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Integrability Formulas. Part I

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Summary. In this article, we give several differentiation and integrability formulas of special and composite functions including the trigonometric function, and the polynomial function.

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The papers [12], [2], [3], [1], [7], [11], [13], [4], [17], [8], [9], [6], [18], [5], [10], [15], [16], and [14] provide the terminology and notation for this paper.

One can check that there exists a subset of $\mathbb{R}$ which is closed-interval.

For simplicity, we use the following convention: a, b, x, r are real numbers, n is an element of $\mathbb{N}$, A is a closed-interval subset of $\mathbb{R}$, f, g, f_1, f_2, g_1, g_2 are partial functions from $\mathbb{R}$ to $\mathbb{R}$, and Z is an open subset of $\mathbb{R}$.

We now state a number of propositions:

(1) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{f_1+f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f_2 = \square^2$. Then $\frac{1}{f_1+f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{f_1+f_2})'_{\restriction Z}(x) = -\frac{2 \cdot x}{(1+x^2)^{\mathbf{2}}}$.

(2) Suppose that $A \subseteq Z$ and $f = \frac{\frac{1}{g_1+g_2}}{f_2}$ and $f_2 =$ the function arccot and $Z \subseteq]-1,1[$ and $g_2 = \square^2$ and for every x such that $x \in Z$ holds $g_1(x) = 1$ and $f_2(x) > 0$ and $Z = \mathrm{dom}\, f$. Then $\int\limits_{A} f(x)dx = (-(\text{the function ln}) \cdot (\text{the function arccot}))(\sup A) - (-(\text{the function ln}) \cdot (\text{the function arccot}))(\inf A)$.

(3) Suppose that

(i) $A \subseteq Z$,

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(ii) for every x such that $x \in Z$ holds (the function exp)$(x) < 1$ and $f_1(x) = 1$,
(iii) $Z \subseteq \mathrm{dom}$((the function arctan) $\cdot$(the function exp)),
(iv) $Z = \mathrm{dom}\, f$, and
(v) $f = \frac{\text{the function exp}}{f_1 + (\text{the function exp})^{\mathbf{2}}}$.

Then $\int\limits_A f(x)dx =$ ((the function arctan) $\cdot$(the function exp))$(\sup A) -$ ((the function arctan) $\cdot$(the function exp))$(\inf A)$.

(4) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds (the function exp)$(x) < 1$ and $f_1(x) = 1$,
(iii) $Z \subseteq \mathrm{dom}$((the function arccot) $\cdot$(the function exp)),
(iv) $Z = \mathrm{dom}\, f$, and
(v) $f = \frac{-\text{the function exp}}{f_1 + (\text{the function exp})^{\mathbf{2}}}$.

Then $\int\limits_A f(x)dx =$ ((the function arccot) $\cdot$(the function exp))$(\sup A) -$((the function arccot) $\cdot$(the function exp))$(\inf A)$.

(5) Suppose that
(i) $A \subseteq Z$,
(ii) $Z = \mathrm{dom}\, f$, and
(iii) $f =$ (the function exp) $\frac{\text{the function sin}}{\text{the function cos}} + \frac{\text{the function exp}}{(\text{the function cos})^{\mathbf{2}}}$.

Then $\int\limits_A f(x)dx =$ ((the function exp) (the function tan))$(\sup A) -$ ((the function exp) (the function tan))$(\inf A)$.

(6) Suppose that
(i) $A \subseteq Z$,
(ii) $Z = \mathrm{dom}\, f$, and
(iii) $f =$ (the function exp) $\frac{\text{the function cos}}{\text{the function sin}} - \frac{\text{the function exp}}{(\text{the function sin})^{\mathbf{2}}}$.

Then $\int\limits_A f(x)dx =$ ((the function exp) (the function cot))$(\sup A) -$ ((the function exp) (the function cot))$(\inf A)$.

(7) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
(iii) $Z \subseteq]-1, 1[$,
(iv) $Z = \mathrm{dom}\, f$, and
(v) $f =$ (the function exp) (the function arctan)$+\frac{\text{the function exp}}{f_1 + \square^2}$.

Then $\int\limits_A f(x)dx = ((\text{the function exp})\ (\text{the function arctan}))(\sup A) - ((\text{the function exp})\ (\text{the function arctan}))(\inf A)$.

(8) Suppose that

(i) $A \subseteq Z$,

(ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,

(iii) $Z \subseteq]-1, 1[$,

(iv) $Z = \operatorname{dom} f$, and

(v) $f = (\text{the function exp})\ (\text{the function arccot}) - \frac{\text{the function exp}}{f_1 + \square^2}$.

