

# Several Classes of BCI-algebras and their Properties

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**Summary.** I have formalized the BCI-algebras closely following the book [6], sections 1.1 to 1.3, 1.6, 2.1 to 2.3, and 2.7. In this article the general theory of BCI-algebras and several classes of BCI-algebras are given.

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The articles [10], [4], [13], [9], [3], [12], [2], [11], [5], [7], [8], [1], and [14] provide the notation and terminology for this paper.

## 1. THE BASICS OF GENERAL THEORY OF BCI-ALGEBRAS

We introduce BCI structures which are extensions of 1-sorted structure and are systems

$\langle$  a carrier, an internal complement  $\rangle$ ,

where the carrier is a set and the internal complement is a binary operation on the carrier.

Let us note that there exists a BCI structure which is non empty and strict.

Let  $A$  be a BCI structure and let  $x, y$  be elements of  $A$ . The functor  $x \setminus y$  yielding an element of  $A$  is defined by:

(Def. 1)  $x \setminus y = (\text{the internal complement of } A)(x, y)$ .

We introduce BCI structures with 0 which are extensions of BCI structure and zero structure and are systems

$\langle$  a carrier, an internal complement, a zero  $\rangle$ ,

where the carrier is a set, the internal complement is a binary operation on the carrier, and the zero is an element of the carrier.

Let us note that there exists a BCI structure with 0 which is non empty and strict.

Let  $I_1$  be a non empty BCI structure with 0 and let  $x$  be an element of  $I_1$ . The functor  $x^c$  yields an element of  $I_1$  and is defined by:

(Def. 2)  $x^c = 0_{(I_1)} \setminus x$ .

Let  $I_1$  be a non empty BCI structure with 0. We say that  $I_1$  is B if and only if:

(Def. 3) For all elements  $x, y, z$  of  $I_1$  holds  $x \setminus y \setminus (z \setminus y) \setminus (x \setminus z) = 0_{(I_1)}$ .

We say that  $I_1$  is C if and only if:

(Def. 4) For all elements  $x, y, z$  of  $I_1$  holds  $x \setminus y \setminus z \setminus (x \setminus z \setminus y) = 0_{(I_1)}$ .

We say that  $I_1$  is I if and only if:

(Def. 5) For every element  $x$  of  $I_1$  holds  $x \setminus x = 0_{(I_1)}$ .

We say that  $I_1$  is K if and only if:

(Def. 6) For all elements  $x, y$  of  $I_1$  holds  $x \setminus y \setminus x = 0_{(I_1)}$ .

We say that  $I_1$  is BCI-4 if and only if:

(Def. 7) For all elements  $x, y$  of  $I_1$  such that  $x \setminus y = 0_{(I_1)}$  and  $y \setminus x = 0_{(I_1)}$  holds  $x = y$ .

We say that  $I_1$  is BCK-5 if and only if:

(Def. 8) For every element  $x$  of  $I_1$  holds  $x^c = 0_{(I_1)}$ .

The BCI structure BCI-EXAMPLE with 0 is defined as follows:

(Def. 9) BCI-EXAMPLE =  $\langle \{\emptyset\}, \text{op}_2, \text{op}_0 \rangle$ .

Let us note that BCI-EXAMPLE is strict and non empty.

One can verify that there exists a non empty BCI structure with 0 which is strict, B, C, I, and BCI-4.

A BCI-algebra is B C I BCI-4 non empty BCI structure with 0.

Let  $X$  be a BCI-algebra. A BCI-algebra is called a subalgebra of  $X$  if it satisfies the conditions (Def. 10).

(Def. 10)(i)  $0_{\text{it}} = 0_X$ ,

(ii) the carrier of it  $\subseteq$  the carrier of  $X$ , and

(iii) the internal complement of it = (the internal complement of  $X$ )  $\upharpoonright$  (the carrier of it).

The following proposition is true

(1) Let  $X$  be a non empty BCI structure with 0. Then  $X$  is a BCI-algebra if and only if the following conditions are satisfied:

(i)  $X$  is I and BCI-4, and

(ii) for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) \setminus (z \setminus y) = 0_X$  and  $x \setminus (x \setminus y) \setminus y = 0_X$ .

