FORMALIZED MATHEMATICS
2007, Vol. 15, No. 3, Pages 73–79
DOI: 10.2478/v10037-007-0009-4

Several Differentiation Formulas of Special Functions. Part V

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Summary. In this article, we give several differentiation formulas of special and composite functions including trigonometric, polynomial and logarithmic functions.

MML identifier: FDIFF_9, version: 7.8.05 4.84.971

The articles [13], [15], [1], [16], [2], [4], [10], [11], [17], [5], [14], [12], [3], [7], [6], [9], and [8] provide the notation and terminology for this paper.

The partial function sec from $\mathbb{R}$ to $\mathbb{R}$ is defined as follows:

(Def. 1) $\sec = \frac{1}{\text{the function cos}}$.

The partial function cosec from $\mathbb{R}$ to $\mathbb{R}$ is defined by:

(Def. 2) $\mathrm{cosec} = \frac{1}{\text{the function sin}}$.

For simplicity, we follow the rules: x, a, b, c are real numbers, n is a natural number, Z is an open subset of $\mathbb{R}$, and f, f_1, f_2 are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) If $(\text{the function cos})(x) \neq 0$, then sec is differentiable in x and $(\sec)'(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(2) If $(\text{the function sin})(x) \neq 0$, then cosec is differentiable in x and $(\mathrm{cosec})'(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(3) $(\frac{1}{x})^{n}_{\mathbb{Z}} = \frac{1}{x^{n}_{\mathbb{Z}}}$.

(4) Suppose $Z \subseteq \mathrm{dom}\sec$. Then sec is differentiable on Z and for every x such that $x \in Z$ holds $(\sec)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

ISSN 1426–2630

(5) Suppose $Z \subseteq \operatorname{dom} \operatorname{cosec}$. Then cosec is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(6) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) $\sec \cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot f)'_{\restriction Z}(x) = \frac{a \cdot (\text{the function sin})(a \cdot x + b)}{(\text{the function cos})(a \cdot x + b)^{\mathbf{2}}}$.

(7) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) $\operatorname{cosec} \cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot f)'_{\restriction Z}(x) = -\frac{a \cdot (\text{the function cos})(a \cdot x + b)}{(\text{the function sin})(a \cdot x + b)^{\mathbf{2}}}$.

(8) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot \frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\sec \cdot \frac{1}{f}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot \frac{1}{f})'_{\restriction Z}(x) = -\frac{(\text{the function sin})(\frac{1}{x})}{x^{\mathbf{2}} \cdot (\text{the function cos})(\frac{1}{x})^{\mathbf{2}}}$.

(9) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot \frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\operatorname{cosec} \cdot \frac{1}{f}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot \frac{1}{f})'_{\restriction Z}(x) = \frac{(\text{the function cos})(\frac{1}{x})}{x^{\mathbf{2}} \cdot (\text{the function sin})(\frac{1}{x})^{\mathbf{2}}}$.

(10) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (f_1 + c\, f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$. Then

(i) $\sec \cdot (f_1 + c\, f_2)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (f_1 + c\, f_2))'_{\restriction Z}(x) = \frac{(b + 2 \cdot c \cdot x) \cdot (\text{the function sin})(a + b \cdot x + c \cdot x^{\mathbf{2}})}{(\text{the function cos})(a + b \cdot x + c \cdot x^{\mathbf{2}})^{\mathbf{2}}}$.

(11) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (f_1 + c\, f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$. Then

(i) $\operatorname{cosec} \cdot (f_1 + c\, f_2)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot (f_1 + c\, f_2))'_{\restriction Z}(x) = -\frac{(b + 2 \cdot c \cdot x) \cdot (\text{the function cos})(a + b \cdot x + c \cdot x^{\mathbf{2}})}{(\text{the function sin})(a + b \cdot x + c \cdot x^{\mathbf{2}})^{\mathbf{2}}}$.

(12) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function exp}))$. Then

(i) $\sec \cdot (\text{the function exp})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (\text{the function exp}))'_{\restriction Z}(x) = \frac{(\text{the function exp})(x) \cdot (\text{the function sin})((\text{the function exp})(x))}{(\text{the function cos})((\text{the function exp})(x))^{\mathbf{2}}}$.

(13) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function exp}))$. Then

(i) $\operatorname{cosec} \cdot (\text{the function exp})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\text{cosec} \cdot (\text{the function exp}))'_{\restriction Z}(x) = -\frac{(\text{the function exp})(x) \cdot (\text{the function cos})((\text{the function exp})(x))}{(\text{the function sin})((\text{the function exp})(x))^{\mathbf{2}}}$.

(14) Suppose $Z \subseteq \text{dom}(\text{sec} \cdot (\text{the function ln}))$. Then

(i) $\text{sec} \cdot (\text{the function ln})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\text{sec} \cdot (\text{the function ln}))'_{\restriction Z}(x) = \frac{(\text{the function sin})((\text{the function ln})(x))}{x \cdot (\text{the function cos})((\text{the function ln})(x))^{\mathbf{2}}}$.