Then $\int\limits_A f(x)dx = ((\text{the function exp})\ (\text{the function arccot}))(\sup A) - ((\text{the function exp})\ (\text{the function arccot}))(\inf A)$.

(9) Suppose $A \subseteq Z = \operatorname{dom} f$ and $f = ((\text{the function exp}) \cdot (\text{the function sin}))\ (\text{the function cos})$. Then $\int\limits_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function sin}))(\sup A) - ((\text{the function exp}) \cdot (\text{the function sin}))(\inf A)$.

(10) Suppose $A \subseteq Z = \operatorname{dom} f$ and $f = ((\text{the function exp}) \cdot (\text{the function cos}))\ (\text{the function sin})$.
Then $\int\limits_A f(x)dx = (-(\text{the function exp}) \cdot (\text{the function cos}))(\sup A) - (-(\text{the function exp}) \cdot (\text{the function cos}))(\inf A)$.

(11) Suppose $A \subseteq Z$ and for every x such that $x \in Z$ holds $x > 0$ and $Z = \operatorname{dom} f$ and $f = ((\text{the function cos}) \cdot (\text{the function ln}))\ \frac{1}{\mathrm{id}_Z}$. Then $\int\limits_A f(x)dx = ((\text{the function sin}) \cdot (\text{the function ln}))(\sup A) - ((\text{the function sin}) \cdot (\text{the function ln}))(\inf A)$.

(12) Suppose $A \subseteq Z$ and for every x such that $x \in Z$ holds $x > 0$ and $Z = \operatorname{dom} f$ and $f = ((\text{the function sin}) \cdot (\text{the function ln}))\ \frac{1}{\mathrm{id}_Z}$. Then $\int\limits_A f(x)dx = (-(\text{the function cos}) \cdot (\text{the function ln}))(\sup A) - (-(\text{the function cos}) \cdot (\text{the function ln}))(\inf A)$.

(13) Suppose $A \subseteq Z = \operatorname{dom} f$ and $f = (\text{the function exp})\ ((\text{the function cos}) \cdot (\text{the function exp}))$. Then $\int\limits_A f(x)dx = ((\text{the function sin}) \cdot (\text{the function exp}))(\sup A) - ((\text{the function sin}) \cdot (\text{the function exp}))(\inf A)$.

(14) Suppose $A \subseteq Z = \operatorname{dom} f$ and $f = (\text{the function exp})\ ((\text{the function sin}) \cdot (\text{the function exp}))$.
Then $\int\limits_A f(x)dx = (-(\text{the function cos}) \cdot (\text{the function exp}))(\sup A) - (-(\text{the function cos}) \cdot (\text{the function exp}))(\inf A)$.

(15) Suppose that $A \subseteq Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot (f_1+f_2))$ and $r \neq 0$ and for every x such that $x \in Z$ holds $g(x) = \frac{x}{r}$ and $g(x) > -1$ and $g(x) < 1$ and $f_1(x) = 1$ and $f_2 = (\square^2) \cdot g$ and $Z = \mathrm{dom}\, f$ and $f = (\text{the function arctan}) \cdot g$. Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\, ((\text{the function arctan}) \cdot g) - \frac{r}{2}\, ((\text{the function ln}) \cdot (f_1 + f_2)))(\sup A) - (\mathrm{id}_Z\, ((\text{the function arctan}) \cdot g) - \frac{r}{2}\, ((\text{the function ln}) \cdot (f_1 + f_2)))(\inf A)$.

(16) Suppose that $A \subseteq Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot (f_1+f_2))$ and $r \neq 0$ and for every x such that $x \in Z$ holds $g(x) = \frac{x}{r}$ and $g(x) > -1$ and $g(x) < 1$ and $f_1(x) = 1$ and $f_2 = (\square^2) \cdot g$ and $Z = \mathrm{dom}\, f$ and $f = (\text{the function arccot}) \cdot g$. Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\, ((\text{the function arccot}) \cdot g) + \frac{r}{2}\, ((\text{the function ln}) \cdot (f_1 + f_2)))(\sup A) - (\mathrm{id}_Z\, ((\text{the function arccot}) \cdot g) + \frac{r}{2}\, ((\text{the function ln}) \cdot (f_1 + f_2)))(\inf A)$.

