

# Cell Petri Net Concepts

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**Summary.** Based on the Petri net definitions and theorems already formalized in [8], with this article, we developed the concept of “Cell Petri Nets”. It is based on [9]. In a cell Petri net we introduce the notions of colors and colored states of a Petri net, connecting mappings for linking two Petri nets, firing rules for transitions, and the synthesis of two or more Petri nets.

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The papers [11], [12], [6], [13], [14], [10], [8], [2], [5], [3], [4], [7], and [1] provide the terminology and notation for this paper.

## 1. PRELIMINARIES: THIN CYLINDER, LOCUS

Let  $A$  be a non empty set, let  $B$  be a set, let  $B_1$  be a set, and let  $y_1$  be a function from  $B_1$  into  $A$ . Let us assume that  $B_1 \subseteq B$ . The functor  $\text{cylinder}_0(A, B, B_1, y_1)$  yields a non empty subset of  $A^B$  and is defined by:

(Def. 1)  $\text{cylinder}_0(A, B, B_1, y_1) = \{y : B \rightarrow A : y|_{B_1} = y_1\}$ .

Let  $A$  be a non empty set and let  $B$  be a set. A non empty subset of  $A^B$  is said to be a thin cylinder of  $A$  and  $B$  if:

(Def. 2) There exists a subset  $B_1$  of  $B$  and there exists a function  $y_1$  from  $B_1$  into  $A$  such that  $B_1$  is finite and it =  $\text{cylinder}_0(A, B, B_1, y_1)$ .

The following propositions are true:

- (1) Let  $A$  be a non empty set,  $B$  be a set, and  $D$  be a thin cylinder of  $A$  and  $B$ . Then there exists a subset  $B_1$  of  $B$  and there exists a function  $y_1$  from  $B_1$  into  $A$  such that  $B_1$  is finite and  $D = \{y : B \rightarrow A: y \upharpoonright B_1 = y_1\}$ .
- (2) Let  $A_1, A_2$  be non empty sets,  $B$  be a set, and  $D_1$  be a thin cylinder of  $A_1$  and  $B$ . If  $A_1 \subseteq A_2$ , then there exists a thin cylinder  $D_2$  of  $A_2$  and  $B$  such that  $D_1 \subseteq D_2$ .

Let  $A$  be a non empty set and let  $B$  be a set. The thin cylinders of  $A$  and  $B$  constitute a non empty family of subsets of  $A^B$  defined by:

(Def. 3) The thin cylinders of  $A$  and  $B = \{D \subseteq A^B: D \text{ is a thin cylinder of } A \text{ and } B\}$ .

We now state three propositions:

- (3) Let  $A$  be a non trivial set,  $B$  be a set,  $B_2$  be a set,  $y_2$  be a function from  $B_2$  into  $A$ ,  $B_3$  be a set, and  $y_3$  be a function from  $B_3$  into  $A$ . If  $B_2 \subseteq B$  and  $B_3 \subseteq B$  and  $\text{cylinder}_0(A, B, B_2, y_2) = \text{cylinder}_0(A, B, B_3, y_3)$ , then  $B_2 = B_3$  and  $y_2 = y_3$ .
- (4) Let  $A_1, A_2$  be non empty sets and  $B_4, B_5$  be sets. Suppose  $A_1 \subseteq A_2$  and  $B_4 \subseteq B_5$ . Then there exists a function  $F$  from the thin cylinders of  $A_1$  and  $B_4$  into the thin cylinders of  $A_2$  and  $B_5$  such that for every set  $x$  if  $x \in$  the thin cylinders of  $A_1$  and  $B_4$ , then there exists a subset  $B_1$  of  $B_4$  and there exists a function  $y_2$  from  $B_1$  into  $A_1$  and there exists a function  $y_3$  from  $B_1$  into  $A_2$  such that  $B_1$  is finite and  $y_2 = y_3$  and  $x = \text{cylinder}_0(A_1, B_4, B_1, y_2)$  and  $F(x) = \text{cylinder}_0(A_2, B_5, B_1, y_3)$ .
- (5) Let  $A_1, A_2$  be non empty sets and  $B_4, B_5$  be sets. Then there exists a function  $G$  from the thin cylinders of  $A_2$  and  $B_5$  into the thin cylinders of  $A_1$  and  $B_4$  such that for every set  $x$  if  $x \in$  the thin cylinders of  $A_2$  and  $B_5$ , then there exists a subset  $B_3$  of  $B_5$  and there exists a subset  $B_2$  of  $B_4$  and there exists a function  $y_2$  from  $B_2$  into  $A_1$  and there exists a function  $y_3$  from  $B_3$  into  $A_2$  such that  $B_2$  is finite and  $B_3$  is finite and  $B_2 = B_4 \cap B_3 \cap y_3^{-1}(A_1)$  and  $y_2 = y_3 \upharpoonright B_2$  and  $x = \text{cylinder}_0(A_2, B_5, B_3, y_3)$  and  $G(x) = \text{cylinder}_0(A_1, B_4, B_2, y_2)$ .

