FORMALIZED MATHEMATICS
Vol. 18, No. 1, Pages 95–106, 2010
DOI: 10.2478/v10037-010-0013-y

Abstract Simplicial Complexes

Karol Pąk
Institute of Informatics
University of Białystok
Poland

Summary. In this article we define the notion of abstract simplicial complexes and operations on them. We introduce the following basic notions: simplex, face, vertex, degree, skeleton, subdivision and substructure, and prove some of their properties.

MML identifier: `SIMPLEX0`, version: `7.11.05 4.134.1080`

The articles [2], [5], [6], [10], [8], [14], [1], [7], [3], [4], [11], [13], [16], [12], [15], and [9] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we adopt the following convention: x, y, X, Y, Z are sets, D is a non empty set, n, k are natural numbers, and i, i_1, i_2 are integers.

Let us consider X. We introduce X has empty element as an antonym of X has non empty elements.

Note that there exists a set which is empty and finite-membered and every set which is empty is also finite-membered. Let X be a finite set. Note that $\{X\}$ is finite-membered and 2^X is finite-membered. Let Y be a finite set. Observe that $\{X, Y\}$ is finite-membered.

Let X be a finite-membered set. Observe that every subset of X is finite-membered. Let Y be a finite-membered set. One can check that $X \cup Y$ is finite-membered.

Let X be a finite finite-membered set. Note that $\bigcup X$ is finite.

One can verify the following observations:

* every set which is empty is also subset-closed,

© 2010 University of Białystok
ISSN 1426–2630(p), 1898-9934(e)

Unauthenticated
Download Date | 5/25/17 5:15 AM

* every set which has empty element is also non empty,
* every set which is non empty and subset-closed has also empty element, and
* there exists a set which has empty element.

Let us consider X. Observe that $\mathrm{SubFin}(X)$ is finite-membered and there exists a family of subsets of X which is subset-closed, finite, and finite-membered.

Let X be a subset-closed set. One can check that $\mathrm{SubFin}(X)$ is subset-closed.

Next we state the proposition

(1) Y is subset-closed iff for every X such that $X \in Y$ holds $2^X \subseteq Y$.

Let A, B be subset-closed sets. Note that $A \cup B$ is subset-closed and $A \cap B$ is subset-closed.

Let us consider X. The subset-closure of X yields a subset-closed set and is defined by the conditions (Def. 1).

(Def. 1)(i) $X \subseteq$ the subset-closure of X, and

(ii) for every Y such that $X \subseteq Y$ and Y is subset-closed holds the subset-closure of $X \subseteq Y$.

The following proposition is true

(2) $x \in$ the subset-closure of X iff there exists y such that $x \subseteq y$ and $y \in X$.

Let us consider X and let F be a family of subsets of X. Then the subset-closure of F is a subset-closed family of subsets of X.

Observe that the subset-closure of $\emptyset$ is empty. Let X be a non empty set. Note that the subset-closure of X is non empty.

Let X be a set with a non-empty element. One can check that the subset-closure of X has a non-empty element.

Let X be a finite-membered set. Note that the subset-closure of X is finite-membered.

The following propositions are true:

(3) If $X \subseteq Y$ and Y is subset-closed, then the subset-closure of $X \subseteq Y$.

(4) The subset-closure of $\{X\} = 2^X$.

(5) The subset-closure of $X \cup Y =$ (the subset-closure of X) $\cup$ (the subset-closure of Y).

(6) X is finer than Y iff the subset-closure of $X \subseteq$ the subset-closure of Y.

(7) If X is subset-closed, then the subset-closure of $X = X$.

(8) If the subset-closure of $X \subseteq X$, then X is subset-closed.

Let us consider Y, X and let n be a set. The subsets of X and Y with cardinality limited by n yields a family of subsets of Y and is defined by the condition (Def. 2).

(Def. 2) Let A be a subset of Y. Then $A \in$ the subsets of X and Y with cardinality limited by n if and only if $A \in X$ and $\mathrm{Card}\, A \subseteq \mathrm{Card}\, n$.

Unauthenticated
Download Date | 5/25/17 5:15 AM

Let us consider D. One can verify that there exists a family of subsets of D which is finite, subset-closed, and finite-membered and has a non-empty element.

Let us consider Y, X and let n be a finite set. One can check that the subsets of X and Y with cardinality limited by n is finite-membered.

Let us consider Y, let X be a subset-closed set, and let n be a set. Note that the subsets of X and Y with cardinality limited by n is subset-closed.

Let us consider Y, let X be a set with empty element, and let n be a set. One can check that the subsets of X and Y with cardinality limited by n has empty element.

Let us consider D, let X be a subset-closed family of subsets of D with a non-empty element, and let n be a non empty set. Note that the subsets of X and D with cardinality limited by n has a non-empty element.

Let us consider X, let Y be a family of subsets of X, and let n be a set. We introduce the subsets of Y with cardinality limited by n as a synonym of the subsets of Y and X with cardinality limited by n.

Let us observe that every set which is empty is also $\subseteq$-linear and there exists a set which is empty and $\subseteq$-linear.

