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# Some Properties of Line and Column Operations on Matrices

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**Summary.** This article describes definitions of elementary operations about matrix and their main properties.

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The articles [8], [13], [17], [11], [1], [18], [5], [6], [2], [7], [15], [16], [9], [10], [20], [4], [3], [21], [12], [14], and [19] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: $j$, $k$, $l$, $n$, $m$, $i$ are natural numbers, $K$ is a field, $a$ is an element of $K$, $M$, $M_1$ are matrices over $K$ of dimension $n \times m$, and $A$ is a matrix over $K$ of dimension $n$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, and let $l$, $k$ be natural numbers. The functor $\mathrm{InterchangeLine}(M, l, k)$ yielding a matrix over $K$ of dimension $n \times m$ is defined by the conditions (Def. 1).

(Def. 1)(i) $\mathrm{len}\,\mathrm{InterchangeLine}(M, l, k) = \mathrm{len}\,M$, and

(ii) for all $i$, $j$ such that $i \in \mathrm{dom}\,M$ and $j \in \mathrm{Seg}\,\mathrm{width}\,M$ holds if $i = l$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{k,j}$ and if $i = k$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{l,j}$ and if $i \neq l$ and $i \neq k$, then $(\mathrm{InterchangeLine}(M, l, k))_{i,j} = M_{i,j}$.




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The following three propositions are true:

(1) For all matrices $M_1$, $M_2$ over $K$ of dimension $n \times m$ holds $\operatorname{width} M_1 = \operatorname{width} M_2$.

(2) Let given $M$, $M_1$, $i$ such that $l \in \operatorname{dom} M$ and $k \in \operatorname{dom} M$ and $i \in \operatorname{dom} M$ and $M_1 = \operatorname{InterchangeLine}(M, l, k)$. Then

(i) if $i = l$, then $\operatorname{Line}(M_1, i) = \operatorname{Line}(M, k)$,

(ii) if $i = k$, then $\operatorname{Line}(M_1, i) = \operatorname{Line}(M, l)$, and

(iii) if $i \neq l$ and $i \neq k$, then $\operatorname{Line}(M_1, i) = \operatorname{Line}(M, i)$.

(3) For all $a$, $i$, $j$, $M$ such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds $(a \cdot \operatorname{Line}(M, i))(j) = a \cdot M_{i,j}$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$ be a natural number, and let $a$ be an element of $K$. The functor $\operatorname{ScalarXLine}(M, l, a)$ yields a matrix over $K$ of dimension $n \times m$ and is defined by the conditions (Def. 2).

(Def. 2)(i) $\operatorname{len} \operatorname{ScalarXLine}(M, l, a) = \operatorname{len} M$, and

(ii) for all $i$, $j$ such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds if $i = l$, then $(\operatorname{ScalarXLine}(M, l, a))_{i,j} = a \cdot M_{l,j}$ and if $i \neq l$, then $(\operatorname{ScalarXLine}(M, l, a))_{i,j} = M_{i,j}$.

We now state the proposition

(4) If $l \in \operatorname{dom} M$ and $i \in \operatorname{dom} M$ and $a \neq 0_K$ and $M_1 = \operatorname{ScalarXLine}(M, l, a)$, then if $i = l$, then $\operatorname{Line}(M_1, i) = a \cdot \operatorname{Line}(M, l)$ and if $i \neq l$, then $\operatorname{Line}(M_1, i) = \operatorname{Line}(M, i)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$, $k$ be natural numbers, and let $a$ be an element of $K$. Let us assume that $l \in \operatorname{dom} M$ and $k \in \operatorname{dom} M$. The functor $\operatorname{RlineXScalar}(M, l, k, a)$ yielding a matrix over $K$ of dimension $n \times m$ is defined by the conditions (Def. 3).

(Def. 3)(i) $\operatorname{len} \operatorname{RlineXScalar}(M, l, k, a) = \operatorname{len} M$, and

(ii) for all $i$, $j$ such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds if $i = l$, then $(\operatorname{RlineXScalar}(M, l, k, a))_{i,j} = a \cdot M_{k,j} + M_{l,j}$ and if $i \neq l$, then $(\operatorname{RlineXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

We now state the proposition

(5) If $l \in \operatorname{dom} M$ and $k \in \operatorname{dom} M$ and $i \in \operatorname{dom} M$ and $M_1 = \operatorname{RlineXScalar}(M, l, k, a)$, then if $i = l$, then $\operatorname{Line}(M_1, i) = a \cdot \operatorname{Line}(M, k) + \operatorname{Line}(M, l)$ and if $i \neq l$, then $\operatorname{Line}(M_1, i) = \operatorname{Line}(M, i)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, and let $l$, $k$ be natural numbers. We introduce $\operatorname{ILine}(M, l, k)$ as a synonym of $\operatorname{InterchangeLine}(M, l, k)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$ be a natural number, and let $a$ be an element of $K$. We introduce SXLine$(M, l, a)$ as a synonym of ScalarXLine$(M, l, a)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$, $k$ be natural numbers, and let $a$ be an element of $K$. We introduce RLineXS$(M, l, k, a)$ as a synonym of RlineXScalar$(M, l, k, a)$.

We now state several propositions:

(6) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$, then $\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k) \cdot A = \mathrm{ILine}(A, l, k)$.

(7) For all $l$, $a$, $A$ such that $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$ holds $\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a) \cdot A = \mathrm{SXLine}(A, l, a)$.

(8) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$, then $\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a) \cdot A = \mathrm{RLineXS}(A, l, k, a)$.

(9) $\mathrm{ILine}(M, k, k) = M$.

