

# Some Properties of Line and Column Operations on Matrices

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**Summary.** This article describes definitions of elementary operations about matrix and their main properties.

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The articles [8], [13], [17], [11], [1], [18], [5], [6], [2], [7], [15], [16], [9], [10], [20], [4], [3], [21], [12], [14], and [19] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention:  $j, k, l, n, m, i$  are natural numbers,  $K$  is a field,  $a$  is an element of  $K$ ,  $M, M_1$  are matrices over  $K$  of dimension  $n \times m$ , and  $A$  is a matrix over  $K$  of dimension  $n$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , and let  $l, k$  be natural numbers. The functor  $\text{InterchangeLine}(M, l, k)$  yielding a matrix over  $K$  of dimension  $n \times m$  is defined by the conditions (Def. 1).

- (Def. 1)(i)  $\text{len InterchangeLine}(M, l, k) = \text{len } M$ , and  
(ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $i = l$ , then  $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{k,j}$  and if  $i = k$ , then  $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{l,j}$  and if  $i \neq l$  and  $i \neq k$ , then  $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{i,j}$ .

The following three propositions are true:

- (1) For all matrices  $M_1, M_2$  over  $K$  of dimension  $n \times m$  holds width  $M_1 =$  width  $M_2$ .
- (2) Let given  $M, M_1, i$  such that  $l \in \text{dom } M$  and  $k \in \text{dom } M$  and  $i \in \text{dom } M$  and  $M_1 = \text{InterchangeLine}(M, l, k)$ . Then
  - (i) if  $i = l$ , then  $\text{Line}(M_1, i) = \text{Line}(M, k)$ ,
  - (ii) if  $i = k$ , then  $\text{Line}(M_1, i) = \text{Line}(M, l)$ , and
  - (iii) if  $i \neq l$  and  $i \neq k$ , then  $\text{Line}(M_1, i) = \text{Line}(M, i)$ .
- (3) For all  $a, i, j, M$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds  $(a \cdot \text{Line}(M, i))(j) = a \cdot M_{i,j}$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l$  be a natural number, and let  $a$  be an element of  $K$ . The functor  $\text{ScalarXLine}(M, l, a)$  yields a matrix over  $K$  of dimension  $n \times m$  and is defined by the conditions (Def. 2).

- (Def. 2)(i)  $\text{len } \text{ScalarXLine}(M, l, a) = \text{len } M$ , and
- (ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $i = l$ , then  $(\text{ScalarXLine}(M, l, a))_{i,j} = a \cdot M_{l,j}$  and if  $i \neq l$ , then  $(\text{ScalarXLine}(M, l, a))_{i,j} = M_{i,j}$ .

We now state the proposition

- (4) If  $l \in \text{dom } M$  and  $i \in \text{dom } M$  and  $a \neq 0_K$  and  $M_1 = \text{ScalarXLine}(M, l, a)$ , then if  $i = l$ , then  $\text{Line}(M_1, i) = a \cdot \text{Line}(M, l)$  and if  $i \neq l$ , then  $\text{Line}(M_1, i) = \text{Line}(M, i)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l, k$  be natural numbers, and let  $a$  be an element of  $K$ . Let us assume that  $l \in \text{dom } M$  and  $k \in \text{dom } M$ . The functor  $\text{RlineXScalar}(M, l, k, a)$  yielding a matrix over  $K$  of dimension  $n \times m$  is defined by the conditions (Def. 3).

- (Def. 3)(i)  $\text{len } \text{RlineXScalar}(M, l, k, a) = \text{len } M$ , and
- (ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $i = l$ , then  $(\text{RlineXScalar}(M, l, k, a))_{i,j} = a \cdot M_{k,j} + M_{l,j}$  and if  $i \neq l$ , then  $(\text{RlineXScalar}(M, l, k, a))_{i,j} = M_{i,j}$ .

We now state the proposition

- (5) If  $l \in \text{dom } M$  and  $k \in \text{dom } M$  and  $i \in \text{dom } M$  and  $M_1 = \text{RlineXScalar}(M, l, k, a)$ , then if  $i = l$ , then  $\text{Line}(M_1, i) = a \cdot \text{Line}(M, k) + \text{Line}(M, l)$  and if  $i \neq l$ , then  $\text{Line}(M_1, i) = \text{Line}(M, i)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , and let  $l, k$  be natural numbers. We introduce  $\text{ILine}(M, l, k)$  as a synonym of  $\text{InterchangeLine}(M, l, k)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l$  be a natural number, and let  $a$  be an element of  $K$ . We

introduce  $\text{SXLine}(M, l, a)$  as a synonym of  $\text{ScalarXLine}(M, l, a)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l, k$  be natural numbers, and let  $a$  be an element of  $K$ . We introduce  $\text{RLineXS}(M, l, k, a)$  as a synonym of  $\text{RlineXScalar}(M, l, k, a)$ .

