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Several Differentiation Formulas of Special Functions. Part VI

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Summary. In this article, we prove a series of differentiation identities [3] involving the secant and cosecant functions and specific combinations of special functions including trigonometric, exponential and logarithmic functions.

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The papers [11], [13], [1], [15], [2], [8], [9], [16], [5], [12], [10], [4], [6], [7], and [14] provide the notation and terminology for this paper.

In this paper x denotes a real number and Z denotes an open subset of $\mathbb{R}$.

One can prove the following propositions:

(1) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function cot}))$. Then

(i) (the function tan) $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{1}{(\text{the function cos})((\text{the function cot})(x))^{\mathbf{2}}} \cdot -\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(2) Suppose $Z \subseteq \mathrm{dom}((\text{the function tan}) \cdot (\text{the function tan}))$. Then

(i) (the function tan) $\cdot$(the function tan) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function tan}))'_{\restriction Z}(x) = \frac{1}{(\text{the function cos})((\text{the function tan})(x))^{\mathbf{2}}} \cdot \frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(3) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot}) \cdot (\text{the function cot}))$. Then

(i) (the function cot) $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot}) \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{1}{(\text{the function sin})((\text{the function cot})(x))^{\mathbf{2}}} \cdot \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(4) Suppose $Z \subseteq \mathrm{dom}((\text{the function cot}) \cdot (\text{the function tan}))$. Then

(i) (the function cot) $\cdot$(the function tan) is differentiable on Z, and

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(ii) for every x such that $x \in Z$ holds ((the function cot) $\cdot$(the function tan))$'_{\restriction Z}(x) = (-\frac{1}{\text{(the function sin)((the function tan)}(x))^{\mathbf{2}}}) \cdot \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}}$.

(5) Suppose $Z \subseteq$ dom((the function tan)$-$(the function cot)). Then
(i) (the function tan)$-$(the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function tan)$-$(the function cot))$'_{\restriction Z}(x) = \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} + \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(6) Suppose $Z \subseteq$ dom((the function tan)+(the function cot)). Then
(i) (the function tan)+(the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function tan)+(the function cot))$'_{\restriction Z}(x) = \frac{1}{\text{(the function cos)}(x)^{\mathbf{2}}} - \frac{1}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(7)(i) (The function sin) $\cdot$(the function sin) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function sin))$'_{\restriction Z}(x) =$ (the function cos)((the function sin)(x)) $\cdot$ (the function cos)(x).

(8)(i) (The function sin) $\cdot$(the function cos) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function cos))$'_{\restriction Z}(x) = -$(the function cos)((the function cos)(x)) $\cdot$ (the function sin) (x).

(9)(i) (The function cos) $\cdot$(the function sin) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function sin))$'_{\restriction Z}(x) = -$(the function sin)((the function sin)(x)) $\cdot$ (the function cos) (x).

(10)(i) (The function cos) $\cdot$(the function cos) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function cos))$'_{\restriction Z}(x) =$ (the function sin)((the function cos)(x)) $\cdot$ (the function sin)(x).

(11) Suppose $Z \subseteq$ dom((the function cos) (the function cot)). Then
(i) (the function cos) (the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) (the function cot))$'_{\restriction Z}(x) = -$(the function cos)$(x) - \frac{\text{(the function cos)}(x)}{\text{(the function sin)}(x)^{\mathbf{2}}}$.

(12) Suppose $Z \subseteq$ dom((the function sin) (the function tan)). Then
(i) (the function sin) (the function tan) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) (the function tan))$'_{\restriction Z}(x) =$ (the function sin)$(x) + \frac{\text{(the function sin)}(x)}{\text{(the function cos)}(x)^{\mathbf{2}}}$.

(13) Suppose $Z \subseteq$ dom((the function sin) (the function cot)). Then
(i) (the function sin) (the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) (the function cot))$'_{\restriction Z}(x) =$ (the function cos)$(x) \cdot$ (the function cot)$(x) - \frac{1}{\text{(the function sin)}(x)}$.

(14) Suppose $Z \subseteq$ dom((the function cos) (the function tan)). Then
 (i) (the function cos) (the function tan) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function cos) (the function tan))$'_{\restriction Z}(x) = -\frac{(\text{the function sin})(x)^{\mathbf{2}}}{(\text{the function cos})(x)} + \frac{1}{(\text{the function cos})(x)}$.

(15) Suppose $Z \subseteq$ dom((the function sin) (the function cos)). Then
 (i) (the function sin) (the function cos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) (the function cos))$'_{\restriction Z}(x) =$ (the function cos)$(x)^{\mathbf{2}}$ − (the function sin)$(x)^{\mathbf{2}}$.

(16) Suppose $Z \subseteq$ dom((the function ln) (the function sin)). Then
 (i) (the function ln) (the function sin) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function sin))$'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{x} +$ (the function ln)$(x)\cdot$(the function cos)(x).

