

Several Differentiation Formulas of Special Functions. Part VI

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Summary. In this article, we prove a series of differentiation identities [3] involving the secant and cosecant functions and specific combinations of special functions including trigonometric, exponential and logarithmic functions.

MML identifier: `FDIFF_10`, version: 7.8.05 4.89.993

The papers [11], [13], [1], [15], [2], [8], [9], [16], [5], [12], [10], [4], [6], [7], and [14] provide the notation and terminology for this paper.

In this paper x denotes a real number and Z denotes an open subset of \mathbb{R} .

One can prove the following propositions:

- (1) Suppose $Z \subseteq \text{dom}(\text{(the function tan)} \cdot \text{(the function cot)})$. Then
 - (i) $\text{(the function tan)} \cdot \text{(the function cot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function tan)} \cdot \text{(the function cot)})'_{|Z}(x) = \frac{1}{(\text{the function cos})((\text{the function cot})(x))^2} \cdot -\frac{1}{(\text{the function sin})(x)^2}$.
- (2) Suppose $Z \subseteq \text{dom}(\text{(the function tan)} \cdot \text{(the function tan)})$. Then
 - (i) $\text{(the function tan)} \cdot \text{(the function tan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function tan)} \cdot \text{(the function tan)})'_{|Z}(x) = \frac{1}{(\text{the function cos})((\text{the function tan})(x))^2} \cdot \frac{1}{(\text{the function cos})(x)^2}$.
- (3) Suppose $Z \subseteq \text{dom}(\text{(the function cot)} \cdot \text{(the function cot)})$. Then
 - (i) $\text{(the function cot)} \cdot \text{(the function cot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function cot)} \cdot \text{(the function cot)})'_{|Z}(x) = \frac{1}{(\text{the function sin})((\text{the function cot})(x))^2} \cdot \frac{1}{(\text{the function sin})(x)^2}$.
- (4) Suppose $Z \subseteq \text{dom}(\text{(the function cot)} \cdot \text{(the function tan)})$. Then
 - (i) $\text{(the function cot)} \cdot \text{(the function tan)}$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $((\text{the function cot}) \cdot (\text{the function tan}))'_{|Z}(x) = \left(-\frac{1}{(\text{the function sin})((\text{the function tan})(x))^2}\right) \cdot \frac{1}{(\text{the function cos})(x)^2}$.
- (5) Suppose $Z \subseteq \text{dom}((\text{the function tan}) - (\text{the function cot}))$. Then
- (i) $(\text{the function tan}) - (\text{the function cot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function tan}) - (\text{the function cot}))'_{|Z}(x) = \frac{1}{(\text{the function cos})(x)^2} + \frac{1}{(\text{the function sin})(x)^2}$.
- (6) Suppose $Z \subseteq \text{dom}((\text{the function tan}) + (\text{the function cot}))$. Then
- (i) $(\text{the function tan}) + (\text{the function cot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function tan}) + (\text{the function cot}))'_{|Z}(x) = \frac{1}{(\text{the function cos})(x)^2} - \frac{1}{(\text{the function sin})(x)^2}$.
- (7)(i) $(\text{The function sin}) \cdot (\text{the function sin})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function sin}))'_{|Z}(x) = (\text{the function cos})((\text{the function sin})(x)) \cdot (\text{the function cos})(x)$.
- (8)(i) $(\text{The function sin}) \cdot (\text{the function cos})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function cos}))'_{|Z}(x) = -(\text{the function cos})((\text{the function cos})(x)) \cdot (\text{the function sin})(x)$.
- (9)(i) $(\text{The function cos}) \cdot (\text{the function sin})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot (\text{the function sin}))'_{|Z}(x) = -(\text{the function sin})((\text{the function sin})(x)) \cdot (\text{the function cos})(x)$.
- (10)(i) $(\text{The function cos}) \cdot (\text{the function cos})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot (\text{the function cos}))'_{|Z}(x) = (\text{the function sin})((\text{the function cos})(x)) \cdot (\text{the function sin})(x)$.
- (11) Suppose $Z \subseteq \text{dom}((\text{the function cos}) (\text{the function cot}))$. Then
- (i) $(\text{the function cos}) (\text{the function cot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function cos}) (\text{the function cot}))'_{|Z}(x) = -(\text{the function cos})(x) - \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.
- (12) Suppose $Z \subseteq \text{dom}((\text{the function sin}) (\text{the function tan}))$. Then
- (i) $(\text{the function sin}) (\text{the function tan})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) (\text{the function tan}))'_{|Z}(x) = (\text{the function sin})(x) + \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.
- (13) Suppose $Z \subseteq \text{dom}((\text{the function sin}) (\text{the function cot}))$. Then
- (i) $(\text{the function sin}) (\text{the function cot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) (\text{the function cot}))'_{|Z}(x) = (\text{the function cos})(x) \cdot (\text{the function cot})(x) - \frac{1}{(\text{the function sin})(x)}$.