We now state several propositions:

$$(6) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right) \cdot A = \text{ILine}(A, l, k).$$

$$(7) \text{ For all } l, a, A \text{ such that } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } a \neq 0_K \text{ holds}$$

$$\text{SXLine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right) \cdot A = \text{SXLine}(A, l, a).$$

$$(8) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{RLineXS}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right) \cdot A = \text{RLineXS}(A, l, k, a).$$

$$(9) \text{ ILine}(M, k, k) = M.$$

$$(10) \text{ ILine}(M, l, k) = \text{ILine}(M, k, l).$$

$$(11) \text{ If } l \in \text{dom } M \text{ and } k \in \text{dom } M, \text{ then } \text{ILine}(\text{ILine}(M, l, k), l, k) = M.$$

$$(12) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right) \text{ is invertible and}$$

$$(\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right))^\sim = \text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right).$$

$$(13) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right)$$

and  $k \neq l$ , then  $\text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right)$  is invertible and

$$\left(\text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right)\right)^{\smile} = \text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, -a\right).$$

$$(14) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } a \neq 0_K, \text{ then}$$

$\text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right)$  is invertible and

$$\left(\text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right)\right)^{\smile} = \text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a^{-1}\right).$$

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , and let  $l, k$  be natural numbers. Let us assume that  $l \in \text{Seg width } M$  and  $k \in \text{Seg width } M$  and  $n > 0$  and  $m > 0$ . The functor  $\text{InterchangeCol}(M, l, k)$  yields a matrix over  $K$  of dimension  $n \times m$  and is defined by the conditions (Def. 4).

(Def. 4)(i)  $\text{len InterchangeCol}(M, l, k) = \text{len } M$ , and

(ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $j = l$ , then  $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,k}$  and if  $j = k$ , then  $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,l}$  and if  $j \neq l$  and  $j \neq k$ , then  $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,j}$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l$  be a natural number, and let  $a$  be an element of  $K$ . Let us assume that  $l \in \text{Seg width } M$  and  $n > 0$  and  $m > 0$ . The functor  $\text{ScalarXCol}(M, l, a)$  yielding a matrix over  $K$  of dimension  $n \times m$  is defined by the conditions (Def. 5).

(Def. 5)(i)  $\text{len ScalarXCol}(M, l, a) = \text{len } M$ , and

(ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $j = l$ , then  $(\text{ScalarXCol}(M, l, a))_{i,j} = a \cdot M_{i,l}$  and if  $j \neq l$ , then  $(\text{ScalarXCol}(M, l, a))_{i,j} = M_{i,j}$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l, k$  be natural numbers, and let  $a$  be an element of  $K$ . Let us assume that  $l \in \text{Seg width } M$  and  $k \in \text{Seg width } M$  and  $n > 0$  and  $m > 0$ . The functor  $\text{RcolXScalar}(M, l, k, a)$  yielding a matrix over  $K$  of dimension  $n \times m$  is defined by the conditions (Def. 6).

(Def. 6)(i)  $\text{len RcolXScalar}(M, l, k, a) = \text{len } M$ , and

(ii) for all  $i, j$  such that  $i \in \text{dom } M$  and  $j \in \text{Seg width } M$  holds if  $j = l$ , then  $(\text{RcolXScalar}(M, l, k, a))_{i,j} = a \cdot M_{i,k} + M_{i,l}$  and if  $j \neq l$ , then  $(\text{RcolXScalar}(M, l, k, a))_{i,j} = M_{i,j}$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , and let  $l, k$  be natural numbers. We introduce  $\text{ICol}(M, l, k)$  as a synonym of  $\text{InterchangeCol}(M, l, k)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l$  be a natural number, and let  $a$  be an element of  $K$ . We introduce  $\text{SXCol}(M, l, a)$  as a synonym of  $\text{ScalarXCol}(M, l, a)$ .

Let us consider  $n, m$ , let us consider  $K$ , let  $M$  be a matrix over  $K$  of dimension  $n \times m$ , let  $l, k$  be natural numbers, and let  $a$  be an element of  $K$ . We introduce  $\text{RColXS}(M, l, k, a)$  as a synonym of  $\text{RcolXScalar}(M, l, k, a)$ .

We now state several propositions:

(15) If  $l \in \text{Seg width } M$  and  $k \in \text{Seg width } M$  and  $n > 0$  and  $m > 0$  and  $M_1 = M^T$ , then  $(\text{ILine}(M_1, l, k))^T = \text{ICol}(M, l, k)$ .

(16) If  $l \in \text{Seg width } M$  and  $a \neq 0_K$  and  $n > 0$  and  $m > 0$  and  $M_1 = M^T$ , then  $(\text{SXLine}(M_1, l, a))^T = \text{SXCol}(M, l, a)$ .

(17) If  $l \in \text{Seg width } M$  and  $k \in \text{Seg width } M$  and  $n > 0$  and  $m > 0$  and  $M_1 = M^T$ , then  $(\text{RLineXS}(M_1, l, k, a))^T = \text{RColXS}(M, l, k, a)$ .

(18) If  $l \in \text{dom} \left( \begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$  and  $k \in \text{dom} \left( \begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$  and

$n > 0$ , then  $A \cdot \text{ICol} \left( \begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k \right) = \text{ICol}(A, l, k)$ .

(19) If  $l \in \text{dom} \left( \begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$  and  $a \neq 0_K$  and  $n > 0$ , then  $A \cdot$

$\text{SXCol} \left( \begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a \right) = \text{SXCol}(A, l, a)$ .

$$(20) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } n > 0, \text{ then } A \cdot \text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, k, a\right) = \text{RColXS}(A, l, k, a).$$

$$(21) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } n > 0, \text{ then } (\text{ICol}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, k\right))^\smile = \text{ICol}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, k\right).$$

$$(22) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } k \neq l \text{ and } n > 0, \text{ then } (\text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, k, a\right))^\smile = \text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, k, -a\right).$$

$$(23) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}\right) \text{ and } a \neq 0_K \text{ and } n > 0, \text{ then } (\text{SXCol}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, a\right))^\smile = \text{SXCol}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_K^{n \times n}, l, a^{-1}\right).$$

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