspace  $X'$  of  $F_1$  holds every subset of  $X'$  is a subset of  $F_1$ .

- (11) For every subset  $P$  of  $F_1$  holds  $P$  is closed iff  $P^c$  is open.
- (12) Let  $A$  be a subset of  $F_1$ . Then  $A$  is open if and only if the following conditions are satisfied:
- (i) for every element  $z$  of  $F_1$  such that  $U(z) \subseteq A$  holds  $z \in A$ , and
  - (ii) for every element  $x$  of  $F_1$  such that  $x \in A$  holds  $U(x) \subseteq A$ .
- (13) Let  $X'$  be a non empty subspace of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $A_1$  be a subset of  $X'$ . If  $A = A_1$ , then  $A_1^b = A^b \cap \Omega_{X'}$ .
- (14) Let  $X'$  be a non empty subspace of  $F_1$ ,  $P_1, Q_1$  be subsets of  $F_1$ , and  $P, Q$  be subsets of  $X'$ . Suppose  $P = P_1$  and  $Q = Q_1$ . If  $P$  and  $Q$  are separated, then  $P_1$  and  $Q_1$  are separated.
- (15) Let  $X'$  be a non empty subspace of  $F_1$ ,  $P, Q$  be subsets of  $F_1$ , and  $P_1, Q_1$  be subsets of  $X'$ . Suppose  $P = P_1$  and  $Q = Q_1$  and  $P \cup Q \subseteq \Omega_{X'}$ . If  $P$  and  $Q$  are separated, then  $P_1$  and  $Q_1$  are separated.
- (16) For every non empty subset  $A$  of  $F_1$  holds  $A$  is connected iff  $F_1 \upharpoonright A$  is connected.
- (17) Let  $F_1$  be a filled non empty finite topology space and  $A$  be a non empty subset of  $F_1$ . Suppose  $F_1$  is symmetric. Then  $A$  is connected if and only if for all subsets  $P, Q$  of  $F_1$  such that  $A = P \cup Q$  and  $P$  misses  $Q$  and  $P$  and  $Q$  are separated holds  $P = \emptyset_{(F_1)}$  or  $Q = \emptyset_{(F_1)}$ .
- (18) For every subset  $A$  of  $F_1$  such that  $F_1$  is filled and connected and  $A \neq \emptyset$  and  $A^c \neq \emptyset$  holds  $A^\delta \neq \emptyset$ .
- (19) For every subset  $A$  of  $F_1$  such that  $F_1$  is filled, symmetric, and connected and  $A \neq \emptyset$  and  $A^c \neq \emptyset$  holds  $A^{\delta_i} \neq \emptyset$ .
- (20) For every subset  $A$  of  $F_1$  such that  $F_1$  is filled, symmetric, and connected and  $A \neq \emptyset$  and  $A^c \neq \emptyset$  holds  $A^{\delta_o} \neq \emptyset$ .
- (21) For every subset  $A$  of  $F_1$  holds  $A^{\delta_i}$  misses  $A^{\delta_o}$ .
- (22) For every filled non empty finite topology space  $F_1$  and for every subset  $A$  of  $F_1$  holds  $A^{\delta_o} = A^b \setminus A$ .
- (23) For all subsets  $A, B$  of  $F_1$  such that  $A$  and  $B$  are separated holds  $A^{\delta_o}$  misses  $B$ .
- (24) Let  $A, B$  be subsets of  $F_1$ . Suppose  $F_1$  is filled and  $A$  misses  $B$  and  $A^{\delta_o}$  misses  $B$  and  $B^{\delta_o}$  misses  $A$ . Then  $A$  and  $B$  are separated.
- (25) For every point  $x$  of  $F_1$  holds  $\{x\}$  is connected.

Let us consider  $F_1$  and let  $x$  be a point of  $F_1$ . Note that  $\{x\}$  is connected.

Let  $F_1$  be a non empty finite topology space and let  $A$  be a subset of  $F_1$ .

We say that  $A$  is a component of  $F_1$  if and only if:

- (Def. 4)  $A$  is connected and for every subset  $B$  of  $F_1$  such that  $B$  is connected holds if  $A \subseteq B$ , then  $A = B$ .