- (14) Suppose $Z \subseteq \text{dom}(\text{(the function cos) (the function tan)})$. Then
- (the function cos) (the function tan) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function cos) (the function tan))' $\Big|_Z(x) = -\frac{(\text{the function sin})(x)^2}{(\text{the function cos})(x)} + \frac{1}{(\text{the function cos})(x)}$.
- (15) Suppose $Z \subseteq \text{dom}(\text{(the function sin) (the function cos)})$. Then
- (the function sin) (the function cos) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function sin) (the function cos))' $\Big|_Z(x) = (\text{the function cos})(x)^2 - (\text{the function sin})(x)^2$.
- (16) Suppose $Z \subseteq \text{dom}(\text{(the function ln) (the function sin)})$. Then
- (the function ln) (the function sin) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function ln) (the function sin))' $\Big|_Z(x) = \frac{(\text{the function sin})(x)}{x} + (\text{the function ln})(x) \cdot (\text{the function cos})(x)$.
- (17) Suppose $Z \subseteq \text{dom}(\text{(the function ln) (the function cos)})$. Then
- (the function ln) (the function cos) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function ln) (the function cos))' $\Big|_Z(x) = \frac{(\text{the function cos})(x)}{x} - (\text{the function ln})(x) \cdot (\text{the function sin})(x)$.
- (18) Suppose $Z \subseteq \text{dom}(\text{(the function ln) (the function exp)})$. Then
- (the function ln) (the function exp) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function ln) (the function exp))' $\Big|_Z(x) = \frac{(\text{the function exp})(x)}{x} + (\text{the function ln})(x) \cdot (\text{the function exp})(x)$.
- (19) Suppose $Z \subseteq \text{dom}(\text{(the function ln) } \cdot \text{(the function ln)})$ and for every x such that $x \in Z$ holds $x > 0$. Then
- (the function ln) \cdot (the function ln) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function ln) \cdot (the function ln))' $\Big|_Z(x) = \frac{1}{(\text{the function ln})(x) \cdot x}$.
- (20) Suppose $Z \subseteq \text{dom}(\text{(the function exp) } \cdot \text{(the function exp)})$. Then
- (the function exp) \cdot (the function exp) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function exp) \cdot (the function exp))' $\Big|_Z(x) = (\text{the function exp})((\text{the function exp})(x)) \cdot (\text{the function exp})(x)$.
- (21) Suppose $Z \subseteq \text{dom}(\text{(the function sin) } \cdot \text{(the function tan)})$. Then
- (the function sin) \cdot (the function tan) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function sin) \cdot (the function tan))' $\Big|_Z(x) = \frac{\cos(\text{the function tan})(x)}{(\text{the function cos})(x)^2}$.
- (22) Suppose $Z \subseteq \text{dom}(\text{(the function sin) } \cdot \text{(the function cot)})$. Then
- (the function sin) \cdot (the function cot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds ((the function sin) \cdot (the function cot))' $\Big|_Z(x) = -\frac{\cos(\text{the function cot})(x)}{(\text{the function sin})(x)^2}$.

- (23) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \cdot \text{(the function tan)})$. Then
- (i) $\text{(the function cos)} \cdot \text{(the function tan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cos)} \cdot \text{(the function tan)}\big|_Z(x) = -\frac{\sin(\text{(the function tan)}(x))}{(\text{(the function cos)}(x))^2}$.
- (24) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \cdot \text{(the function cot)})$. Then
- (i) $\text{(the function cos)} \cdot \text{(the function cot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cos)} \cdot \text{(the function cot)}\big|_Z(x) = \frac{\sin(\text{(the function cot)}(x))}{(\text{(the function sin)}(x))^2}$.
- (25) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \text{ ((the function tan)+(the function cot))})$. Then
- (i) $\text{(the function sin)} \text{ ((the function tan)+(the function cot))}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sin)} \text{ ((the function tan)+(the function cot))}\big|_Z(x) = (\text{(the function cos)}(x) \cdot ((\text{(the function tan)}(x) + \text{(the function cot)}(x)) + (\text{(the function sin)}(x) \cdot (\frac{1}{(\text{(the function cos)}(x))^2} - \frac{1}{(\text{(the function sin)}(x))^2}))$.
- (26) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \text{ ((the function tan)+(the function cot))})$. Then
- (i) $\text{(the function cos)} \text{ ((the function tan)+(the function cot))}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cos)} \text{ ((the function tan)+(the function cot))}\big|_Z(x) = -(\text{(the function sin)}(x) \cdot ((\text{(the function tan)}(x) + \text{(the function cot)}(x)) + (\text{(the function cos)}(x) \cdot (\frac{1}{(\text{(the function cos)}(x))^2} - \frac{1}{(\text{(the function sin)}(x))^2}))$.
- (27) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \text{ ((the function tan)-(the function cot))})$. Then
- (i) $\text{(the function sin)} \text{ ((the function tan)-(the function cot))}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sin)} \text{ ((the function tan)-(the function cot))}\big|_Z(x) = (\text{(the function cos)}(x) \cdot ((\text{(the function tan)}(x) - \text{(the function cot)}(x)) + (\text{(the function sin)}(x) \cdot (\frac{1}{(\text{(the function cos)}(x))^2} + \frac{1}{(\text{(the function sin)}(x))^2}))$.
- (28) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \text{ ((the function tan)-(the function cot))})$. Then
- (i) $\text{(the function cos)} \text{ ((the function tan)-(the function cot))}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cos)} \text{ ((the function tan)-(the function cot))}\big|_Z(x) = -(\text{(the function sin)}(x) \cdot ((\text{(the function tan)}(x) - \text{(the function cot)}(x)) + (\text{(the function cos)}(x) \cdot (\frac{1}{(\text{(the function cos)}(x))^2} + \frac{1}{(\text{(the function sin)}(x))^2}))$.

