

## Integrability Formulas. Part I

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**Summary.** In this article, we give several differentiation and integrability formulas of special and composite functions including the trigonometric function, and the polynomial function.

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The papers [12], [2], [3], [1], [7], [11], [13], [4], [17], [8], [9], [6], [18], [5], [10], [15], [16], and [14] provide the terminology and notation for this paper.

One can check that there exists a subset of  $\mathbb{R}$  which is closed-interval.

For simplicity, we use the following convention:  $a, b, x, r$  are real numbers,  $n$  is an element of  $\mathbb{N}$ ,  $A$  is a closed-interval subset of  $\mathbb{R}$ ,  $f, g, f_1, f_2, g_1, g_2$  are partial functions from  $\mathbb{R}$  to  $\mathbb{R}$ , and  $Z$  is an open subset of  $\mathbb{R}$ .

We now state a number of propositions:

- (1) Suppose  $Z \subseteq \text{dom}(\frac{1}{f_1+f_2})$  and for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$  and  $f_2 = \square^2$ . Then  $\frac{1}{f_1+f_2}$  is differentiable on  $Z$  and for every  $x$  such that  $x \in Z$  holds  $(\frac{1}{f_1+f_2})'_{|Z}(x) = -\frac{2 \cdot x}{(1+x^2)^2}$ .
- (2) Suppose that  $A \subseteq Z$  and  $f = \frac{1}{\frac{g_1+g_2}{f_2}}$  and  $f_2 =$  the function arccot and  $Z \subseteq ]-1, 1[$  and  $g_2 = \square^2$  and for every  $x$  such that  $x \in Z$  holds  $g_1(x) = 1$  and  $f_2(x) > 0$  and  $Z = \text{dom } f$ . Then  $\int_A f(x)dx = (-(\text{the function } \ln) \cdot (\text{the function arccot}))(\text{sup } A) - (-(\text{the function } \ln) \cdot (\text{the function arccot}))(\text{inf } A)$ .
- (3) Suppose that
  - (i)  $A \subseteq Z$ ,

- (ii) for every  $x$  such that  $x \in Z$  holds  $(\text{the function exp})(x) < 1$  and  $f_1(x) = 1$ ,
- (iii)  $Z \subseteq \text{dom}((\text{the function arctan}) \cdot (\text{the function exp}))$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f = \frac{\text{the function exp}}{f_1 + (\text{the function exp})^2}$ .

$$\text{Then } \int_A f(x) dx = ((\text{the function arctan}) \cdot (\text{the function exp}))(\sup A) - ((\text{the function arctan}) \cdot (\text{the function exp}))(\inf A).$$

- (4) Suppose that
  - (i)  $A \subseteq Z$ ,
  - (ii) for every  $x$  such that  $x \in Z$  holds  $(\text{the function exp})(x) < 1$  and  $f_1(x) = 1$ ,
  - (iii)  $Z \subseteq \text{dom}((\text{the function arccot}) \cdot (\text{the function exp}))$ ,
  - (iv)  $Z = \text{dom } f$ , and
  - (v)  $f = \frac{-\text{the function exp}}{f_1 + (\text{the function exp})^2}$ .

$$\text{Then } \int_A f(x) dx = ((\text{the function arccot}) \cdot (\text{the function exp}))(\sup A) - ((\text{the function arccot}) \cdot (\text{the function exp}))(\inf A).$$

- (5) Suppose that
  - (i)  $A \subseteq Z$ ,
  - (ii)  $Z = \text{dom } f$ , and
  - (iii)  $f = (\text{the function exp}) \frac{\text{the function sin}}{\text{the function cos}} + \frac{\text{the function exp}}{(\text{the function cos})^2}$ .

$$\text{Then } \int_A f(x) dx = ((\text{the function exp}) (\text{the function tan}))(\sup A) - ((\text{the function exp}) (\text{the function tan}))(\inf A).$$

- (6) Suppose that
  - (i)  $A \subseteq Z$ ,
  - (ii)  $Z = \text{dom } f$ , and
  - (iii)  $f = (\text{the function exp}) \frac{\text{the function cos}}{\text{the function sin}} - \frac{\text{the function exp}}{(\text{the function sin})^2}$ .

$$\text{Then } \int_A f(x) dx = ((\text{the function exp}) (\text{the function cot}))(\sup A) - ((\text{the function exp}) (\text{the function cot}))(\inf A).$$

- (7) Suppose that
  - (i)  $A \subseteq Z$ ,
  - (ii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$ ,
  - (iii)  $Z \subseteq ]-1, 1[$ ,
  - (iv)  $Z = \text{dom } f$ , and
  - (v)  $f = (\text{the function exp}) (\text{the function arctan}) + \frac{\text{the function exp}}{f_1 + \square^2}$ .