(17) Suppose that
- (i) $A \subseteq Z$,
- (ii) $f = (\text{the function arctan}) \cdot f_1 + \frac{\mathrm{id}_Z}{r\,(g+{f_1}^{\mathbf{2}})}$,
- (iii) for every x such that $x \in Z$ holds $g(x) = 1$ and $f_1(x) = \frac{x}{r}$ and $f_1(x) > -1$ and $f_1(x) < 1$,
- (iv) $Z = \mathrm{dom}\, f$, and
- (v) f is continuous on A.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\, ((\text{the function arctan}) \cdot f_1))(\sup A) - (\mathrm{id}_Z\, ((\text{the function arctan}) \cdot f_1))(\inf A)$.

(18) Suppose that
- (i) $A \subseteq Z$,
- (ii) $f = (\text{the function arccot}) \cdot f_1 - \frac{\mathrm{id}_Z}{r\,(g+{f_1}^{\mathbf{2}})}$,
- (iii) for every x such that $x \in Z$ holds $g(x) = 1$ and $f_1(x) = \frac{x}{r}$ and $f_1(x) > -1$ and $f_1(x) < 1$,
- (iv) $Z = \mathrm{dom}\, f$, and
- (v) f is continuous on A.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\, ((\text{the function arccot}) \cdot f_1))(\sup A) - (\mathrm{id}_Z\, ((\text{the function arccot}) \cdot f_1))(\inf A)$.

(19) Suppose that $A \subseteq Z \subseteq\,]-1, 1[$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z = \mathrm{dom}\, f$ and $Z \subseteq \mathrm{dom}((\square^n) \cdot (\text{the function arcsin}))$ and $1 < n$ and $f = \frac{n\,((\square^{n-1}) \cdot (\text{the function arcsin}))}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$. Then $\int\limits_A f(x)dx = ((\square^n) \cdot (\text{the function arcsin}))(\sup A) - ((\square^n) \cdot (\text{the function arcsin}))(\inf A)$.

(20) Suppose that $A \subseteq Z \subseteq\,]-1, 1[$ and for every x such that $x \in Z$ holds

$f_1(x) = 1$ and $Z \subseteq \operatorname{dom}((\square^n) \cdot (\text{the function arccos}))$ and $Z = \operatorname{dom} f$ and $1 < n$ and $f = \frac{n\,((\square^{n-1}) \cdot (\text{the function arccos}))}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$. Then $\int\limits_A f(x)dx = (-(\square^n) \cdot (\text{the function arccos}))(\sup A) - (-(\square^n) \cdot (\text{the function arccos}))(\inf A)$.

(21) Suppose $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z \subseteq]-1, 1[$ and $Z = \operatorname{dom} f$ and $f = (\text{the function arcsin}) + \frac{\mathrm{id}_Z}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$. Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\,(\text{the function arcsin}))(\sup A) - (\mathrm{id}_Z\,(\text{the function arcsin}))(\inf A)$.

(22) Suppose $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z \subseteq]-1, 1[$ and $Z = \operatorname{dom} f$ and $f = (\text{the function arccos}) - \frac{\mathrm{id}_Z}{(\square^{\frac{1}{2}}) \cdot (f_1 - \square^2)}$. Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\,(\text{the function arccos}))(\sup A) - (\mathrm{id}_Z\,(\text{the function arccos}))(\inf A)$.

(23) Suppose that
 (i) $A \subseteq Z$,
 (ii) $Z \subseteq]-1, 1[$,
 (iii) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x + b$ and $f_2(x) = 1$,
 (iv) $Z = \operatorname{dom} f$, and
 (v) $f = a\,(\text{the function arcsin}) + \frac{f_1}{(\square^{\frac{1}{2}}) \cdot (f_2 - \square^2)}$.

Then $\int\limits_A f(x)dx = (f_1\,(\text{the function arcsin}))(\sup A) - (f_1\,(\text{the function arcsin}))(\inf A)$.

(24) Suppose that
 (i) $A \subseteq Z$,
 (ii) $Z \subseteq]-1, 1[$,
 (iii) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x + b$ and $f_2(x) = 1$,
 (iv) $Z = \operatorname{dom} f$, and
 (v) $f = a\,(\text{the function arccos}) - \frac{f_1}{(\square^{\frac{1}{2}}) \cdot (f_2 - \square^2)}$.

Then $\int\limits_A f(x)dx = (f_1\,(\text{the function arccos}))(\sup A) - (f_1\,(\text{the function arccos}))(\inf A)$.