Let  $A_1, A_2$  be non trivial sets and let  $B_4, B_5$  be sets. Let us assume that there exist sets  $x, y$  such that  $x \neq y$  and  $x, y \in A_1$  and  $A_1 \subseteq A_2$  and  $B_4 \subseteq B_5$ . The functor  $\text{Extcylinders}(A_1, B_4, A_2, B_5)$  yielding a function from the thin cylinders of  $A_1$  and  $B_4$  into the thin cylinders of  $A_2$  and  $B_5$  is defined by the condition (Def. 4).

(Def. 4) Let  $x$  be a set. Suppose  $x \in$  the thin cylinders of  $A_1$  and  $B_4$ . Then there exists a subset  $B_1$  of  $B_4$  and there exists a function  $y_2$  from  $B_1$  into  $A_1$  and there exists a function  $y_3$  from  $B_1$  into  $A_2$  such that  $B_1$  is finite and  $y_2 = y_3$  and  $x = \text{cylinder}_0(A_1, B_4, B_1, y_2)$  and  $(\text{Extcylinders}(A_1, B_4, A_2, B_5))(x) = \text{cylinder}_0(A_2, B_5, B_1, y_3)$ .

Let  $A_1$  be a non empty set, let  $A_2$  be a non trivial set, and let  $B_4, B_5$  be sets. Let us assume that  $A_1 \subseteq A_2$  and  $B_4 \subseteq B_5$ . The functor  $\text{Ristcylinders}(A_1, B_4, A_2, B_5)$  yields a function from the thin cylinders of  $A_2$  and  $B_5$  into the thin cylinders of  $A_1$  and  $B_4$  and is defined by the condition (Def. 5).

(Def. 5) Let  $x$  be a set. Suppose  $x \in$  the thin cylinders of  $A_2$  and  $B_5$ . Then there exists a subset  $B_3$  of  $B_5$  and there exists a subset  $B_2$  of  $B_4$  and there exists a function  $y_2$  from  $B_2$  into  $A_1$  and there exists a function  $y_3$  from  $B_3$  into  $A_2$  such that  $B_2$  is finite and  $B_3$  is finite and  $B_2 = B_4 \cap B_3 \cap y_3^{-1}(A_1)$  and  $y_2 = y_3 \upharpoonright B_2$  and  $x = \text{cylinder}_0(A_2, B_5, B_3, y_3)$  and  $(\text{Ristcylinders}(A_1, B_4, A_2, B_5))(x) = \text{cylinder}_0(A_1, B_4, B_2, y_2)$ .

Let  $A$  be a non trivial set, let  $B$  be a set, and let  $D$  be a thin cylinder of  $A$  and  $B$ . The functor  $\text{loc } D$  yielding a finite subset of  $B$  is defined by the condition (Def. 6).

(Def. 6) There exists a subset  $B_1$  of  $B$  and there exists a function  $y_1$  from  $B_1$  into  $A$  such that  $B_1$  is finite and  $D = \{y : B \rightarrow A: y \upharpoonright B_1 = y_1\}$  and  $\text{loc } D = B_1$ .

## 2. COLORED PETRI NETS

Let  $A_1, A_2$  be non trivial sets, let  $B_4, B_5$  be sets, let  $C_1, C_2$  be non trivial sets, let  $D_1, D_2$  be sets, and let  $F$  be a function from the thin cylinders of  $A_1$  and  $B_4$  into the thin cylinders of  $C_1$  and  $D_1$ . The functor  $\text{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F)$  yielding a function from the thin cylinders of  $A_2$  and  $B_5$  into the thin cylinders of  $C_2$  and  $D_2$  is defined as follows:

(Def. 7)  $\text{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F) = \text{Extcylinders}(C_1, D_1, C_2, D_2) \cdot F \cdot \text{Ristcylinders}(A_1, B_4, A_2, B_5)$ .