Then  $\int_A f(x)dx = ((\text{the function exp}) (\text{the function arctan}))(\text{sup } A) - ((\text{the function exp}) (\text{the function arctan}))(\text{inf } A)$ .

(8) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$ ,
- (iii)  $Z \subseteq ]-1, 1[$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f = (\text{the function exp}) (\text{the function arccot}) - \frac{\text{the function exp}}{f_1 + \square^2}$ .

Then  $\int_A f(x)dx = ((\text{the function exp}) (\text{the function arccot}))(\text{sup } A) - ((\text{the function exp}) (\text{the function arccot}))(\text{inf } A)$ .

(9) Suppose  $A \subseteq Z = \text{dom } f$  and  $f = ((\text{the function exp}) \cdot (\text{the function sin})) (\text{the function cos})$ . Then  $\int_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function sin}))(\text{sup } A) - ((\text{the function exp}) \cdot (\text{the function sin}))(\text{inf } A)$ .

(10) Suppose  $A \subseteq Z = \text{dom } f$  and  $f = ((\text{the function exp}) \cdot (\text{the function cos})) (\text{the function sin})$ .

Then  $\int_A f(x)dx = (-(\text{the function exp}) \cdot (\text{the function cos}))(\text{sup } A) - (-(\text{the function exp}) \cdot (\text{the function cos}))(\text{inf } A)$ .

(11) Suppose  $A \subseteq Z$  and for every  $x$  such that  $x \in Z$  holds  $x > 0$  and  $Z = \text{dom } f$  and  $f = ((\text{the function cos}) \cdot (\text{the function ln})) \frac{1}{\text{id}_Z}$ . Then  $\int_A f(x)dx = ((\text{the function sin}) \cdot (\text{the function ln}))(\text{sup } A) - ((\text{the function sin}) \cdot (\text{the function ln}))(\text{inf } A)$ .

(12) Suppose  $A \subseteq Z$  and for every  $x$  such that  $x \in Z$  holds  $x > 0$  and  $Z = \text{dom } f$  and  $f = ((\text{the function sin}) \cdot (\text{the function ln})) \frac{1}{\text{id}_Z}$ . Then  $\int_A f(x)dx = (-(\text{the function cos}) \cdot (\text{the function ln}))(\text{sup } A) - (-(\text{the function cos}) \cdot (\text{the function ln}))(\text{inf } A)$ .

(13) Suppose  $A \subseteq Z = \text{dom } f$  and  $f = (\text{the function exp}) ((\text{the function cos}) \cdot (\text{the function exp}))$ . Then  $\int_A f(x)dx = ((\text{the function sin}) \cdot (\text{the function exp}))(\text{sup } A) - ((\text{the function sin}) \cdot (\text{the function exp}))(\text{inf } A)$ .

(14) Suppose  $A \subseteq Z = \text{dom } f$  and  $f = (\text{the function exp}) ((\text{the function sin}) \cdot (\text{the function exp}))$ .

Then  $\int_A f(x)dx = (-(\text{the function cos}) \cdot (\text{the function exp}))(\text{sup } A) - (-(\text{the function cos}) \cdot (\text{the function exp}))(\text{inf } A)$ .