(15) Suppose $Z \subseteq \text{dom}(\text{cosec} \cdot (\text{the function ln}))$. Then

(i) $\text{cosec} \cdot (\text{the function ln})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\text{cosec} \cdot (\text{the function ln}))'_{\restriction Z}(x) = -\frac{(\text{the function cos})((\text{the function ln})(x))}{x \cdot (\text{the function sin})((\text{the function ln})(x))^{\mathbf{2}}}$.

(16) Suppose $Z \subseteq \text{dom}((\text{the function exp}) \cdot \text{sec})$. Then

(i) $(\text{the function exp}) \cdot \text{sec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function exp}) \cdot \text{sec})'_{\restriction Z}(x) = \frac{(\text{the function exp})((\text{sec})(x)) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(17) Suppose $Z \subseteq \text{dom}((\text{the function exp}) \cdot \text{cosec})$. Then

(i) $(\text{the function exp}) \cdot \text{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function exp}) \cdot \text{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function exp})((\text{cosec})(x)) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(18) Suppose $Z \subseteq \text{dom}((\text{the function ln}) \cdot \text{sec})$. Then

(i) $(\text{the function ln}) \cdot \text{sec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function ln}) \cdot \text{sec})'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)}$.

(19) Suppose $Z \subseteq \text{dom}((\text{the function ln}) \cdot \text{cosec})$. Then

(i) $(\text{the function ln}) \cdot \text{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function ln}) \cdot \text{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)}$.

(20) Suppose $Z \subseteq \text{dom}((^{n}_{\mathbb{Z}}) \cdot \text{sec})$ and $1 \leq n$. Then

(i) $(^{n}_{\mathbb{Z}}) \cdot \text{sec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((^{n}_{\mathbb{Z}}) \cdot \text{sec})'_{\restriction Z}(x) = \frac{n \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}_{\mathbb{Z}}}$.

(21) Suppose $Z \subseteq \text{dom}((^{n}_{\mathbb{Z}}) \cdot \text{cosec})$ and $1 \leq n$. Then

(i) $(^{n}_{\mathbb{Z}}) \cdot \text{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((^{n}_{\mathbb{Z}}) \cdot \text{cosec})'_{\restriction Z}(x) = -\frac{n \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}_{\mathbb{Z}}}$.

(22) Suppose $Z \subseteq \text{dom}(\text{sec} - \text{id}_Z)$. Then

(i) $\text{sec} - \text{id}_Z$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\text{sec} - \text{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x) - (\text{the function cos})(x)^{\mathbf{2}}}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq \mathrm{dom}(-\mathrm{cosec} - \mathrm{id}_Z)$. Then
(i) $-\mathrm{cosec} - \mathrm{id}_Z$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(-\mathrm{cosec} - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function cos})(x) - (\text{the function sin})(x)^{2}}{(\text{the function sin})(x)^{2}}$.

(24) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp})\ \mathrm{sec})$. Then
(i) (the function exp) sec is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\text{the function exp})\ \mathrm{sec})'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{(\text{the function cos})(x)} + \frac{(\text{the function exp})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{2}}$.

(25) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp})\ \mathrm{cosec})$. Then
(i) (the function exp) cosec is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\text{the function exp})\ \mathrm{cosec})'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{(\text{the function sin})(x)} - \frac{(\text{the function exp})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{2}}$.

(26) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x$ and $a \neq 0$. Then
(i) $\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function sin})(a \cdot x) - (\text{the function cos})(a \cdot x)^{2}}{(\text{the function cos})(a \cdot x)^{2}}$.

(27) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x$ and $a \neq 0$. Then
(i) $(-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function cos})(a \cdot x) - (\text{the function sin})(a \cdot x)^{2}}{(\text{the function sin})(a \cdot x)^{2}}$.

(28) Suppose $Z \subseteq \mathrm{dom}(f\ \mathrm{sec})$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
(i) f sec is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(f\ \mathrm{sec})'_{\restriction Z}(x) = \frac{a}{(\text{the function cos})(x)} + \frac{(a \cdot x + b) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{2}}$.

(29) Suppose $Z \subseteq \mathrm{dom}(f\ \mathrm{cosec})$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
(i) f cosec is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(f\ \mathrm{cosec})'_{\restriction Z}(x) = \frac{a}{(\text{the function sin})(x)} - \frac{(a \cdot x + b) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{2}}$.

(30) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ \mathrm{sec})$. Then
(i) (the function ln) sec is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $((\text{the function ln})\ \mathrm{sec})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)}}{x} + \frac{(\text{the function ln})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{2}}$.