(10) $\mathrm{ILine}(M, l, k) = \mathrm{ILine}(M, k, l)$.

(11) If $l \in \mathrm{dom}\, M$ and $k \in \mathrm{dom}\, M$, then $\mathrm{ILine}(\mathrm{ILine}(M, l, k), l, k) = M$.

(12) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$, then $\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k)$ is invertible and $(\mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k))^{\smile} = \mathrm{ILine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k)$.

(13) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \neq l$, then $\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a)$ is invertible and $(\mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a))^{\smile} = \mathrm{RLineXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n},$ $l, k, -a)$.

(14) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$, then $\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a)$ is invertible and $(\mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a))^{\smile} = \mathrm{SXLine}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a^{-1})$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, and let $l$, $k$ be natural numbers. Let us assume that $l \in \operatorname{Seg} \operatorname{width} M$ and $k \in \operatorname{Seg} \operatorname{width} M$ and $n > 0$ and $m > 0$. The functor $\mathrm{InterchangeCol}(M, l, k)$ yields a matrix over $K$ of dimension $n \times m$ and is defined by the conditions (Def. 4).

(Def. 4)(i) $\operatorname{len} \mathrm{InterchangeCol}(M, l, k) = \operatorname{len} M$, and

(ii) for all $i$, $j$ such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds if $j = l$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,k}$ and if $j = k$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,l}$ and if $j \neq l$ and $j \neq k$, then $(\mathrm{InterchangeCol}(M, l, k))_{i,j} = M_{i,j}$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$ be a natural number, and let $a$ be an element of $K$. Let us assume that $l \in \operatorname{Seg} \operatorname{width} M$ and $n > 0$ and $m > 0$. The functor $\mathrm{ScalarXCol}(M, l, a)$ yielding a matrix over $K$ of dimension $n \times m$ is defined by the conditions (Def. 5).

(Def. 5)(i) $\operatorname{len} \mathrm{ScalarXCol}(M, l, a) = \operatorname{len} M$, and

(ii) for all $i$, $j$ such that $i \in \operatorname{dom} M$ and $j \in \operatorname{Seg} \operatorname{width} M$ holds if $j = l$, then $(\mathrm{ScalarXCol}(M, l, a))_{i,j} = a \cdot M_{i,l}$ and if $j \neq l$, then $(\mathrm{ScalarXCol}(M, l, a))_{i,j} = M_{i,j}$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$, $k$ be natural numbers, and let $a$ be an element of $K$. Let us assume that $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $k \in \mathrm{Seg}\,\mathrm{width}\,M$ and $n > 0$ and $m > 0$. The functor $\mathrm{RcolXScalar}(M, l, k, a)$ yielding a matrix over $K$ of dimension $n \times m$ is defined by the conditions (Def. 6).

(Def. 6)(i) $\mathrm{len}\,\mathrm{RcolXScalar}(M, l, k, a) = \mathrm{len}\,M$, and

(ii) for all $i$, $j$ such that $i \in \mathrm{dom}\,M$ and $j \in \mathrm{Seg}\,\mathrm{width}\,M$ holds if $j = l$, then $(\mathrm{RcolXScalar}(M, l, k, a))_{i,j} = a \cdot M_{i,k} + M_{i,l}$ and if $j \neq l$, then $(\mathrm{RcolXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, and let $l$, $k$ be natural numbers. We introduce $\mathrm{ICol}(M, l, k)$ as a synonym of $\mathrm{InterchangeCol}(M, l, k)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$ be a natural number, and let $a$ be an element of $K$. We introduce $\mathrm{SXCol}(M, l, a)$ as a synonym of $\mathrm{ScalarXCol}(M, l, a)$.

Let us consider $n$, $m$, let us consider $K$, let $M$ be a matrix over $K$ of dimension $n \times m$, let $l$, $k$ be natural numbers, and let $a$ be an element of $K$. We introduce $\mathrm{RColXS}(M, l, k, a)$ as a synonym of $\mathrm{RcolXScalar}(M, l, k, a)$.

We now state several propositions:

(15) If $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $k \in \mathrm{Seg}\,\mathrm{width}\,M$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\mathrm{ILine}(M_1, l, k))^{\mathrm{T}} = \mathrm{ICol}(M, l, k)$.

(16) If $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $a \neq 0_K$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\mathrm{SXLine}(M_1, l, a))^{\mathrm{T}} = \mathrm{SXCol}(M, l, a)$.

(17) If $l \in \mathrm{Seg}\,\mathrm{width}\,M$ and $k \in \mathrm{Seg}\,\mathrm{width}\,M$ and $n > 0$ and $m > 0$ and $M_1 = M^{\mathrm{T}}$, then $(\mathrm{RLineXS}(M_1, l, k, a))^{\mathrm{T}} = \mathrm{RColXS}(M, l, k, a)$.

(18) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $A \cdot \mathrm{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k) = \mathrm{ICol}(A, l, k)$.

(19) If $l \in \mathrm{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$ and $n > 0$, then $A \cdot \mathrm{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a) = \mathrm{SXCol}(A, l, a)$.

(20) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $A \cdot \mathrm{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a) = \mathrm{RColXS}(A, l, k, a)$.

(21) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $n > 0$, then $(\mathrm{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k))^{\smile} = \mathrm{ICol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k)$.

(22) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $k \neq l$ and $n > 0$, then $(\mathrm{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, a))^{\smile} = \mathrm{RColXS}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, k, -a)$.

(23) If $l \in \operatorname{dom}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n})$ and $a \neq 0_K$ and $n > 0$, then $(\mathrm{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a))^{\smile} = \mathrm{SXCol}(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_K^{n \times n}, l, a^{-1})$.

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