- (15) Suppose that  $A \subseteq Z \subseteq \text{dom}((\text{the function } \ln) \cdot (f_1 + f_2))$  and  $r \neq 0$  and for every  $x$  such that  $x \in Z$  holds  $g(x) = \frac{x}{r}$  and  $g(x) > -1$  and  $g(x) < 1$  and  $f_1(x) = 1$  and  $f_2 = (\square^2) \cdot g$  and  $Z = \text{dom } f$  and  $f = (\text{the function } \arctan) \cdot g$ . Then  $\int_A f(x) dx = (\text{id}_Z ((\text{the function } \arctan) \cdot g) - \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))(\text{sup } A) - (\text{id}_Z ((\text{the function } \arctan) \cdot g) - \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))(\text{inf } A)$ .
- (16) Suppose that  $A \subseteq Z \subseteq \text{dom}((\text{the function } \ln) \cdot (f_1 + f_2))$  and  $r \neq 0$  and for every  $x$  such that  $x \in Z$  holds  $g(x) = \frac{x}{r}$  and  $g(x) > -1$  and  $g(x) < 1$  and  $f_1(x) = 1$  and  $f_2 = (\square^2) \cdot g$  and  $Z = \text{dom } f$  and  $f = (\text{the function } \text{arccot}) \cdot g$ . Then  $\int_A f(x) dx = (\text{id}_Z ((\text{the function } \text{arccot}) \cdot g) + \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))(\text{sup } A) - (\text{id}_Z ((\text{the function } \text{arccot}) \cdot g) + \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))(\text{inf } A)$ .
- (17) Suppose that
- (i)  $A \subseteq Z$ ,
  - (ii)  $f = (\text{the function } \arctan) \cdot f_1 + \frac{\text{id}_Z}{r(g+f_1^2)}$ ,
  - (iii) for every  $x$  such that  $x \in Z$  holds  $g(x) = 1$  and  $f_1(x) = \frac{x}{r}$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
  - (iv)  $Z = \text{dom } f$ , and
  - (v)  $f$  is continuous on  $A$ .
- Then  $\int_A f(x) dx = (\text{id}_Z ((\text{the function } \arctan) \cdot f_1))(\text{sup } A) - (\text{id}_Z ((\text{the function } \arctan) \cdot f_1))(\text{inf } A)$ .
- (18) Suppose that
- (i)  $A \subseteq Z$ ,
  - (ii)  $f = (\text{the function } \text{arccot}) \cdot f_1 - \frac{\text{id}_Z}{r(g+f_1^2)}$ ,
  - (iii) for every  $x$  such that  $x \in Z$  holds  $g(x) = 1$  and  $f_1(x) = \frac{x}{r}$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
  - (iv)  $Z = \text{dom } f$ , and
  - (v)  $f$  is continuous on  $A$ .
- Then  $\int_A f(x) dx = (\text{id}_Z ((\text{the function } \text{arccot}) \cdot f_1))(\text{sup } A) - (\text{id}_Z ((\text{the function } \text{arccot}) \cdot f_1))(\text{inf } A)$ .
- (19) Suppose that  $A \subseteq Z \subseteq ]-1, 1[$  and for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$  and  $Z = \text{dom } f$  and  $Z \subseteq \text{dom}((\square^n) \cdot (\text{the function } \arcsin))$  and  $1 < n$  and  $f = \frac{n((\square^{n-1}) \cdot (\text{the function } \arcsin))}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$ . Then  $\int_A f(x) dx = ((\square^n) \cdot (\text{the function } \arcsin))(\text{sup } A) - ((\square^n) \cdot (\text{the function } \arcsin))(\text{inf } A)$ .
- (20) Suppose that  $A \subseteq Z \subseteq ]-1, 1[$  and for every  $x$  such that  $x \in Z$  holds

$f_1(x) = 1$  and  $Z \subseteq \text{dom}((\square^n) \cdot (\text{the function arccos}))$  and  $Z = \text{dom } f$  and  $1 < n$  and  $f = \frac{n \cdot ((\square^{n-1}) \cdot (\text{the function arccos}))}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$ . Then  $\int_A f(x) dx = ((-\square^n) \cdot (\text{the function arccos}))(\text{sup } A) - ((-\square^n) \cdot (\text{the function arccos}))(\text{inf } A)$ .

(21) Suppose  $A \subseteq Z$  and for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$  and  $Z \subseteq ]-1, 1[$  and  $Z = \text{dom } f$  and  $f = (\text{the function arcsin}) + \frac{\text{id}_Z}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$ .

Then  $\int_A f(x) dx = (\text{id}_Z (\text{the function arcsin}))(\text{sup } A) - (\text{id}_Z (\text{the function arcsin}))(\text{inf } A)$ .

(22) Suppose  $A \subseteq Z$  and for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$  and  $Z \subseteq ]-1, 1[$  and  $Z = \text{dom } f$  and  $f = (\text{the function arccos}) - \frac{\text{id}_Z}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$ .

Then  $\int_A f(x) dx = (\text{id}_Z (\text{the function arccos}))(\text{sup } A) - (\text{id}_Z (\text{the function arccos}))(\text{inf } A)$ .

(23) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $Z \subseteq ]-1, 1[$ ,
- (iii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x + b$  and  $f_2(x) = 1$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f = a (\text{the function arcsin}) + \frac{f_1}{(\square^{\frac{1}{2}}) \cdot (f_2 - \square^2)}$ .