(17) Suppose $Z \subseteq$ dom((the function ln) (the function cos)). Then
 (i) (the function ln) (the function cos) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function cos))$'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)}{x} -$ (the function ln)$(x)\cdot$(the function sin)(x).

(18) Suppose $Z \subseteq$ dom((the function ln) (the function exp)). Then
 (i) (the function ln) (the function exp) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) (the function exp))$'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{x} +$ (the function ln)$(x) \cdot$ (the function exp)(x).

(19) Suppose $Z \subseteq$ dom((the function ln) $\cdot$(the function ln)) and for every x such that $x \in Z$ holds $x > 0$. Then
 (i) (the function ln) $\cdot$(the function ln) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function ln) $\cdot$(the function ln))$'_{\restriction Z}(x) = \frac{1}{(\text{the function ln})(x)\cdot x}$.

(20) Suppose $Z \subseteq$ dom((the function exp) $\cdot$(the function exp)). Then
 (i) (the function exp) $\cdot$(the function exp) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$(the function exp))$'_{\restriction Z}(x) =$ (the function exp)((the function exp)(x)) $\cdot$ (the function exp)(x).

(21) Suppose $Z \subseteq$ dom((the function sin) $\cdot$(the function tan)). Then
 (i) (the function sin) $\cdot$(the function tan) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function tan))$'_{\restriction Z}(x) = \frac{\cos(\text{the function tan})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(22) Suppose $Z \subseteq$ dom((the function sin) $\cdot$(the function cot)). Then
 (i) (the function sin) $\cdot$(the function cot) is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$(the function cot))$'_{\restriction Z}(x) = -\frac{\cos(\text{the function cot})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq$ dom((the function cos) $\cdot$(the function tan)). Then
(i) (the function cos) $\cdot$(the function tan) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function tan))$'_{\restriction Z}(x) = -\frac{\sin(\text{the function tan})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.
(24) Suppose $Z \subseteq$ dom((the function cos) $\cdot$(the function cot)). Then
(i) (the function cos) $\cdot$(the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$(the function cot))$'_{\restriction Z}(x) = \frac{\sin(\text{the function cot})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.
(25) Suppose $Z \subseteq$ dom((the function sin) ((the function tan)+(the function cot))). Then
(i) (the function sin) ((the function tan)+(the function cot)) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) ((the function tan)+(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)$(x) \cdot$ ((the function tan)(x) + (the function cot)(x)) + (the function sin)$(x) \cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.
(26) Suppose $Z \subseteq$ dom((the function cos) ((the function tan)+(the function cot))). Then
(i) (the function cos) ((the function tan)+(the function cot)) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) ((the function tan)+(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)$(x) \cdot$ ((the function tan)(x) + (the function cot)(x))+(the function cos)$(x)\cdot(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.
(27) Suppose $Z \subseteq$ dom((the function sin) ((the function tan)$-$(the function cot))). Then
(i) (the function sin) ((the function tan)$-$(the function cot)) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function sin) ((the function tan)$-$(the function cot)))$'_{\restriction Z}(x)$ = (the function cos)$(x) \cdot$ ((the function tan)(x) $-$ (the function cot)(x)) + (the function sin)$(x) \cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.
(28) Suppose $Z \subseteq$ dom((the function cos) ((the function tan)$-$(the function cot))). Then
(i) (the function cos) ((the function tan)$-$(the function cot)) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ((the function cos) ((the function tan)$-$(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)$(x) \cdot$ ((the function tan)(x) $-$ (the function cot)(x))+(the function cos)$(x)\cdot(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(29) Suppose $Z \subseteq \mathrm{dom}(($the function exp$)$ $(($the function tan$)+($the function cot$))))$. Then

(i) (the function exp) ((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function exp$)$ $(($the function tan$)+($the function cot$)))'_{\restriction Z}(x) = ($the function exp$)(x) \cdot (($the function tan$)(x) + ($the function cot$)(x)) + ($the function exp$)(x) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(30) Suppose $Z \subseteq \mathrm{dom}(($the function exp$)$ $(($the function tan$)-($the function cot$))))$. Then

(i) (the function exp) ((the function tan)−(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function exp$)$ $(($the function tan$)-($the function cot$)))'_{\restriction Z}(x) = ($the function exp$)(x) \cdot (($the function tan$)(x) - ($the function cot$)(x)) + ($the function exp$)(x) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(31) Suppose $Z \subseteq \mathrm{dom}(($the function sin$)$ $(($the function sin$)+($the function cos$))))$. Then

(i) (the function sin) ((the function sin)+(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function sin$)$ $(($the function sin$)+($the function cos$)))'_{\restriction Z}(x) = (($the function cos$)(x)^{\mathbf{2}} + 2 \cdot ($the function sin$)(x) \cdot ($the function cos$)(x)) - ($the function sin$)(x)^{\mathbf{2}}$.