(25) Suppose that
 (i) $A \subseteq Z$,
 (ii) for every x such that $x \in Z$ holds $g(x) = 1$ and $f_1(x) = \frac{x}{a}$ and $f_1(x) > -1$ and $f_1(x) < 1$,
 (iii) $Z = \operatorname{dom} f$,
 (iv) f is continuous on A, and

(v) $f =$ (the function arcsin) $\cdot f_1 + \frac{\mathrm{id}_Z}{a\,((\square^{\frac{1}{2}})\cdot(g-f_1{}^{\mathbf{2}}))}$.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z$ ((the function arcsin) $\cdot f_1))(\sup A) - (\mathrm{id}_Z$ ((the function arcsin) $\cdot f_1))(\inf A)$.

(26) Suppose that

(i) $A \subseteq Z$,

(ii) for every x such that $x \in Z$ holds $g(x) = 1$ and $f_1(x) = \frac{x}{a}$ and $f_1(x) > -1$ and $f_1(x) < 1$,

(iii) $Z = \mathrm{dom}\, f$,

(iv) f is continuous on A, and

(v) $f =$ (the function arccos) $\cdot f_1 - \frac{\mathrm{id}_Z}{a\,((\square^{\frac{1}{2}})\cdot(g-f_1{}^{\mathbf{2}}))}$.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z$ ((the function arccos) $\cdot f_1))(\sup A) - (\mathrm{id}_Z$ ((the function arccos) $\cdot f_1))(\inf A)$.

(27) Suppose $A \subseteq Z$ and $f = \frac{n\,((\square^{n-1})\cdot(\text{the function sin}))}{(\square^{n+1})\cdot(\text{the function cos})}$ and $1 \leq n$ and $Z \subseteq \mathrm{dom}((\square^n) \cdot$ (the function tan)) and $Z = \mathrm{dom}\, f$. Then $\int\limits_A f(x)dx = ((\square^n) \cdot$ (the function tan))$(\sup A) - ((\square^n) \cdot$ (the function tan))$(\inf A)$.

(28) Suppose $A \subseteq Z$ and $f = \frac{n\,((\square^{n-1})\cdot(\text{the function cos}))}{(\square^{n+1})\cdot(\text{the function sin})}$ and $1 \leq n$ and $Z \subseteq \mathrm{dom}((\square^n) \cdot$ (the function cot)) and $Z = \mathrm{dom}\, f$. Then $\int\limits_A f(x)dx = (-(\square^n) \cdot$ (the function cot))$(\sup A) - (-(\square^n) \cdot$ (the function cot))$(\inf A)$.

(29) Suppose that

(i) $A \subseteq Z$,

(ii) $Z \subseteq \mathrm{dom}$((the function tan) $\cdot f_1$),

(iii) $f = \frac{((\text{the function sin})\cdot f_1)^{\mathbf{2}}}{((\text{the function cos})\cdot f_1)^{\mathbf{2}}}$,

(iv) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x$ and $a \neq 0$, and

(v) $Z = \mathrm{dom}\, f$.

Then $\int\limits_A f(x)dx = (\frac{1}{a}$ ((the function tan) $\cdot f_1) - \mathrm{id}_Z)(\sup A) - (\frac{1}{a}$ ((the function tan) $\cdot f_1) - \mathrm{id}_Z)(\inf A)$.

(30) Suppose that

(i) $A \subseteq Z$,

(ii) $Z \subseteq \mathrm{dom}$((the function cot) $\cdot f_1$),

(iii) $f = \frac{((\text{the function cos})\cdot f_1)^{\mathbf{2}}}{((\text{the function sin})\cdot f_1)^{\mathbf{2}}}$,

(iv) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x$ and $a \neq 0$, and

(v) $Z = \mathrm{dom}\, f$.

Then $\int\limits_A f(x)dx = ((-\frac{1}{a})((\text{the function cot}) \cdot f_1) - \mathrm{id}_Z)(\sup A) - ((-\frac{1}{a})((\text{the function cot}) \cdot f_1) - \mathrm{id}_Z)(\inf A)$.

(31) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x + b$,
(iii) $Z = \mathrm{dom}\, f$, and
(iv) $f = a \frac{\text{the function sin}}{\text{the function cos}} + \frac{f_1}{(\text{the function cos})^2}$.

Then $\int\limits_A f(x)dx = (f_1\,(\text{the function tan}))(\sup A) - (f_1\,(\text{the function tan}))(\inf A)$.

(32) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f_1(x) = a \cdot x + b$,
(iii) $Z = \mathrm{dom}\, f$, and
(iv) $f = a \frac{\text{the function cos}}{\text{the function sin}} - \frac{f_1}{(\text{the function sin})^2}$.