(31) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ \mathrm{cosec})$. Then

(i) (the function ln) cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function ln) cosec)$'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function sin})(x)}}{x} - \frac{(\text{the function ln})(x)\cdot(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(32) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{f}\ \mathrm{sec})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\frac{1}{f}$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{f}\ \mathrm{sec})'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function cos})(x)}}{x^{\mathbf{2}}} + \frac{\frac{(\text{the function sin})(x)}{x}}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(33) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{f}\ \mathrm{cosec})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\frac{1}{f}$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{f}\ \mathrm{cosec})'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)}}{x^{\mathbf{2}}} - \frac{\frac{(\text{the function cos})(x)}{x}}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(34) Suppose $Z \subseteq \mathrm{dom}(\mathrm{sec}\cdot(\text{the function sin}))$. Then

(i) sec $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{sec}\cdot(\text{the function sin}))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)\cdot(\text{the function sin})((\text{the function sin})(x))}{(\text{the function cos})((\text{the function sin})(x))^{\mathbf{2}}}$.

(35) Suppose $Z \subseteq \mathrm{dom}(\mathrm{sec}\cdot(\text{the function cos}))$. Then

(i) sec $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{sec}\cdot(\text{the function cos}))'_{\restriction Z}(x) = -\frac{(\text{the function sin})(x)\cdot(\text{the function sin})((\text{the function cos})(x))}{(\text{the function cos})((\text{the function cos})(x))^{\mathbf{2}}}$.

(36) Suppose $Z \subseteq \mathrm{dom}(\mathrm{cosec}\cdot(\text{the function sin}))$. Then

(i) cosec $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec}\cdot(\text{the function sin}))'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)\cdot(\text{the function cos})((\text{the function sin})(x))}{(\text{the function sin})((\text{the function sin})(x))^{\mathbf{2}}}$.

(37) Suppose $Z \subseteq \mathrm{dom}(\mathrm{cosec}\cdot(\text{the function cos}))$. Then

(i) cosec $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec}\cdot(\text{the function cos}))'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)\cdot(\text{the function cos})((\text{the function cos})(x))}{(\text{the function sin})((\text{the function cos})(x))^{\mathbf{2}}}$.

(38) Suppose $Z \subseteq \mathrm{dom}(\mathrm{sec}\cdot(\text{the function tan}))$. Then

(i) sec $\cdot$(the function tan) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{sec}\cdot(\text{the function tan}))'_{\restriction Z}(x) = \frac{\frac{(\text{the function sin})((\text{the function tan})(x))}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function cos})((\text{the function tan})(x))^{\mathbf{2}}}$.

(39) Suppose $Z \subseteq \mathrm{dom}(\mathrm{sec}\cdot(\text{the function cot}))$. Then

(i) sec $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{sec} \cdot (\text{the function cot}))'_{\restriction Z}(x) = -\frac{\frac{(\text{the function sin})((\text{the function cot})(x))}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function cos})((\text{the function cot})(x))^{\mathbf{2}}}$.

(40) Suppose $Z \subseteq \mathrm{dom}(\mathrm{cosec} \cdot (\text{the function tan}))$. Then

(i) $\mathrm{cosec} \cdot (\text{the function tan})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec} \cdot (\text{the function tan}))'_{\restriction Z}(x) = -\frac{\frac{(\text{the function cos})((\text{the function tan})(x))}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function sin})((\text{the function tan})(x))^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq \mathrm{dom}(\mathrm{cosec} \cdot (\text{the function cot}))$. Then

(i) $\mathrm{cosec} \cdot (\text{the function cot})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec} \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{\frac{(\text{the function cos})((\text{the function cot})(x))}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function sin})((\text{the function cot})(x))^{\mathbf{2}}}$.

(42) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan})\ \ \mathrm{sec})$. Then

(i) $(\text{the function tan})\ \ \mathrm{sec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan})\ \ \mathrm{sec})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function cos})(x)} + \frac{(\text{the function tan})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(43) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot})\ \ \mathrm{sec})$. Then

(i) $(\text{the function cot})\ \ \mathrm{sec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot})\ \ \mathrm{sec})'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function cos})(x)} + \frac{(\text{the function cot})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(44) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan})\ \ \mathrm{cosec})$. Then

(i) $(\text{the function tan})\ \ \mathrm{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan})\ \ \mathrm{cosec})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function sin})(x)} - \frac{(\text{the function tan})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(45) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot})\ \ \mathrm{cosec})$. Then

(i) $(\text{the function cot})\ \ \mathrm{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot})\ \ \mathrm{cosec})'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function sin})(x)} - \frac{(\text{the function cot})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

References

[1] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[2] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.

[3] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[4] Jarosław Kotowicz. Partial functions from a domain to a domain. *Formalized Mathematics*, 1(**4**):697–702, 1990.

[5] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 1(**4**):703–709, 1990.

[6] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.

[7] Rafał Kwiatek. Factorial and Newton coefficients. *Formalized Mathematics*, 1(**5**):887–890, 1990.
[8] Konrad Raczkowski. Integer and rational exponents. *Formalized Mathematics*, 2(**1**):125–130, 1991.
[9] Konrad Raczkowski and Paweł Sadowski. Real function differentiability. *Formalized Mathematics*, 1(**4**):797–801, 1990.
[10] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.
[11] Yasunari Shidama. The Taylor expansions. *Formalized Mathematics*, 12(**2**):195–200, 2004.
[12] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics.*
[13] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[14] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[15] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[16] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.
[17] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

Received July 9, 2007