One can prove the following propositions:

- (26) For every subset  $A$  of  $F_1$  such that  $A$  is a component of  $F_1$  holds  $A \neq \emptyset_{(F_1)}$ .
- (27) If  $A$  is closed and  $B$  is closed and  $A$  misses  $B$ , then  $A$  and  $B$  are separated.
- (28) If  $F_1$  is filled and  $\Omega_{(F_1)} = A \cup B$  and  $A$  and  $B$  are separated, then  $A$  is open and closed.
- (29) For all subsets  $A, B, A_1, B_1$  of  $F_1$  such that  $A$  and  $B$  are separated and  $A_1 \subseteq A$  and  $B_1 \subseteq B$  holds  $A_1$  and  $B_1$  are separated.
- (30) If  $A$  and  $B$  are separated and  $A$  and  $C$  are separated, then  $A$  and  $B \cup C$  are separated.
- (31) Suppose that
- (i)  $F_1$  is filled and symmetric, and
  - (ii) for all subsets  $A, B$  of  $F_1$  such that  $\Omega_{(F_1)} = A \cup B$  and  $A \neq \emptyset_{(F_1)}$  and  $B \neq \emptyset_{(F_1)}$  and  $A$  is closed and  $B$  is closed holds  $A$  meets  $B$ .
- Then  $F_1$  is connected.
- (32) Suppose  $F_1$  is connected. Let  $A, B$  be subsets of  $F_1$ . Suppose  $\Omega_{(F_1)} = A \cup B$  and  $A \neq \emptyset_{(F_1)}$  and  $B \neq \emptyset_{(F_1)}$  and  $A$  is closed and  $B$  is closed. Then  $A$  meets  $B$ .
- (33) If  $F_1$  is filled and  $A$  is connected and  $A \subseteq B \cup C$  and  $B$  and  $C$  are separated, then  $A \subseteq B$  or  $A \subseteq C$ .
- (34) Let  $A, B$  be subsets of  $F_1$ . Suppose  $F_1$  is symmetric and  $A$  is connected and  $B$  is connected and  $A$  and  $B$  are not separated. Then  $A \cup B$  is connected.
- (35) For all subsets  $A, C$  of  $F_1$  such that  $F_1$  is symmetric and  $C$  is connected and  $C \subseteq A$  and  $A \subseteq C^b$  holds  $A$  is connected.
- (36) For every subset  $C$  of  $F_1$  such that  $F_1$  is filled and symmetric and  $C$  is connected holds  $C^b$  is connected.
- (37) Suppose  $F_1$  is filled, symmetric, and connected and  $A$  is connected and  $\Omega_{(F_1)} \setminus A = B \cup C$  and  $B$  and  $C$  are separated. Then  $A \cup B$  is connected.
- (38) Let  $X'$  be a non empty subspace of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $B$  be a subset of  $X'$ . Suppose  $F_1$  is symmetric and  $A = B$ . Then  $A$  is connected if and only if  $B$  is connected.
- (39) For every subset  $A$  of  $F_1$  such that  $F_1$  is filled and symmetric and  $A$  is a component of  $F_1$  holds  $A$  is closed.
- (40) Let  $A, B$  be subsets of  $F_1$ . Suppose  $F_1$  is symmetric and  $A$  is a component of  $F_1$  and  $B$  is a component of  $F_1$ . Then  $A = B$  or  $A$  and  $B$  are separated.
- (41) Let  $A, B$  be subsets of  $F_1$ . Suppose  $F_1$  is filled and symmetric and  $A$  is a component of  $F_1$  and  $B$  is a component of  $F_1$ . Then  $A = B$  or  $A$  misses  $B$ .

- (42) Let  $C$  be a subset of  $F_1$ . Suppose  $F_1$  is filled and symmetric and  $C$  is connected. Let  $S$  be a subset of  $F_1$ . If  $S$  is a component of  $F_1$ , then  $C$  misses  $S$  or  $C \subseteq S$ .

Let  $F_1$  be a non empty finite topology space, let  $A$  be a non empty subset of  $F_1$ , and let  $B$  be a subset of  $F_1$ . We say that  $B$  is a component of  $A$  if and only if:

- (Def. 5) There exists a subset  $B_1$  of  $F_1 \upharpoonright A$  such that  $B_1 = B$  and  $B_1$  is a component of  $F_1 \upharpoonright A$ .

We now state the proposition

- (43) Let  $D$  be a non empty subset of  $F_1$ . Suppose  $F_1$  is filled and symmetric and  $D = \Omega_{(F_1)} \setminus A$ . Suppose  $F_1$  is connected and  $A$  is connected and  $C$  is a component of  $D$ . Then  $\Omega_{(F_1)} \setminus C$  is connected.

## 2. CONTINUOUS FINITE SEQUENCES AND MINIMUM PATH

Let us consider  $F_1$  and let  $f$  be a finite sequence of elements of  $F_1$ . We say that  $f$  is continuous if and only if the conditions (Def. 6) are satisfied.