We consider colored place/transition net structures as extensions of place/transition net structure as systems

$\langle \text{places, transitions, S-T arcs, T-S arcs, a colored set, a firing-rule} \rangle$ ,

where the places and the transitions constitute non empty sets, the S-T arcs constitute a non empty relation between the places and the transitions, the T-S arcs constitute a non empty relation between the transitions and the places, the colored set is a non empty finite set, and the firing-rule is a function.

Let  $C_3$  be a colored place/transition net structure and let  $t_0$  be a transition of  $C_3$ . We say that  $t_0$  is outbound if and only if:

(Def. 8)  $\overline{\{t_0\}} = \emptyset$ .

Let  $C_4$  be a colored place/transition net structure. The functor  $\text{Outbds } C_4$  yielding a subset of the transitions of  $C_4$  is defined by:

(Def. 9)  $\text{Outbds } C_4 = \{x; x \text{ ranges over transitions of } C_4: x \text{ is outbound}\}$ .

Let  $C_3$  be a colored place/transition net structure. We say that  $C_3$  is colored-PT-net-like if and only if the conditions (Def. 10) are satisfied.

- (Def. 10)(i)  $\text{dom}(\text{the firing-rule of } C_3) \subseteq (\text{the transitions of } C_3) \setminus \text{Outbds } C_3$ , and  
(ii) for every transition  $t$  of  $C_3$  such that  $t \in \text{dom}(\text{the firing-rule of } C_3)$  there exists a non empty subset  $C_5$  of the colored set of  $C_3$  and there exists a subset  $I$  of  $^*\{t\}$  and there exists a subset  $O$  of  $\overline{\{t\}}$  such that (the firing-rule of  $C_3$ )( $t$ ) is a function from the thin cylinders of  $C_5$  and  $I$  into the thin cylinders of  $C_5$  and  $O$ .

We now state two propositions:

- (6) Let  $C_3$  be a colored place/transition net structure and  $t$  be a transition of  $C_3$ . Suppose  $C_3$  is colored-PT-net-like and  $t \in \text{dom}(\text{the firing-rule of } C_3)$ . Then there exists a non empty subset  $C_5$  of the colored set of  $C_3$  and there exists a subset  $I$  of  $^*\{t\}$  and there exists a subset  $O$  of  $\overline{\{t\}}$  such that (the firing-rule of  $C_3$ )( $t$ ) is a function from the thin cylinders of  $C_5$  and  $I$  into the thin cylinders of  $C_5$  and  $O$ .
- (7) Let  $C_4, C_6$  be colored place/transition net structures,  $t_1$  be a transition of  $C_4$ , and  $t_2$  be a transition of  $C_6$ . Suppose that
- (i) the places of  $C_4 \subseteq$  the places of  $C_6$ ,
  - (ii) the transitions of  $C_4 \subseteq$  the transitions of  $C_6$ ,
  - (iii) the S-T arcs of  $C_4 \subseteq$  the S-T arcs of  $C_6$ ,
  - (iv) the T-S arcs of  $C_4 \subseteq$  the T-S arcs of  $C_6$ , and
  - (v)  $t_1 = t_2$ .

Then  $^*\{t_1\} \subseteq ^*\{t_2\}$  and  $\overline{\{t_1\}} \subseteq \overline{\{t_2\}}$ .

One can verify that there exists a colored place/transition net structure which is strict and colored-PT-net-like.

A colored place/transition net is a colored-PT-net-like colored place/transition net structure.

### 3. COLOR COUNTS OF CPNT

Let  $C_4, C_6$  be colored place/transition net structures. We say that  $C_4$  misses  $C_6$  if and only if:

- (Def. 11) (The places of  $C_4 \cap$  (the places of  $C_6) = \emptyset$  and (the transitions of  $C_4 \cap$  (the transitions of  $C_6) = \emptyset$ ).

Let us note that the predicate  $C_4$  misses  $C_6$  is symmetric.

### 4. COLORED STATES OF CPNT

Let  $C_4$  be a colored place/transition net structure and let  $C_6$  be a colored place/transition net structure. Connecting mapping of  $C_4$  and  $C_6$  is defined by the condition (Def. 12).

- (Def. 12) There exists a function  $O_{12}$  from  $\text{Outbds } C_4$  into the places of  $C_6$  and there exists a function  $O_{21}$  from  $\text{Outbds } C_6$  into the places of  $C_4$  such that  $\text{it} = \langle O_{12}, O_{21} \rangle$ .

