

Some Properties of Line and Column Operations on Matrices

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Summary. This article describes definitions of elementary operations about matrix and their main properties.

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The articles [8], [13], [17], [11], [1], [18], [5], [6], [2], [7], [15], [16], [9], [10], [20], [4], [3], [21], [12], [14], and [19] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: j, k, l, n, m, i are natural numbers, K is a field, a is an element of K , M, M_1 are matrices over K of dimension $n \times m$, and A is a matrix over K of dimension n .

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. The functor $\text{InterchangeLine}(M, l, k)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 1).

- (Def. 1)(i) $\text{len InterchangeLine}(M, l, k) = \text{len } M$, and
(ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $i = l$, then $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{k,j}$ and if $i = k$, then $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{l,j}$ and if $i \neq l$ and $i \neq k$, then $(\text{InterchangeLine}(M, l, k))_{i,j} = M_{i,j}$.

The following three propositions are true:

- (1) For all matrices M_1, M_2 over K of dimension $n \times m$ holds width $M_1 =$ width M_2 .
- (2) Let given M, M_1, i such that $l \in \text{dom } M$ and $k \in \text{dom } M$ and $i \in \text{dom } M$ and $M_1 = \text{InterchangeLine}(M, l, k)$. Then
 - (i) if $i = l$, then $\text{Line}(M_1, i) = \text{Line}(M, k)$,
 - (ii) if $i = k$, then $\text{Line}(M_1, i) = \text{Line}(M, l)$, and
 - (iii) if $i \neq l$ and $i \neq k$, then $\text{Line}(M_1, i) = \text{Line}(M, i)$.
- (3) For all a, i, j, M such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds $(a \cdot \text{Line}(M, i))(j) = a \cdot M_{i,j}$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K . The functor $\text{ScalarXLine}(M, l, a)$ yields a matrix over K of dimension $n \times m$ and is defined by the conditions (Def. 2).

- (Def. 2)(i) $\text{len } \text{ScalarXLine}(M, l, a) = \text{len } M$, and
- (ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $i = l$, then $(\text{ScalarXLine}(M, l, a))_{i,j} = a \cdot M_{l,j}$ and if $i \neq l$, then $(\text{ScalarXLine}(M, l, a))_{i,j} = M_{i,j}$.

We now state the proposition

- (4) If $l \in \text{dom } M$ and $i \in \text{dom } M$ and $a \neq 0_K$ and $M_1 = \text{ScalarXLine}(M, l, a)$, then if $i = l$, then $\text{Line}(M_1, i) = a \cdot \text{Line}(M, l)$ and if $i \neq l$, then $\text{Line}(M_1, i) = \text{Line}(M, i)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K . Let us assume that $l \in \text{dom } M$ and $k \in \text{dom } M$. The functor $\text{RlineXScalar}(M, l, k, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 3).

- (Def. 3)(i) $\text{len } \text{RlineXScalar}(M, l, k, a) = \text{len } M$, and
- (ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $i = l$, then $(\text{RlineXScalar}(M, l, k, a))_{i,j} = a \cdot M_{k,j} + M_{l,j}$ and if $i \neq l$, then $(\text{RlineXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

We now state the proposition

- (5) If $l \in \text{dom } M$ and $k \in \text{dom } M$ and $i \in \text{dom } M$ and $M_1 = \text{RlineXScalar}(M, l, k, a)$, then if $i = l$, then $\text{Line}(M_1, i) = a \cdot \text{Line}(M, k) + \text{Line}(M, l)$ and if $i \neq l$, then $\text{Line}(M_1, i) = \text{Line}(M, i)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. We introduce $\text{ILine}(M, l, k)$ as a synonym of $\text{InterchangeLine}(M, l, k)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K . We

introduce $\text{SXLine}(M, l, a)$ as a synonym of $\text{ScalarXLine}(M, l, a)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K . We introduce $\text{RLineXS}(M, l, k, a)$ as a synonym of $\text{RlineXScalar}(M, l, k, a)$.

We now state several propositions:

$$(6) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right) \cdot A = \text{ILine}(A, l, k).$$

$$(7) \text{ For all } l, a, A \text{ such that } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } a \neq 0_K \text{ holds}$$

$$\text{SXLine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right) \cdot A = \text{SXLine}(A, l, a).$$

$$(8) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{RLineXS}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right) \cdot A = \text{RLineXS}(A, l, k, a).$$

$$(9) \text{ ILine}(M, k, k) = M.$$

$$(10) \text{ ILine}(M, l, k) = \text{ILine}(M, k, l).$$

$$(11) \text{ If } l \in \text{dom } M \text{ and } k \in \text{dom } M, \text{ then } \text{ILine}(\text{ILine}(M, l, k), l, k) = M.$$

$$(12) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right), \text{ then}$$

$$\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right) \text{ is invertible and}$$

$$(\text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right))^\sim = \text{ILine}\left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right).$$

$$(13) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right)$$

and $k \neq l$, then $\text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right)$ is invertible and

$$\left(\text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right)\right)^{\smile} = \text{RLineXS}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, k, -a\right).$$

$$(14) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } a \neq 0_K, \text{ then}$$

$\text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right)$ is invertible and

$$\left(\text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right)\right)^{\smile} = \text{SXLine}\left(\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}_{K}^{n \times n}, l, a^{-1}\right).$$

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. Let us assume that $l \in \text{Seg width } M$ and $k \in \text{Seg width } M$ and $n > 0$ and $m > 0$. The functor $\text{InterchangeCol}(M, l, k)$ yields a matrix over K of dimension $n \times m$ and is defined by the conditions (Def. 4).