One can check that there exists a BCI-algebra which is strict and BCK-5.

A BCK-algebra is BCK-5 BCI-algebra.

Let  $I_1$  be a non empty BCI structure with 0 and let  $x, y$  be elements of  $I_1$ .

The predicate  $x \leq y$  is defined as follows:

(Def. 11)  $x \setminus y = 0_{(I_1)}$ .

We use the following convention:  $X$  denotes a BCI-algebra,  $x, y, z, u, a, b$  denote elements of  $X$ , and  $I_1$  denotes a non empty subset of  $X$ .

We now state a number of propositions:

- (2)  $x \setminus 0_X = x$ .
- (3) If  $x \setminus y = 0_X$  and  $y \setminus z = 0_X$ , then  $x \setminus z = 0_X$ .
- (4) If  $x \setminus y = 0_X$ , then  $x \setminus z \setminus (y \setminus z) = 0_X$  and  $z \setminus y \setminus (z \setminus x) = 0_X$ .
- (5) If  $x \leq y$ , then  $x \setminus z \leq y \setminus z$  and  $z \setminus y \leq z \setminus x$ .
- (6) If  $x \setminus y = 0_X$ , then  $(y \setminus x)^c = 0_X$ .
- (7)  $x \setminus y \setminus z = x \setminus z \setminus y$ .
- (8)  $x \setminus (x \setminus (x \setminus y)) = x \setminus y$ .
- (9)  $(x \setminus y)^c = x^c \setminus y^c$ .
- (10)  $x \setminus (x \setminus y) \setminus (y \setminus x) \setminus (x \setminus (x \setminus (y \setminus (y \setminus x)))) = 0_X$ .
- (11) Let  $X$  be a non empty BCI structure with 0. Then  $X$  is a BCI-algebra if and only if the following conditions are satisfied:
  - (i)  $X$  is BCI-4, and
  - (ii) for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) \setminus (z \setminus y) = 0_X$  and  $x \setminus 0_X = x$ .
- (12) If for every BCI-algebra  $X$  and for all elements  $x, y$  of  $X$  holds  $x \setminus (x \setminus y) = y \setminus (y \setminus x)$ , then  $X$  is a BCK-algebra.
- (13) If for every BCI-algebra  $X$  and for all elements  $x, y$  of  $X$  holds  $x \setminus y \setminus y = x \setminus y$ , then  $X$  is a BCK-algebra.
- (14) If for every BCI-algebra  $X$  and for all elements  $x, y$  of  $X$  holds  $x \setminus (y \setminus x) = x$ , then  $X$  is a BCK-algebra.
- (15) If for every BCI-algebra  $X$  and for all elements  $x, y, z$  of  $X$  holds  $(x \setminus y) \setminus y = x \setminus z \setminus (y \setminus z)$ , then  $X$  is a BCK-algebra.
- (16) If for every BCI-algebra  $X$  and for all elements  $x, y$  of  $X$  holds  $x \setminus y \setminus (y \setminus x) = x \setminus y$ , then  $X$  is a BCK-algebra.
- (17) If for every BCI-algebra  $X$  and for all elements  $x, y$  of  $X$  holds  $x \setminus y \setminus (x \setminus y \setminus (y \setminus x)) = 0_X$ , then  $X$  is a BCK-algebra.
- (18) For every BCI-algebra  $X$  holds  $X$  is K iff  $X$  is a BCK-algebra.

Let  $X$  be a BCI-algebra. The functor BCK-part  $X$  yielding a non empty subset of  $X$  is defined by:

(Def. 12) BCK-part  $X = \{x; x \text{ ranges over elements of } X: 0_X \leq x\}$ .

Next we state the proposition

(19)  $0_X \in \text{BCK-part } X$ .

Let us consider  $X$ . Note that  $0_X$

Next we state three propositions:

(20) For all elements  $x, y$  of BCK-part  $X$  holds  $x \setminus y \in \text{BCK-part } X$ .

(21) For every element  $x$  of  $X$  and for every element  $y$  of BCK-part  $X$  holds  $x \setminus y \leq x$ .

(22)  $X$  is a subalgebra of  $X$ .