## 5. OUTBOUND TRANSITIONS OF CPNT

Let  $C_4, C_6$  be colored place/transition nets and let  $O$  be a connecting mapping of  $C_4$  and  $C_6$ . Connecting firing rule of  $C_4, C_6$ , and  $O$  is defined by the condition (Def. 13).

- (Def. 13) There exist functions  $q_{12}, q_{21}$  and there exists a function  $O_{12}$  from  $\text{Outbds } C_4$  into the places of  $C_6$  and there exists a function  $O_{21}$  from  $\text{Outbds } C_6$  into the places of  $C_4$  such that
- (i)  $O = \langle O_{12}, O_{21} \rangle$ ,
  - (ii)  $\text{dom } q_{12} = \text{Outbds } C_4$ ,
  - (iii)  $\text{dom } q_{21} = \text{Outbds } C_6$ ,
  - (iv) for every transition  $t_3$  of  $C_4$  such that  $t_3$  is outbound holds  $q_{12}(t_3)$  is a function from the thin cylinders of the colored set of  $C_4$  and  $^*\{t_3\}$  into the thin cylinders of the colored set of  $C_4$  and  $O_{12} \circ t_3$ ,
  - (v) for every transition  $t_4$  of  $C_6$  such that  $t_4$  is outbound holds  $q_{21}(t_4)$  is a function from the thin cylinders of the colored set of  $C_6$  and  $^*\{t_4\}$  into the thin cylinders of the colored set of  $C_6$  and  $O_{21} \circ t_4$ , and
  - (vi)  $\text{it} = \langle q_{12}, q_{21} \rangle$ .

## 6. CONNECTING MAPPING FOR CPNT1, CPNT2

Let  $C_4, C_6$  be colored place/transition nets, let  $O$  be a connecting mapping of  $C_4$  and  $C_6$ , and let  $q$  be a connecting firing rule of  $C_4, C_6$ , and  $O$ . Let us assume that  $C_4$  misses  $C_6$ . The functor  $\text{synthesis}(C_4, C_6, O, q)$  yielding a strict colored place/transition net is defined by the condition (Def. 14).

- (Def. 14) There exist functions  $q_{12}, q_{21}$  and there exists a function  $O_{12}$  from  $\text{Outbds } C_4$  into the places of  $C_6$  and there exists a function  $O_{21}$  from  $\text{Outbds } C_6$  into the places of  $C_4$  such that  $O = \langle O_{12}, O_{21} \rangle$  and  $\text{dom } q_{12} = \text{Outbds } C_4$  and  $\text{dom } q_{21} = \text{Outbds } C_6$  and for every transition  $t_3$  of  $C_4$  such that  $t_3$  is outbound holds  $q_{12}(t_3)$  is a function from the thin cylinders of the colored set of  $C_4$  and  $^*\{t_3\}$  into the thin cylinders of the colored set of  $C_4$  and  $O_{12} \circ t_3$  and for every transition  $t_4$  of  $C_6$  such that  $t_4$  is outbound holds  $q_{21}(t_4)$  is a function from the thin cylinders of the colored set of  $C_6$  and  $^*\{t_4\}$  into the thin cylinders of the colored set of  $C_6$  and  $O_{21} \circ t_4$  and  $q = \langle q_{12}, q_{21} \rangle$  and the places of  $\text{synthesis}(C_4, C_6, O, q) = (\text{the places of } C_4) \cup (\text{the places of } C_6)$  and the

transitions of synthesis  $(C_4, C_6, O, q) = (\text{the transitions of } C_4) \cup (\text{the transitions of } C_6)$  and the S-T arcs of synthesis  $(C_4, C_6, O, q) = (\text{the S-T arcs of } C_4) \cup (\text{the S-T arcs of } C_6)$  and the T-S arcs of synthesis  $(C_4, C_6, O, q) = (\text{the T-S arcs of } C_4) \cup (\text{the T-S arcs of } C_6) \cup O_{12} \cup O_{21}$  and the colored set of synthesis  $(C_4, C_6, O, q) = (\text{the colored set of } C_4) \cup (\text{the colored set of } C_6)$  and the firing-rule of synthesis  $(C_4, C_6, O, q) = (\text{the firing-rule of } C_4) + (\text{the firing-rule of } C_6) + q_{12} + q_{21}$ .

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