(32) Suppose $Z \subseteq \mathrm{dom}(($the function sin$)$ $(($the function sin$)-($the function cos$))))$. Then

(i) (the function sin) ((the function sin)−(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function sin$)$ $(($the function sin$)-($the function cos$)))'_{\restriction Z}(x) = (($the function sin$)(x)^{\mathbf{2}} + 2 \cdot ($the function sin$)(x) \cdot ($the function cos$)(x)) - ($the function cos$)(x)^{\mathbf{2}}$.

(33) Suppose $Z \subseteq \mathrm{dom}(($the function cos$)$ $(($the function sin$)-($the function cos$))))$. Then

(i) (the function cos) ((the function sin)−(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function cos$)$ $(($the function sin$)-($the function cos$)))'_{\restriction Z}(x) = (($the function cos$)(x)^{\mathbf{2}} + 2 \cdot ($the function sin$)(x) \cdot ($the function cos$)(x)) - ($the function sin$)(x)^{\mathbf{2}}$.

(34) Suppose $Z \subseteq \mathrm{dom}(($the function cos$)$ $(($the function sin$)+($the function cos$))))$. Then

(i) (the function cos) ((the function sin)+(the function cos)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) ((the function sin)+(the function cos)))$'_{\restriction Z}(x) =$ (the function cos)$(x)^{\mathbf{2}} - 2 \cdot$ (the function sin)$(x) \cdot$ (the function cos)$(x) -$ (the function sin)$(x)^{\mathbf{2}}$.

(35) Suppose $Z \subseteq$ dom((the function sin) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function sin) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x) =$ (the function cos)((the function tan)$(x)+$ (the function cot)$(x)) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(36) Suppose $Z \subseteq$ dom((the function sin) $\cdot$((the function tan)$-$(the function cot))). Then

(i) (the function sin) $\cdot$((the function tan)$-$(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$((the function tan)$-$(the function cot)))$'_{\restriction Z}(x) =$ (the function cos)((the function tan)$(x)-$ (the function cot)$(x)) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(37) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function tan)$-$(the function cot))). Then

(i) (the function cos) $\cdot$((the function tan)$-$(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$((the function tan)$-$(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)((the function tan)$(x) -$ (the function cot)$(x)) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(38) Suppose $Z \subseteq$ dom((the function cos) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function cos) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x) = -$(the function sin)((the function tan) $(x) +$ (the function cot)$(x)) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(39) Suppose $Z \subseteq$ dom((the function exp) $\cdot$((the function tan)+(the function cot))). Then

(i) (the function exp) $\cdot$((the function tan)+(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$((the function tan)+(the function cot)))$'_{\restriction Z}(x) =$ (the function exp)((the function tan)$(x)+$ (the function cot)$(x)) \cdot (\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} - \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(40) Suppose $Z \subseteq$ dom((the function exp) $\cdot$((the function tan)$-$(the function cot))). Then

(i) (the function exp) $\cdot$((the function tan)$-$(the function cot)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function exp) $\cdot$((the function tan)$-$(the function cot)))$'_{\restriction Z}(x)$ = (the function exp)((the function tan)(x) $-$ (the function cot)(x)) $\cdot$ $(\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}} + \frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})$.

(41) Suppose $Z \subseteq \mathrm{dom}(\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}})$. Then

(i) $\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function tan})-(\text{the function cot})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{((\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}+\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}})-(\text{the function tan})(x))+(\text{the function cot})(x)}{(\text{the function exp})(x)}$.

(42) Suppose $Z \subseteq \mathrm{dom}(\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}})$. Then

(i) $\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function tan})+(\text{the function cot})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}-\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}-(\text{the function tan})(x)-(\text{the function cot})(x)}{(\text{the function exp})(x)}$.

(43) Suppose $Z \subseteq \mathrm{dom}$((the function sin) $\cdot$ sec). Then

(i) (the function sin) $\cdot$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$ sec)$'_{\restriction Z}(x) = \frac{(\text{the function cos})((\text{sec})(x))\cdot(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(44) Suppose $Z \subseteq \mathrm{dom}$((the function cos) $\cdot$ sec). Then

(i) (the function cos) $\cdot$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$ sec)$'_{\restriction Z}(x) = -\frac{(\text{the function sin})((\text{sec})(x))\cdot(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(45) Suppose $Z \subseteq \mathrm{dom}$((the function sin) $\cdot$ cosec). Then

(i) (the function sin) $\cdot$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function sin) $\cdot$ cosec)$'_{\restriction Z}(x) = -\frac{(\text{the function cos})((\text{cosec})(x))\cdot(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(46) Suppose $Z \subseteq \mathrm{dom}$((the function cos) $\cdot$ cosec). Then

(i) (the function cos) $\cdot$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function cos) $\cdot$ cosec)$'_{\restriction Z}(x) = \frac{(\text{the function sin})((\text{cosec})(x))\cdot(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

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