Then $\int\limits_A f(x)dx = (f_1\,(\text{the function cot}))(\sup A) - (f_1\,(\text{the function cot}))(\inf A)$.

(33) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{(\text{the function sin})(x)^2}{(\text{the function cos})(x)^2}$,
(iii) $Z \subseteq \mathrm{dom}((\text{the function tan}) - \mathrm{id}_Z)$,
(iv) $Z = \mathrm{dom}\, f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function tan}) - \mathrm{id}_Z)(\sup A) - ((\text{the function tan}) - \mathrm{id}_Z)(\inf A)$.

(34) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{(\text{the function cos})(x)^2}{(\text{the function sin})(x)^2}$,
(iii) $Z \subseteq \mathrm{dom}(-\text{the function cot} - \mathrm{id}_Z)$,
(iv) $Z = \mathrm{dom}\, f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = (-\text{the function cot} - \mathrm{id}_Z)(\sup A) - (-\text{the function cot} - \mathrm{id}_Z)(\inf A)$.

(35) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{1}{x \cdot (1 + (\text{the function ln})(x)^2)}$ and $(\text{the function ln})(x) > -1$ and $(\text{the function ln})(x) < 1$,

(iii) $Z \subseteq \operatorname{dom}((\text{the function arctan}) \cdot (\text{the function ln}))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.
Then $\int\limits_A f(x)dx = ((\text{the function arctan}) \cdot (\text{the function ln}))(\sup A) - ((\text{the function arctan}) \cdot (\text{the function ln}))(\inf A)$.

(36) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = -\frac{1}{x \cdot (1 + (\text{the function ln})(x)^{\mathbf{2}})}$ and (the function ln)$(x) > -1$ and (the function ln)$(x) < 1$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function arccot}) \cdot (\text{the function ln}))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.
Then $\int\limits_A f(x)dx = ((\text{the function arccot}) \cdot (\text{the function ln}))(\sup A) - ((\text{the function arccot}) \cdot (\text{the function ln}))(\inf A)$.

(37) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{a}{\sqrt{1 - (a \cdot x + b)^{\mathbf{2}}}}$ and $f_1(x) = a \cdot x + b$ and $f_1(x) > -1$ and $f_1(x) < 1$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function arcsin}) \cdot f_1)$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.
Then $\int\limits_A f(x)dx = ((\text{the function arcsin}) \cdot f_1)(\sup A) - ((\text{the function arcsin}) \cdot f_1)(\inf A)$.

(38) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{a}{\sqrt{1 - (a \cdot x + b)^{\mathbf{2}}}}$ and $f_1(x) = a \cdot x + b$ and $f_1(x) > -1$ and $f_1(x) < 1$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function arccos}) \cdot f_1)$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.
Then $\int\limits_A f(x)dx = (-(\text{the function arccos}) \cdot f_1)(\sup A) - (-(\text{the function arccos}) \cdot f_1)(\inf A)$.

(39) Suppose that $A \subseteq Z$ and $f_1 = g - f_2$ and $f_2 = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x \cdot (1 - x^2)^{-\frac{1}{2}}$ and $g(x) = 1$ and $f_1(x) > 0$ and $Z \subseteq \operatorname{dom}((\square^{\frac{1}{2}}) \cdot f_1)$ and $Z = \operatorname{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx =$

$(-(\square^{\frac{1}{2}}) \cdot f_1)(\sup A) - (-(\square^{\frac{1}{2}}) \cdot f_1)(\inf A)$.

(40) Suppose that $A \subseteq Z$ and $g = f_1 - f_2$ and $f_2 = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x \cdot (a^{\mathbf{2}} - x^2)^{-\frac{1}{2}}$ and $f_1(x) = a^{\mathbf{2}}$ and $g(x) > 0$ and $Z \subseteq \operatorname{dom}((\square^{\frac{1}{2}}) \cdot g)$ and $Z = \operatorname{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (-(\square^{\frac{1}{2}}) \cdot g)(\sup A) - (-(\square^{\frac{1}{2}}) \cdot g)(\inf A)$.

(41) Suppose that

(i) $A \subseteq Z$,

(ii) $n > 0$,

(iii) for every x such that $x \in Z$ holds $f(x) = \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}}$ and (the function sin)$(x) \neq 0$,

(iv) $Z \subseteq \operatorname{dom}((\square^n) \cdot \frac{1}{\text{the function sin}})$,

(v) $Z = \operatorname{dom} f$, and

(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = ((-\frac{1}{n})((\square^n) \cdot \frac{1}{\text{the function sin}}))(\sup A) - ((-\frac{1}{n})((\square^n) \cdot \frac{1}{\text{the function sin}}))(\inf A)$.