- (Def. 6)(i)  $1 \leq \text{len } f$ , and  
(ii) for every natural number  $i$  and for every element  $x_1$  of  $F_1$  such that  $1 \leq i$  and  $i < \text{len } f$  and  $x_1 = f(i)$  holds  $f(i+1) \in U(x_1)$ .

Let us consider  $F_1$  and let  $x$  be an element of  $F_1$ . Observe that  $\langle x \rangle$  is continuous.

One can prove the following two propositions:

- (44) Let  $f$  be a finite sequence of elements of  $F_1$  and  $x, y$  be elements of  $F_1$ . If  $f$  is continuous and  $y = f(\text{len } f)$  and  $x \in U(y)$ , then  $f \hat{\ } \langle x \rangle$  is continuous.  
(45) Let  $f, g$  be finite sequences of elements of  $F_1$ . Suppose  $f$  is continuous and  $g$  is continuous and  $g(1) \in U(f_{\text{len } f})$ . Then  $f \hat{\ } g$  is continuous.

Let us consider  $F_1$  and let  $A$  be a subset of  $F_1$ . We say that  $A$  is arcwise connected if and only if the condition (Def. 7) is satisfied.

- (Def. 7) Let  $x_1, x_2$  be elements of  $F_1$ . Suppose  $x_1 \in A$  and  $x_2 \in A$ . Then there exists a finite sequence  $f$  of elements of  $F_1$  such that  $f$  is continuous and  $\text{rng } f \subseteq A$  and  $f(1) = x_1$  and  $f(\text{len } f) = x_2$ .

Let us consider  $F_1$ . Observe that  $\emptyset_{(F_1)}$  is arcwise connected.

Let us consider  $F_1$  and let  $x$  be an element of  $F_1$ . One can verify that  $\{x\}$  is arcwise connected.

The following three propositions are true:

- (46) For every subset  $A$  of  $F_1$  such that  $F_1$  is symmetric holds  $A$  is connected iff  $A$  is arcwise connected.  
(47) Let  $g$  be a finite sequence of elements of  $F_1$  and  $k$  be a natural number. If  $g$  is continuous and  $1 \leq k$ , then  $g \upharpoonright k$  is continuous.

- (48) Let  $g$  be a finite sequence of elements of  $F_1$  and  $k$  be an element of  $\mathbb{N}$ .  
If  $g$  is continuous and  $k < \text{len } g$ , then  $g \downarrow k$  is continuous.

Let us consider  $F_1$ , let  $g$  be a finite sequence of elements of  $F_1$ , let  $A$  be a subset of  $F_1$ , and let  $x_1, x_2$  be elements of  $F_1$ . We say that  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  if and only if the conditions (Def. 8) are satisfied.

- (Def. 8)(i)  $g$  is continuous,  
(ii)  $\text{rng } g \subseteq A$ ,  
(iii)  $g(1) = x_1$ ,  
(iv)  $g(\text{len } g) = x_2$ , and  
(v) for every finite sequence  $h$  of elements of  $F_1$  such that  $h$  is continuous and  $\text{rng } h \subseteq A$  and  $h(1) = x_1$  and  $h(\text{len } h) = x_2$  holds  $\text{len } g \leq \text{len } h$ .

One can prove the following propositions:

- (49) For every subset  $A$  of  $F_1$  and for every element  $x$  of  $F_1$  such that  $x \in A$  holds  $\langle x \rangle$  is minimum path in  $A$  between  $x$  and  $x$ .
- (50) Let  $A$  be a subset of  $F_1$ . Then  $A$  is arcwise connected if and only if for all elements  $x_1, x_2$  of  $F_1$  such that  $x_1 \in A$  and  $x_2 \in A$  holds there exists a finite sequence of elements of  $F_1$  which is minimum path in  $A$  between  $x_1$  and  $x_2$ .
- (51) Let  $A$  be a subset of  $F_1$  and  $x_1, x_2$  be elements of  $F_1$ . Given a finite sequence  $f$  of elements of  $F_1$  such that  $f$  is continuous and  $\text{rng } f \subseteq A$  and  $f(1) = x_1$  and  $f(\text{len } f) = x_2$ . Then there exists a finite sequence of elements of  $F_1$  which is minimum path in  $A$  between  $x_1$  and  $x_2$ .
- (52) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ ,  $x_1, x_2$  be elements of  $F_1$ , and  $k$  be an element of  $\mathbb{N}$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  and  $1 \leq k$  and  $k \leq \text{len } g$ . Then  $g \uparrow k$  is continuous and  $\text{rng}(g \uparrow k) \subseteq A$  and  $(g \uparrow k)(1) = x_1$  and  $(g \uparrow k)(\text{len}(g \uparrow k)) = g_k$ .
- (53) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ ,  $x_1, x_2$  be elements of  $F_1$ , and  $k$  be an element of  $\mathbb{N}$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  and  $k < \text{len } g$ . Then  $g \downarrow k$  is continuous and  $\text{rng}(g \downarrow k) \subseteq A$  and  $g \downarrow k(1) = g_{1+k}$  and  $g \downarrow k(\text{len}(g \downarrow k)) = x_2$ .
- (54) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $x_1, x_2$  be elements of  $F_1$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$ . Let  $k$  be a natural number. If  $1 \leq k$  and  $k \leq \text{len } g$ , then  $g \uparrow k$  is minimum path in  $A$  between  $x_1$  and  $g_k$ .
- (55) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $x_1, x_2$  be elements of  $F_1$ . If  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$ , then  $g$  is one-to-one.