Then  $\int_A f(x) dx = (f_1 (\text{the function arcsin}))(\text{sup } A) - (f_1 (\text{the function arcsin}))(\text{inf } A)$ .

(24) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $Z \subseteq ]-1, 1[$ ,
- (iii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x + b$  and  $f_2(x) = 1$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f = a (\text{the function arccos}) - \frac{f_1}{(\square^{\frac{1}{2}}) \cdot (f_2 - \square^2)}$ .

Then  $\int_A f(x) dx = (f_1 (\text{the function arccos}))(\text{sup } A) - (f_1 (\text{the function arccos}))(\text{inf } A)$ .

(25) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $g(x) = 1$  and  $f_1(x) = \frac{x}{a}$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
- (iii)  $Z = \text{dom } f$ ,
- (iv)  $f$  is continuous on  $A$ , and

$$(v) \quad f = (\text{the function arcsin}) \cdot f_1 + \frac{\text{id}_Z}{a((\square^{\frac{1}{2}}) \cdot (g - f_1^2))}.$$

$$\text{Then } \int_A f(x) dx = (\text{id}_Z((\text{the function arcsin}) \cdot f_1))(\text{sup } A) - (\text{id}_Z((\text{the function arcsin}) \cdot f_1))(\text{inf } A).$$

(26) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $g(x) = 1$  and  $f_1(x) = \frac{x}{a}$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
- (iii)  $Z = \text{dom } f$ ,
- (iv)  $f$  is continuous on  $A$ , and
- (v)  $f = (\text{the function arccos}) \cdot f_1 - \frac{\text{id}_Z}{a((\square^{\frac{1}{2}}) \cdot (g - f_1^2))}.$

$$\text{Then } \int_A f(x) dx = (\text{id}_Z((\text{the function arccos}) \cdot f_1))(\text{sup } A) - (\text{id}_Z((\text{the function arccos}) \cdot f_1))(\text{inf } A).$$

(27) Suppose  $A \subseteq Z$  and  $f = \frac{n((\square^{n-1}) \cdot (\text{the function sin}))}{(\square^{n+1}) \cdot (\text{the function cos})}$  and  $1 \leq n$  and  $Z \subseteq \text{dom}((\square^n) \cdot (\text{the function tan}))$  and  $Z = \text{dom } f$ . Then  $\int_A f(x) dx = ((\square^n) \cdot (\text{the function tan}))(\text{sup } A) - ((\square^n) \cdot (\text{the function tan}))(\text{inf } A).$

(28) Suppose  $A \subseteq Z$  and  $f = \frac{n((\square^{n-1}) \cdot (\text{the function cos}))}{(\square^{n+1}) \cdot (\text{the function sin})}$  and  $1 \leq n$  and  $Z \subseteq \text{dom}((\square^n) \cdot (\text{the function cot}))$  and  $Z = \text{dom } f$ . Then  $\int_A f(x) dx = (-((\square^n) \cdot (\text{the function cot}))(\text{sup } A) - (-((\square^n) \cdot (\text{the function cot}))(\text{inf } A)).$

(29) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $Z \subseteq \text{dom}((\text{the function tan}) \cdot f_1)$ ,
- (iii)  $f = \frac{((\text{the function sin}) \cdot f_1)^2}{((\text{the function cos}) \cdot f_1)^2}$ ,
- (iv) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x$  and  $a \neq 0$ , and
- (v)  $Z = \text{dom } f$ .

$$\text{Then } \int_A f(x) dx = \left(\frac{1}{a}((\text{the function tan}) \cdot f_1) - \text{id}_Z\right)(\text{sup } A) - \left(\frac{1}{a}((\text{the function tan}) \cdot f_1) - \text{id}_Z\right)(\text{inf } A).$$

(30) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $Z \subseteq \text{dom}((\text{the function cot}) \cdot f_1)$ ,
- (iii)  $f = \frac{((\text{the function cos}) \cdot f_1)^2}{((\text{the function sin}) \cdot f_1)^2}$ ,
- (iv) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x$  and  $a \neq 0$ , and
- (v)  $Z = \text{dom } f$ .

Then  $\int_A f(x)dx = ((-\frac{1}{a})((\text{the function cot}) \cdot f_1) - \text{id}_Z)(\text{sup } A) - ((-\frac{1}{a})((\text{the function cot}) \cdot f_1) - \text{id}_Z)(\text{inf } A)$ .