Let  $X$  be a BCI-algebra and let  $I_1$  be a subalgebra of  $X$ . We say that  $I_1$  is proper if and only if:

(Def. 13)  $I_1 \neq X$ .

Let us consider  $X$ . Note that there exists a subalgebra of  $X$  which is non proper.

Let  $X$  be a BCI-algebra and let  $I_1$  be an element of  $X$ . We say that  $I_1$  is atom if and only if:

(Def. 14) For every element  $z$  of  $X$  such that  $z \setminus I_1 = 0_X$  holds  $z = I_1$ .

Let  $X$  be a BCI-algebra. The functor  $\text{AtomSet } X$  yields a non empty subset of  $X$  and is defined by:

(Def. 15)  $\text{AtomSet } X = \{x; x \text{ ranges over elements of } X: x \text{ is atom}\}$ .

One can prove the following propositions:

(23)  $0_X \in \text{AtomSet } X$ .

(24) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for every element  $z$  of  $X$  holds  $z \setminus (z \setminus x) = x$ .

(25) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for all elements  $u, z$  of  $X$  holds  $z \setminus u \setminus (z \setminus x) = x \setminus u$ .

(26) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for all elements  $y, z$  of  $X$  holds  $x \setminus (z \setminus y) \leq y \setminus (z \setminus x)$ .

(27) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for all elements  $y, z, u$  of  $X$  holds  $(x \setminus u) \setminus (z \setminus y) \leq y \setminus u \setminus (z \setminus x)$ .

(28) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for every element  $z$  of  $X$  holds  $z^c \setminus x^c = x \setminus z$ .

(29) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff  $(x^c)^c = x$ .

(30) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for every element  $z$  of  $X$  holds  $(z \setminus x)^c = x \setminus z$ .

(31) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for every element  $z$  of  $X$  holds  $((x \setminus z)^c)^c = x \setminus z$ .

(32) For every element  $x$  of  $X$  holds  $x \in \text{AtomSet } X$  iff for all elements  $z, u$  of  $X$  holds  $z \setminus (z \setminus (x \setminus u)) = x \setminus u$ .

- (33) For every element  $a$  of  $\text{AtomSet } X$  and for every element  $x$  of  $X$  holds  $a \setminus x \in \text{AtomSet } X$ .

Let  $X$  be a BCI-algebra and let  $a, b$  be elements of  $\text{AtomSet } X$ . Then  $a \setminus b$  is an element of  $\text{AtomSet } X$ .

One can prove the following propositions:

- (34) For every element  $x$  of  $X$  holds  $x^c \in \text{AtomSet } X$ .  
 (35) For every element  $x$  of  $X$  there exists an element  $a$  of  $\text{AtomSet } X$  such that  $a \leq x$ .

Let  $X$  be a BCI-algebra. We say that  $X$  is generated by atom if and only if:

- (Def. 16) For every element  $x$  of  $X$  there exists an element  $a$  of  $\text{AtomSet } X$  such that  $a \leq x$ .

Let  $X$  be a BCI-algebra and let  $a$  be an element of  $\text{AtomSet } X$ . The functor  $\text{BranchV } a$  yields a non empty subset of  $X$  and is defined as follows:

- (Def. 17)  $\text{BranchV } a = \{x; x \text{ ranges over elements of } X: a \leq x\}$ .

We now state several propositions:

- (36) Every BCI-algebra is generated by atom.  
 (37) For all elements  $a, b$  of  $\text{AtomSet } X$  and for every element  $x$  of  $\text{BranchV } b$  holds  $a \setminus x = a \setminus b$ .  
 (38) For every element  $a$  of  $\text{AtomSet } X$  and for every element  $x$  of  $\text{BCK-part } X$  holds  $a \setminus x = a$ .  
 (39) For all elements  $a, b$  of  $\text{AtomSet } X$  and for every element  $x$  of  $\text{BranchV } a$  and for every element  $y$  of  $\text{BranchV } b$  holds  $x \setminus y \in \text{BranchV}(a \setminus b)$ .  
 (40) For every element  $a$  of  $\text{AtomSet } X$  and for all elements  $x, y$  of  $\text{BranchV } a$  holds  $x \setminus y \in \text{BCK-part } X$ .  
 (41) For all elements  $a, b$  of  $\text{AtomSet } X$  and for every element  $x$  of  $\text{BranchV } a$  and for every element  $y$  of  $\text{BranchV } b$  such that  $a \neq b$  holds  $x \setminus y \notin \text{BCK-part } X$ .  
 (42) For all elements  $a, b$  of  $\text{AtomSet } X$  such that  $a \neq b$  holds  $\text{BranchV } a \cap \text{BranchV } b = \emptyset$ .