- (29) Suppose $Z \subseteq \text{dom}(\text{(the function exp)} \text{(the function tan)} + \text{(the function cot)})$. Then
- (i) $\text{(the function exp)} \text{(the function tan)} + \text{(the function cot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function exp)} \text{(the function tan)} + \text{(the function cot)})'_{|Z}(x) = (\text{the function exp})(x) \cdot (\text{(the function tan)}(x) + \text{(the function cot)}(x)) + (\text{the function exp})(x) \cdot \left(\frac{1}{(\text{the function cos})(x)^2} - \frac{1}{(\text{the function sin})(x)^2} \right)$.
- (30) Suppose $Z \subseteq \text{dom}(\text{(the function exp)} \text{(the function tan)} - \text{(the function cot)})$. Then
- (i) $\text{(the function exp)} \text{(the function tan)} - \text{(the function cot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function exp)} \text{(the function tan)} - \text{(the function cot)})'_{|Z}(x) = (\text{the function exp})(x) \cdot (\text{(the function tan)}(x) - \text{(the function cot)}(x)) + (\text{the function exp})(x) \cdot \left(\frac{1}{(\text{the function cos})(x)^2} + \frac{1}{(\text{the function sin})(x)^2} \right)$.
- (31) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \text{(the function sin)} + \text{(the function cos)})$. Then
- (i) $\text{(the function sin)} \text{(the function sin)} + \text{(the function cos)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function sin)} \text{(the function sin)} + \text{(the function cos)})'_{|Z}(x) = ((\text{the function cos})(x))^2 + 2 \cdot (\text{the function sin})(x) \cdot (\text{the function cos})(x) - (\text{the function sin})(x)^2$.
- (32) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \text{(the function sin)} - \text{(the function cos)})$. Then
- (i) $\text{(the function sin)} \text{(the function sin)} - \text{(the function cos)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function sin)} \text{(the function sin)} - \text{(the function cos)})'_{|Z}(x) = ((\text{the function sin})(x))^2 + 2 \cdot (\text{the function sin})(x) \cdot (\text{the function cos})(x) - (\text{the function cos})(x)^2$.
- (33) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \text{(the function sin)} - \text{(the function cos)})$. Then
- (i) $\text{(the function cos)} \text{(the function sin)} - \text{(the function cos)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function cos)} \text{(the function sin)} - \text{(the function cos)})'_{|Z}(x) = ((\text{the function cos})(x))^2 + 2 \cdot (\text{the function sin})(x) \cdot (\text{the function cos})(x) - (\text{the function sin})(x)^2$.
- (34) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \text{(the function sin)} + \text{(the function cos)})$. Then
- (i) $\text{(the function cos)} \text{(the function sin)} + \text{(the function cos)}$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \sin) + (\text{the function } \cos)))'_{|Z}(x) = (\text{the function } \cos)(x)^2 - 2 \cdot (\text{the function } \sin)(x) \cdot (\text{the function } \cos)(x) - (\text{the function } \sin)(x)^2$.
- (35) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))$. Then
- (i) $(\text{the function } \sin) \cdot ((\text{the function } \tan) + (\text{the function } \cot))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))'_{|Z}(x) = (\text{the function } \cos)((\text{the function } \tan)(x) + (\text{the function } \cot)(x)) \cdot (\frac{1}{(\text{the function } \cos)(x)^2} - \frac{1}{(\text{the function } \sin)(x)^2})$.
- (36) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) \cdot ((\text{the function } \tan) - (\text{the function } \cot)))$. Then
- (i) $(\text{the function } \sin) \cdot ((\text{the function } \tan) - (\text{the function } \cot))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot ((\text{the function } \tan) - (\text{the function } \cot)))'_{|Z}(x) = (\text{the function } \cos)((\text{the function } \tan)(x) - (\text{the function } \cot)(x)) \cdot (\frac{1}{(\text{the function } \cos)(x)^2} + \frac{1}{(\text{the function } \sin)(x)^2})$.
- (37) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) \cdot ((\text{the function } \tan) - (\text{the function } \cot)))$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \tan) - (\text{the function } \cot))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \tan) - (\text{the function } \cot)))'_{|Z}(x) = -(\text{the function } \sin)((\text{the function } \tan)(x) - (\text{the function } \cot)(x)) \cdot (\frac{1}{(\text{the function } \cos)(x)^2} + \frac{1}{(\text{the function } \sin)(x)^2})$.
- (38) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \tan) + (\text{the function } \cot))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))'_{|Z}(x) = -(\text{the function } \sin)((\text{the function } \tan)(x) + (\text{the function } \cot)(x)) \cdot (\frac{1}{(\text{the function } \cos)(x)^2} - \frac{1}{(\text{the function } \sin)(x)^2})$.
- (39) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))$. Then
- (i) $(\text{the function } \exp) \cdot ((\text{the function } \tan) + (\text{the function } \cot))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) \cdot ((\text{the function } \tan) + (\text{the function } \cot)))'_{|Z}(x) = (\text{the function } \exp)((\text{the function } \tan)(x) + (\text{the function } \cot)(x)) \cdot (\frac{1}{(\text{the function } \cos)(x)^2} - \frac{1}{(\text{the function } \sin)(x)^2})$.
- (40) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) \cdot ((\text{the function } \tan) - (\text{the function } \cot)))$. Then