(31) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x + b$ ,
- (iii)  $Z = \text{dom } f$ , and
- (iv)  $f = a \frac{\text{the function sin}}{\text{the function cos}} + \frac{f_1}{(\text{the function cos})^2}$ .

Then  $\int_A f(x)dx = (f_1 (\text{the function tan}))(\text{sup } A) - (f_1 (\text{the function tan}))(\text{inf } A)$ .

(32) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = a \cdot x + b$ ,
- (iii)  $Z = \text{dom } f$ , and
- (iv)  $f = a \frac{\text{the function cos}}{\text{the function sin}} - \frac{f_1}{(\text{the function sin})^2}$ .

Then  $\int_A f(x)dx = (f_1 (\text{the function cot}))(\text{sup } A) - (f_1 (\text{the function cot}))(\text{inf } A)$ .

(33) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{(\text{the function sin})(x)^2}{(\text{the function cos})(x)^2}$ ,
- (iii)  $Z \subseteq \text{dom}((\text{the function tan}) - \text{id}_Z)$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = ((\text{the function tan}) - \text{id}_Z)(\text{sup } A) - ((\text{the function tan}) - \text{id}_Z)(\text{inf } A)$ .

(34) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{(\text{the function cos})(x)^2}{(\text{the function sin})(x)^2}$ ,
- (iii)  $Z \subseteq \text{dom}(-\text{the function cot} - \text{id}_Z)$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = (-\text{the function cot} - \text{id}_Z)(\text{sup } A) - (-\text{the function cot} - \text{id}_Z)(\text{inf } A)$ .

(35) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{1}{x \cdot (1 + (\text{the function ln})(x)^2)}$  and  $(\text{the function ln})(x) > -1$  and  $(\text{the function ln})(x) < 1$ ,

- (iii)  $Z \subseteq \text{dom}(\text{(the function arctan)} \cdot \text{(the function ln)})$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = (\text{(the function arctan)} \cdot \text{(the function ln)})(\text{sup } A) - (\text{(the function arctan)} \cdot \text{(the function ln)})(\text{inf } A)$ .

(36) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = -\frac{1}{x \cdot (1 + \text{(the function ln)}(x)^2)}$  and  $\text{(the function ln)}(x) > -1$  and  $\text{(the function ln)}(x) < 1$ ,
- (iii)  $Z \subseteq \text{dom}(\text{(the function arccot)} \cdot \text{(the function ln)})$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = (\text{(the function arccot)} \cdot \text{(the function ln)})(\text{sup } A) - (\text{(the function arccot)} \cdot \text{(the function ln)})(\text{inf } A)$ .

(37) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{a}{\sqrt{1 - (a \cdot x + b)^2}}$  and  $f_1(x) = a \cdot x + b$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
- (iii)  $Z \subseteq \text{dom}(\text{(the function arcsin)} \cdot f_1)$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = (\text{(the function arcsin)} \cdot f_1)(\text{sup } A) - (\text{(the function arcsin)} \cdot f_1)(\text{inf } A)$ .

(38) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{a}{\sqrt{1 - (a \cdot x + b)^2}}$  and  $f_1(x) = a \cdot x + b$  and  $f_1(x) > -1$  and  $f_1(x) < 1$ ,
- (iii)  $Z \subseteq \text{dom}(\text{(the function arccos)} \cdot f_1)$ ,
- (iv)  $Z = \text{dom } f$ , and
- (v)  $f$  is continuous on  $A$ .

Then  $\int_A f(x)dx = (-\text{(the function arccos)} \cdot f_1)(\text{sup } A) - (-\text{(the function arccos)} \cdot f_1)(\text{inf } A)$ .

(39) Suppose that  $A \subseteq Z$  and  $f_1 = g - f_2$  and  $f_2 = \square^2$  and for every  $x$  such that  $x \in Z$  holds  $f(x) = x \cdot (1 - x^2)^{-\frac{1}{2}}$  and  $g(x) = 1$  and  $f_1(x) > 0$  and  $Z \subseteq \text{dom}(\square^{\frac{1}{2}} \cdot f_1)$  and  $Z = \text{dom } f$  and  $f$  is continuous on  $A$ . Then  $\int_A f(x)dx =$

$$(-(\square^{\frac{1}{2}}) \cdot f_1)(\sup A) - (-(\square^{\frac{1}{2}}) \cdot f_1)(\inf A).$$