Let  $X$  be a BCI-algebra. A non empty subset of  $X$  is said to be an ideal of  $X$  if:

- (Def. 18)  $0_X \in \text{it}$  and for all elements  $x, y$  of  $X$  such that  $x \setminus y \in \text{it}$  and  $y \in \text{it}$  holds  $x \in \text{it}$ .

Let  $X$  be a BCI-algebra and let  $I_1$  be an ideal of  $X$ . We say that  $I_1$  is closed if and only if:

- (Def. 19) For every element  $x$  of  $I_1$  holds  $x^c \in I_1$ .

Let us consider  $X$ . Note that there exists an ideal of  $X$  which is closed.

Next we state four propositions:

- (43)  $\{0_X\}$  is a closed ideal of  $X$ .

- (44) The carrier of  $X$  is a closed ideal of  $X$ .
- (45) BCK-part  $X$  is a closed ideal of  $X$ .
- (46) If  $I_1$  is an ideal of  $X$ , then for all elements  $x, y$  of  $X$  such that  $x \in I_1$  and  $y \leq x$  holds  $y \in I_1$ .

## 2. ASSOCIATIVE BCI-ALGEBRAS

Let  $I_1$  be a BCI-algebra. We say that  $I_1$  is associative if and only if:

(Def. 20) For all elements  $x, y, z$  of  $I_1$  holds  $(x \setminus y) \setminus z = x \setminus (y \setminus z)$ .

We say that  $I_1$  is quasi-associative if and only if:

(Def. 21) For every element  $x$  of  $I_1$  holds  $(x^c)^c = x^c$ .

We say that  $I_1$  is positive-implicative if and only if:

(Def. 22) For all elements  $x, y$  of  $I_1$  holds  $(x \setminus (x \setminus y)) \setminus (y \setminus x) = x \setminus (x \setminus (y \setminus (y \setminus x)))$ .

We say that  $I_1$  is weakly-positive-implicative if and only if:

(Def. 23) For all elements  $x, y, z$  of  $I_1$  holds  $(x \setminus y) \setminus z = x \setminus z \setminus z \setminus (y \setminus z)$ .

We say that  $I_1$  is implicative if and only if:

(Def. 24) For all elements  $x, y$  of  $I_1$  holds  $(x \setminus (x \setminus y)) \setminus (y \setminus x) = y \setminus (y \setminus x)$ .

We say that  $I_1$  is weakly-implicative if and only if:

(Def. 25) For all elements  $x, y$  of  $I_1$  holds  $x \setminus (y \setminus x) \setminus (y \setminus x)^c = x$ .

We say that  $I_1$  is  $p$ -semisimple if and only if:

(Def. 26) For all elements  $x, y$  of  $I_1$  holds  $x \setminus (x \setminus y) = y$ .

We say that  $I_1$  is alternative if and only if:

(Def. 27) For all elements  $x, y$  of  $I_1$  holds  $x \setminus (x \setminus y) = (x \setminus x) \setminus y$  and  $(x \setminus y) \setminus y = x \setminus (y \setminus y)$ .

One can check that there exists a BCI-algebra which is implicative, positive-implicative,  $p$ -semisimple, associative, weakly-implicative, and weakly-positive-implicative.

Next we state several propositions:

- (47)  $X$  is associative iff for every element  $x$  of  $X$  holds  $x^c = x$ .
- (48) For all elements  $x, y$  of  $X$  holds  $y \setminus x = x \setminus y$  iff  $X$  is associative.
- (49) Let  $X$  be a non empty BCI structure with 0. Then  $X$  is an associative BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $y \setminus x \setminus (z \setminus x) = z \setminus y$  and  $x \setminus 0_X = x$ .
- (50) Let  $X$  be a non empty BCI structure with 0. Then  $X$  is an associative BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) = z \setminus y$  and  $x^c = x$ .