- (Def. 4)(i) $\text{len InterchangeCol}(M, l, k) = \text{len } M$, and
- (ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $j = l$, then $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,k}$ and if $j = k$, then $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,l}$ and if $j \neq l$ and $j \neq k$, then $(\text{InterchangeCol}(M, l, k))_{i,j} = M_{i,j}$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K . Let us assume that $l \in \text{Seg width } M$ and $n > 0$ and $m > 0$. The functor $\text{ScalarXCol}(M, l, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 5).

- (Def. 5)(i) $\text{len ScalarXCol}(M, l, a) = \text{len } M$, and
- (ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $j = l$, then $(\text{ScalarXCol}(M, l, a))_{i,j} = a \cdot M_{i,l}$ and if $j \neq l$, then $(\text{ScalarXCol}(M, l, a))_{i,j} = M_{i,j}$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K . Let us assume that $l \in \text{Seg width } M$ and $k \in \text{Seg width } M$ and $n > 0$ and $m > 0$. The functor $\text{RcolXScalar}(M, l, k, a)$ yielding a matrix over K of dimension $n \times m$ is defined by the conditions (Def. 6).

(Def. 6)(i) $\text{len RcolXScalar}(M, l, k, a) = \text{len } M$, and

(ii) for all i, j such that $i \in \text{dom } M$ and $j \in \text{Seg width } M$ holds if $j = l$, then $(\text{RcolXScalar}(M, l, k, a))_{i,j} = a \cdot M_{i,k} + M_{i,l}$ and if $j \neq l$, then $(\text{RcolXScalar}(M, l, k, a))_{i,j} = M_{i,j}$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, and let l, k be natural numbers. We introduce $\text{ICol}(M, l, k)$ as a synonym of $\text{InterchangeCol}(M, l, k)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l be a natural number, and let a be an element of K . We introduce $\text{SXCol}(M, l, a)$ as a synonym of $\text{ScalarXCol}(M, l, a)$.

Let us consider n, m , let us consider K , let M be a matrix over K of dimension $n \times m$, let l, k be natural numbers, and let a be an element of K . We introduce $\text{RColXS}(M, l, k, a)$ as a synonym of $\text{RcolXScalar}(M, l, k, a)$.

We now state several propositions:

(15) If $l \in \text{Seg width } M$ and $k \in \text{Seg width } M$ and $n > 0$ and $m > 0$ and $M_1 = M^T$, then $(\text{ILine}(M_1, l, k))^T = \text{ICol}(M, l, k)$.

(16) If $l \in \text{Seg width } M$ and $a \neq 0_K$ and $n > 0$ and $m > 0$ and $M_1 = M^T$, then $(\text{SXLine}(M_1, l, a))^T = \text{SXCol}(M, l, a)$.

(17) If $l \in \text{Seg width } M$ and $k \in \text{Seg width } M$ and $n > 0$ and $m > 0$ and $M_1 = M^T$, then $(\text{RLineXS}(M_1, l, k, a))^T = \text{RColXS}(M, l, k, a)$.

(18) If $l \in \text{dom} \left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$ and $k \in \text{dom} \left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$ and

$n > 0$, then $A \cdot \text{ICol} \left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k \right) = \text{ICol}(A, l, k)$.

(19) If $l \in \text{dom} \left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n} \right)$ and $a \neq 0_K$ and $n > 0$, then $A \cdot$

$\text{SXCol} \left(\begin{pmatrix} 1 & 0 \\ & \ddots \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a \right) = \text{SXCol}(A, l, a)$.

$$(20) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } n > 0, \text{ then } A \cdot \text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right) = \text{RColXS}(A, l, k, a).$$

$$(21) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } n > 0, \text{ then } (\text{ICol}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right))^{\smile} = \text{ICol}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k\right).$$

$$(22) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } k \neq l \text{ and } n > 0, \text{ then } (\text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k, a\right))^{\smile} = \text{RColXS}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, k, -a\right).$$

$$(23) \text{ If } l \in \text{dom}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}\right) \text{ and } a \neq 0_K \text{ and } n > 0, \text{ then } (\text{SXCol}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a\right))^{\smile} = \text{SXCol}\left(\begin{pmatrix} 1 & 0 \\ \cdot & \cdot \\ 0 & 1 \end{pmatrix}_{K}^{n \times n}, l, a^{-1}\right).$$

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