(40) Suppose that  $A \subseteq Z$  and  $g = f_1 - f_2$  and  $f_2 = \square^2$  and for every  $x$  such that  $x \in Z$  holds  $f(x) = x \cdot (a^2 - x^2)^{-\frac{1}{2}}$  and  $f_1(x) = a^2$  and  $g(x) > 0$  and  $Z \subseteq \text{dom}((\square^{\frac{1}{2}}) \cdot g)$  and  $Z = \text{dom } f$  and  $f$  is continuous on  $A$ . Then  $\int_A f(x)dx =$

$$(-(\square^{\frac{1}{2}}) \cdot g)(\sup A) - (-(\square^{\frac{1}{2}}) \cdot g)(\inf A).$$

(41) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $n > 0$ ,
- (iii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}}$  and (the function sin)( $x$ )  $\neq 0$ ,
- (iv)  $Z \subseteq \text{dom}((\square^n) \cdot \frac{1}{\text{the function sin}})$ ,
- (v)  $Z = \text{dom } f$ , and
- (vi)  $f$  is continuous on  $A$ .

$$\text{Then } \int_A f(x)dx = ((-\frac{1}{n})((\square^n) \cdot \frac{1}{\text{the function sin}}))(\sup A) - ((-\frac{1}{n})((\square^n) \cdot \frac{1}{\text{the function sin}}))(\inf A).$$

(42) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $n > 0$ ,
- (iii) for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}}$  and (the function cos)( $x$ )  $\neq 0$ ,
- (iv)  $Z \subseteq \text{dom}((\square^n) \cdot \frac{1}{\text{the function cos}})$ ,
- (v)  $Z = \text{dom } f$ , and
- (vi)  $f$  is continuous on  $A$ .

$$\text{Then } \int_A f(x)dx = (\frac{1}{n}((\square^n) \cdot \frac{1}{\text{the function cos}}))(\sup A) - (\frac{1}{n}((\square^n) \cdot \frac{1}{\text{the function cos}}))(\inf A).$$

(43) Suppose that  $A \subseteq Z$  and  $f = \frac{g_1+g_2}{f_2}$  and  $f_2 =$  the function arccot and  $Z \subseteq ]-1, 1[$  and  $g_2 = \square^2$  and for every  $x$  such that  $x \in Z$  holds  $f(x) = \frac{1}{(1+x^2) \cdot (\text{the function arccot})(x)}$  and  $g_1(x) = 1$  and  $f_2(x) > 0$  and  $Z = \text{dom } f$ . Then  $\int_A f(x)dx =$   
 $(-(\text{the function ln}) \cdot (\text{the function arccot}))(\sup A) -$   
 $(-(\text{the function ln}) \cdot (\text{the function arccot}))(\inf A).$

(44) Suppose that

- (i)  $A \subseteq Z$ ,
- (ii)  $Z \subseteq ]-1, 1[$ ,
- (iii) for every  $x$  such that  $x \in Z$  holds (the function arcsin)( $x$ )  $> 0$  and  $f_1(x) = 1$ ,

- (iv)  $Z \subseteq \text{dom}(\text{(the function ln)} \cdot \text{(the function arcsin)})$ ,  
 (v)  $Z = \text{dom } f$ , and  
 (vi)  $f = \frac{1}{((\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)) \text{(the function arcsin)}}$ .

Then  $\int_A f(x)dx = (\text{(the function ln)} \cdot \text{(the function arcsin)})(\text{sup } A) - (\text{(the function ln)} \cdot \text{(the function arcsin)})(\text{inf } A)$ .

(45) Suppose that

- (i)  $A \subseteq Z$ ,  
 (ii)  $Z \subseteq ]-1, 1[$ ,  
 (iii) for every  $x$  such that  $x \in Z$  holds  $f_1(x) = 1$  and  $\text{(the function arccos)}(x) > 0$ ,  
 (iv)  $Z \subseteq \text{dom}(\text{(the function ln)} \cdot \text{(the function arccos)})$ ,  
 (v)  $Z = \text{dom } f$ , and  
 (vi)  $f = \frac{1}{((\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)) \text{(the function arccos)}}$ .

Then  $\int_A f(x)dx = (-\text{(the function ln)} \cdot \text{(the function arccos)})(\text{sup } A) - (-\text{(the function ln)} \cdot \text{(the function arccos)})(\text{inf } A)$ .

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