- (51) Let  $X$  be a non empty BCI structure with  $0$ . Then  $X$  is an associative BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) = y \setminus z$  and  $x \setminus 0_X = x$ .

### 3. $p$ -SEMISIMPLE BCI-ALGEBRAS

One can prove the following propositions:

- (52)  $X$  is  $p$ -semisimple iff every element of  $X$  is atom.  
 (53) If  $X$  is  $p$ -semisimple, then BCK-part  $X = \{0_X\}$ .  
 (54)  $X$  is  $p$ -semisimple iff for every element  $x$  of  $X$  holds  $(x^c)^c = x$ .  
 (55)  $X$  is  $p$ -semisimple iff for all  $x, y$  holds  $y \setminus (y \setminus x) = x$ .  
 (56)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  holds  $z \setminus y \setminus (z \setminus x) = x \setminus y$ .  
 (57)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  holds  $x \setminus (z \setminus y) = y \setminus (z \setminus x)$ .  
 (58)  $X$  is  $p$ -semisimple iff for all  $x, y, z, u$  holds  $(x \setminus u) \setminus (z \setminus y) = y \setminus u \setminus (z \setminus x)$ .  
 (59)  $X$  is  $p$ -semisimple iff for all  $x, z$  holds  $z^c \setminus x^c = x \setminus z$ .  
 (60)  $X$  is  $p$ -semisimple iff for all  $x, z$  holds  $((x \setminus z)^c)^c = x \setminus z$ .  
 (61)  $X$  is  $p$ -semisimple iff for all  $x, u, z$  holds  $z \setminus (z \setminus (x \setminus u)) = x \setminus u$ .  
 (62)  $X$  is  $p$ -semisimple iff for every  $x$  such that  $x^c = 0_X$  holds  $x = 0_X$ .  
 (63)  $X$  is  $p$ -semisimple iff for all  $x, y$  holds  $x \setminus y^c = y \setminus x^c$ .  
 (64)  $X$  is  $p$ -semisimple iff for all  $x, y, z, u$  holds  $(x \setminus y) \setminus (z \setminus u) = x \setminus z \setminus (y \setminus u)$ .  
 (65)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  holds  $x \setminus y \setminus (z \setminus y) = x \setminus z$ .  
 (66)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  holds  $x \setminus (y \setminus z) = (z \setminus y) \setminus x^c$ .  
 (67)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  such that  $y \setminus x = z \setminus x$  holds  $y = z$ .  
 (68)  $X$  is  $p$ -semisimple iff for all  $x, y, z$  such that  $x \setminus y = x \setminus z$  holds  $y = z$ .  
 (69) Let  $X$  be a non empty BCI structure with  $0$ . Then  $X$  is a  $p$ -semisimple BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) = z \setminus y$  and  $x \setminus 0_X = x$ .  
 (70) Let  $X$  be a non empty BCI structure with  $0$ . Then  $X$  is a  $p$ -semisimple BCI-algebra if and only if  $X$  is I and for all elements  $x, y, z$  of  $X$  holds  $x \setminus (y \setminus z) = z \setminus (y \setminus x)$  and  $x \setminus 0_X = x$ .

### 4. QUASI-ASSOCIATIVE BCI-ALGEBRAS

Next we state several propositions:

- (71)  $X$  is quasi-associative iff for every element  $x$  of  $X$  holds  $x^c \leq x$ .  
 (72)  $X$  is quasi-associative iff for all elements  $x, y$  of  $X$  holds  $(x \setminus y)^c = (y \setminus x)^c$ .  
 (73)  $X$  is quasi-associative iff for all elements  $x, y$  of  $X$  holds  $x^c \setminus y = (x \setminus y)^c$ .

- (74)  $X$  is quasi-associative iff for all elements  $x, y$  of  $X$  holds  $x \setminus y \setminus (y \setminus x) \in$  BCK-part  $X$ .
- (75)  $X$  is quasi-associative iff for all elements  $x, y, z$  of  $X$  holds  $(x \setminus y) \setminus z \leq x \setminus (y \setminus z)$ .