- (i) (the function exp) · ((the function tan) – (the function cot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function exp) · ((the function tan) – (the function cot)))' $\upharpoonright_Z(x) =$ (the function exp)((the function tan)(x) – (the function cot)(x)) · ($\frac{1}{(\text{the function cos})(x)^2} + \frac{1}{(\text{the function sin})(x)^2}$).
- (41) Suppose $Z \subseteq \text{dom}(\frac{(\text{the function tan}) - (\text{the function cot})}{\text{the function exp}})$. Then
- (i) $\frac{(\text{the function tan}) - (\text{the function cot})}{\text{the function exp}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ($\frac{(\text{the function tan}) - (\text{the function cot})}{\text{the function exp}}$)' $\upharpoonright_Z(x) =$ $\frac{((\frac{1}{(\text{the function cos})(x)^2} + \frac{1}{(\text{the function sin})(x)^2}) - (\text{the function tan})(x) + (\text{the function cot})(x))}{(\text{the function exp})(x)}$.
- (42) Suppose $Z \subseteq \text{dom}(\frac{(\text{the function tan}) + (\text{the function cot})}{\text{the function exp}})$. Then
- (i) $\frac{(\text{the function tan}) + (\text{the function cot})}{\text{the function exp}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ($\frac{(\text{the function tan}) + (\text{the function cot})}{\text{the function exp}}$)' $\upharpoonright_Z(x) =$ $\frac{\frac{1}{(\text{the function cos})(x)^2} - \frac{1}{(\text{the function sin})(x)^2} - (\text{the function tan})(x) - (\text{the function cot})(x)}{(\text{the function exp})(x)}}$.
- (43) Suppose $Z \subseteq \text{dom}((\text{the function sin}) \cdot \text{sec})$. Then
- (i) (the function sin) · sec is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function sin) · sec)' $\upharpoonright_Z(x) =$ $\frac{(\text{the function cos})((\text{sec})(x)) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.
- (44) Suppose $Z \subseteq \text{dom}((\text{the function cos}) \cdot \text{sec})$. Then
- (i) (the function cos) · sec is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function cos) · sec)' $\upharpoonright_Z(x) =$ $-\frac{(\text{the function sin})((\text{sec})(x)) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.
- (45) Suppose $Z \subseteq \text{dom}((\text{the function sin}) \cdot \text{cosec})$. Then
- (i) (the function sin) · cosec is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function sin) · cosec)' $\upharpoonright_Z(x) =$ $-\frac{(\text{the function cos})((\text{cosec})(x)) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.
- (46) Suppose $Z \subseteq \text{dom}((\text{the function cos}) \cdot \text{cosec})$. Then
- (i) (the function cos) · cosec is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function cos) · cosec)' $\upharpoonright_Z(x) =$ $\frac{(\text{the function sin})((\text{cosec})(x)) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.

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Received September 19, 2007