Let us consider  $F_1$  and let  $f$  be a finite sequence of elements of  $F_1$ . We say that  $f$  is inversely continuous if and only if the conditions (Def. 9) are satisfied.

- (Def. 9)(i)  $1 \leq \text{len } f$ , and

- (ii) for all natural numbers  $i, j$  and for every element  $y$  of  $F_1$  such that  $1 \leq i$  and  $i \leq \text{len } f$  and  $1 \leq j$  and  $j \leq \text{len } f$  and  $y = f(i)$  and  $i \neq j$  and  $f(j) \in U(y)$  holds  $i = j + 1$  or  $j = i + 1$ .

We now state three propositions:

- (56) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $x_1, x_2$  be elements of  $F_1$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  and  $F_1$  is symmetric. Then  $g$  is inversely continuous.
- (57) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a subset of  $F_1$ , and  $x_1, x_2$  be elements of  $F_1$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  and  $F_1$  is filled and symmetric and  $x_1 \neq x_2$ . Then
- (i) for every natural number  $i$  such that  $1 < i$  and  $i < \text{len } g$  holds  $\text{rng } g \cap U(g_i) = \{g(i-1), g(i), g(i+1)\}$ ,
- (ii)  $\text{rng } g \cap U(g_1) = \{g(1), g(2)\}$ , and
- (iii)  $\text{rng } g \cap U(g_{\text{len } g}) = \{g(\text{len } g - 1), g(\text{len } g)\}$ .
- (58) Let  $g$  be a finite sequence of elements of  $F_1$ ,  $A$  be a non empty subset of  $F_1$ ,  $x_1, x_2$  be elements of  $F_1$ , and  $B_0$  be a subset of  $F_1 \setminus A$ . Suppose  $g$  is minimum path in  $A$  between  $x_1$  and  $x_2$  and  $F_1$  is filled and symmetric and  $x_1 \neq x_2$  and  $B_0 = \{x_1\}$ . Let  $i$  be an element of  $\mathbb{N}$ . If  $i < \text{len } g$ , then  $g(i+1) \in \text{Finf}(B_0, i)$  and if  $i \geq 1$ , then  $g(i+1) \notin \text{Finf}(B_0, i-1)$ .

## REFERENCES

- [1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(1):41–46, 1990.
- [2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(1):107–114, 1990.
- [3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(1):55–65, 1990.
- [4] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(1):153–164, 1990.
- [5] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(1):47–53, 1990.
- [6] Hiroshi Imura and Masayoshi Eguchi. Finite topological spaces. *Formalized Mathematics*, 3(2):189–193, 1992.
- [7] Hiroshi Imura, Masami Tanaka, and Yatsuka Nakamura. Continuous mappings between finite and one-dimensional finite topological spaces. *Formalized Mathematics*, 12(3):381–384, 2004.
- [8] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Formalized Mathematics*, 3(2):275–278, 1992.
- [9] Yatsuka Nakamura. Finite topology concept for discrete spaces. In H. Umegaki, editor, *Proceedings of the Eleventh Symposium on Applied Functional Analysis*, pages 111–116, Noda-City, Chiba, Japan, 1988. Science University of Tokyo.
- [10] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(1):83–86, 1993.
- [11] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(1):223–230, 1990.
- [12] Masami Tanaka and Yatsuka Nakamura. Some set series in finite topological spaces. Fundamental concepts for image processing. *Formalized Mathematics*, 12(2):125–129, 2004.
- [13] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics*.

- [14] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(1):115–122, 1990.
- [15] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(1):25–34, 1990.
- [16] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(1):9–11, 1990.
- [17] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(3):575–579, 1990.
- [18] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(1):67–71, 1990.
- [19] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(1):73–83, 1990.

*Received August 18, 2006*

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