## 5. ALTERNATIVE BCI-ALGEBRAS

We now state several propositions:

- (76) If  $X$  is alternative, then  $x^c = x$  and  $x \setminus (x \setminus y) = y$  and  $x \setminus y \setminus y = x$ .
- (77) If  $X$  is alternative and  $x \setminus a = x \setminus b$ , then  $a = b$ .
- (78) If  $X$  is alternative and  $a \setminus x = b \setminus x$ , then  $a = b$ .
- (79) If  $X$  is alternative and  $x \setminus y = 0_X$ , then  $x = y$ .
- (80) If  $X$  is alternative and  $x \setminus a \setminus b = 0_X$ , then  $a = x \setminus b$  and  $b = x \setminus a$ .

One can check the following observations:

- \* every BCI-algebra which is alternative is also associative,
- \* every BCI-algebra which is associative is also alternative, and
- \* every BCI-algebra which is alternative is also implicative.

The following two propositions are true:

- (81) If  $X$  is alternative, then  $x \setminus (x \setminus y) \setminus (y \setminus x) = x$ .
- (82) If  $X$  is alternative, then  $y \setminus (y \setminus (x \setminus (x \setminus y))) = y$ .

## 6. IMPLICATIVE, POSITIVE-IMPLICATIVE, AND WEAKLY-POSITIVE-IMPLICATIVE BCI-ALGEBRAS

Let us observe that every BCI-algebra which is associative is also weakly-positive-implicative and every BCI-algebra which is  $p$ -semisimple is also weakly-positive-implicative.

We now state two propositions:

- (83) Let  $X$  be a non empty BCI structure with  $0$ . Then  $X$  is an implicative BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) \setminus (z \setminus y) = 0_X$  and  $x \setminus 0_X = x$  and  $(x \setminus (x \setminus y)) \setminus (y \setminus x) = y \setminus (y \setminus x)$ .
- (84)  $X$  is weakly-positive-implicative iff for all elements  $x, y$  of  $X$  holds  $x \setminus y = x \setminus y \setminus y \setminus y^c$ .

One can verify that every BCI-algebra which is positive-implicative is also weakly-positive-implicative and every BCI-algebra which is alternative is also weakly-positive-implicative.

One can prove the following two propositions:

- (85) Suppose  $X$  is a weakly-positive-implicative BCI-algebra. Let  $x, y$  be elements of  $X$ . Then  $(x \setminus (x \setminus y)) \setminus (y \setminus x) = y \setminus (y \setminus x) \setminus (y \setminus x) \setminus (x \setminus y)$ .



- (86) Let  $X$  be a non empty BCI structure with  $0$ . Then  $X$  is a positive-implicative BCI-algebra if and only if for all elements  $x, y, z$  of  $X$  holds  $x \setminus y \setminus (x \setminus z) \setminus (z \setminus y) = 0_X$  and  $x \setminus 0_X = x$  and  $x \setminus y = x \setminus y \setminus y \setminus y^c$  and  $(x \setminus (x \setminus y)) \setminus (y \setminus x) = y \setminus (y \setminus x) \setminus (y \setminus x) \setminus (x \setminus y)$ .

## REFERENCES

- [1] Józef Białas. Group and field definitions. *Formalized Mathematics*, 1(3):433–439, 1990.
- [2] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(1):175–180, 1990.
- [3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(1):55–65, 1990.
- [4] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(1):47–53, 1990.
- [5] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Formalized Mathematics*, 1(2):335–342, 1990.
- [6] Jie Meng and YoungLin Liu. *An Introduction to BCI-algebras*. Shaanxi Scientific and Technological Press, 2001.
- [7] Michał Muzalewski. Midpoint algebras. *Formalized Mathematics*, 1(3):483–488, 1990.
- [8] Michał Muzalewski. Construction of rings and left-, right-, and bi-modules over a ring. *Formalized Mathematics*, 2(1):3–11, 1991.
- [9] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics*.
- [10] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(1):9–11, 1990.
- [11] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(5):821–827, 1990.
- [12] Wojciech A. Trybulec. Vectors in real linear space. *Formalized Mathematics*, 1(2):291–296, 1990.
- [13] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(1):67–71, 1990.
- [14] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(1):73–83, 1990.

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