(42) Suppose that

(i) $A \subseteq Z$,

(ii) $n > 0$,

(iii) for every x such that $x \in Z$ holds $f(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}}$ and (the function cos)$(x) \neq 0$,

(iv) $Z \subseteq \operatorname{dom}((\square^n) \cdot \frac{1}{\text{the function cos}})$,

(v) $Z = \operatorname{dom} f$, and

(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (\frac{1}{n}((\square^n) \cdot \frac{1}{\text{the function cos}}))(\sup A) - (\frac{1}{n}((\square^n) \cdot \frac{1}{\text{the function cos}}))(\inf A)$.

(43) Suppose that $A \subseteq Z$ and $f = \frac{\frac{1}{g_1+g_2}}{f_2}$ and $f_2 =$ the function arccot and $Z \subseteq]{-1}, 1[$ and $g_2 = \square^2$ and for every x such that $x \in Z$ holds $f(x) = \frac{1}{(1+x^{\mathbf{2}}) \cdot (\text{the function arccot})(x)}$ and $g_1(x) = 1$ and $f_2(x) > 0$ and $Z = \operatorname{dom} f$. Then $\int\limits_A f(x)dx = (-(\text{the function ln}) \cdot (\text{the function arccot}))(\sup A) - (-(\text{the function ln}) \cdot (\text{the function arccot}))(\inf A)$.

(44) Suppose that

(i) $A \subseteq Z$,

(ii) $Z \subseteq]{-1}, 1[$,

(iii) for every x such that $x \in Z$ holds (the function arcsin)$(x) > 0$ and $f_1(x) = 1$,

(iv) $Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot (\text{the function arcsin}))$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) $f = \frac{1}{((\square^{\frac{1}{2}}) \cdot (f_1 - \square^2))\ (\text{the function arcsin})}$.

Then $\int\limits_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function arcsin}))(\sup A) - ((\text{the function ln}) \cdot (\text{the function arcsin}))(\inf A)$.

(45) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1, 1[$,
(iii) for every x such that $x \in Z$ holds $f_1(x) = 1$ and $(\text{the function arccos})(x) > 0$,
(iv) $Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot (\text{the function arccos}))$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) $f = \frac{1}{((\square^{\frac{1}{2}}) \cdot (f_1 - \square^2))\ (\text{the function arccos})}$.

Then $\int\limits_A f(x)dx = (-(\text{the function ln}) \cdot (\text{the function arccos}))(\sup A) - (-(\text{the function ln}) \cdot (\text{the function arccos}))(\inf A)$.

References

[1] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[2] Noboru Endou and Artur Korniłowicz. The definition of the Riemann definite integral and some related lemmas. *Formalized Mathematics*, 8(**1**):93–102, 1999.
[3] Noboru Endou, Katsumi Wasaki, and Yasunari Shidama. Definition of integrability for partial functions from $\mathbb{R}$ to $\mathbb{R}$ and integrability for continuous functions. *Formalized Mathematics*, 9(**2**):281–284, 2001.
[4] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[5] Artur Korniłowicz and Yasunari Shidama. Inverse trigonometric functions arcsin and arccos. *Formalized Mathematics*, 13(**1**):73–79, 2005.
[6] Jarosław Kotowicz. Convergent real sequences. Upper and lower bound of sets of real numbers. *Formalized Mathematics*, 1(**3**):477–481, 1990.
[7] Jarosław Kotowicz. Partial functions from a domain to a domain. *Formalized Mathematics*, 1(**4**):697–702, 1990.
[8] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 1(**4**):703–709, 1990.
[9] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.
[10] Xiquan Liang and Bing Xie. Inverse trigonometric functions arctan and arccot. *Formalized Mathematics*, 16(**2**):147–158, 2008, doi:10.2478/v10037-008-0021-3.
[11] Konrad Raczkowski. Integer and rational exponents. *Formalized Mathematics*, 2(**1**):125–130, 1991.
[12] Konrad Raczkowski and Paweł Sadowski. Real function continuity. *Formalized Mathematics*, 1(**4**):787–791, 1990.
[13] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.
[14] Yasunari Shidama. The Taylor expansions. *Formalized Mathematics*, 12(**2**):195–200, 2004.
[15] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[16] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[17] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.
[18] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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