Let X be a $\subseteq$-linear set. Note that every subset of X is $\subseteq$-linear.

The following propositions are true:

(9) If X is non empty, finite, and $\subseteq$-linear, then $\bigcup X \in X$.

(10) For every finite $\subseteq$-linear set X such that X has non empty elements holds $\mathrm{Card}\, X \subseteq \mathrm{Card} \bigcup X$.

(11) If X is $\subseteq$-linear and $\bigcup X$ misses x, then $X \cup \{\bigcup X \cup x\}$ is $\subseteq$-linear.

(12) Let X be a non empty set. Then there exists a family Y of subsets of X such that

(i) Y is $\subseteq$-linear and has non empty elements,

(ii) $X \in Y$,

(iii) $\mathrm{Card}\, X = \mathrm{Card}\, Y$, and

(iv) for every Z such that $Z \in Y$ and $\mathrm{Card}\, Z \neq 1$ there exists x such that $x \in Z$ and $Z \setminus \{x\} \in Y$.

(13) Let Y be a family of subsets of X. Suppose Y is finite and $\subseteq$-linear and has non empty elements and $X \in Y$. Then there exists a family Y' of subsets of X such that

(i) $Y \subseteq Y'$,

(ii) Y' is $\subseteq$-linear and has non empty elements,

(iii) $\mathrm{Card}\, X = \mathrm{Card}\, Y'$, and

(iv) for every Z such that $Z \in Y'$ and $\mathrm{Card}\, Z \neq 1$ there exists x such that $x \in Z$ and $Z \setminus \{x\} \in Y'$.

Unauthenticated
Download Date | 5/25/17 5:15 AM

2. Simplicial Complexes

A simplicial complex structure is a topological structure.

In the sequel K denotes a simplicial complex structure.

Let us consider K and let A be a subset of K. We introduce A is simplex-like as a synonym of A is open.

Let us consider K and let S be a family of subsets of K. We introduce S is simplex-like as a synonym of S is open.

Let us consider K. One can check that there exists a family of subsets of K which is empty and simplex-like.

The following proposition is true

(14) For every family S of subsets of K holds S is simplex-like iff $S \subseteq$ the topology of K.

Let us consider K and let v be an element of K. We say that v is vertex-like if and only if:

(Def. 3) There exists a subset S of K such that S is simplex-like and $v \in S$.

Let us consider K. The functor Vertices K yielding a subset of K is defined by:

(Def. 4) For every element v of K holds $v \in \mathrm{Vertices}\,K$ iff v is vertex-like.

Let K be a simplicial complex structure. A vertex of K is an element of Vertices K.

Let K be a simplicial complex structure. We say that K is finite-vertices if and only if:

(Def. 5) Vertices K is finite.

Let us consider K. We say that K is locally-finite if and only if:

(Def. 6) For every vertex v of K holds $\{S \subseteq K\colon S$ is simplex-like $\wedge\ v \in S\}$ is finite.

Let us consider K. We say that K is empty-membered if and only if:

(Def. 7) The topology of K is empty-membered.

We say that K has non empty elements if and only if:

(Def. 8) The topology of K has non empty elements.

Let us consider K. We introduce K has a non-empty element as an antonym of K is empty-membered. We introduce K has empty element as an antonym of K has non empty elements.

Let us consider X. A simplicial complex structure is said to be a simplicial complex structure of X if:

(Def. 9) $\Omega_{\mathrm{it}} \subseteq X$.

Let us consider X and let K_1 be a simplicial complex structure of X. We say that K_1 is total if and only if:

Unauthenticated
Download Date | 5/25/17 5:15 AM

(Def. 10) $\Omega_{(K_1)} = X$.

One can check the following observations:

* every simplicial complex structure which has empty element is also non void,
* every simplicial complex structure which has a non-empty element is also non void,
* every simplicial complex structure which is non void and empty-membered has also empty element,
* every simplicial complex structure which is non void and subset-closed has also empty element,
* every simplicial complex structure which is empty-membered is also subset-closed and finite-vertices,
* every simplicial complex structure which is finite-vertices is also locally-finite and finite-degree, and
* every simplicial complex structure which is locally-finite and subset-closed is also finite-membered.

Let us consider X. Observe that there exists a simplicial complex structure of X which is empty, void, empty-membered, and strict.

Let us consider D. Note that there exists a simplicial complex structure of D which is non empty, non void, total, empty-membered, and strict and there exists a simplicial complex structure of D which is non empty, total, finite-vertices, subset-closed, and strict and has empty element and a non-empty element.

Let us observe that there exists a simplicial complex structure which is non empty, finite-vertices, subset-closed, and strict and has empty element and a non-empty element.

Let K be a simplicial complex structure with a non-empty element. Observe that $\mathrm{Vertices}\, K$ is non empty.

Let K be a finite-vertices simplicial complex structure. Note that every family of subsets of K which is simplex-like is also finite.

Let K be a finite-membered simplicial complex structure. Note that every family of subsets of K which is simplex-like is also finite-membered.

Next we state several propositions:

(15) $\mathrm{Vertices}\, K$ is empty iff K is empty-membered.

(16) $\mathrm{Vertices}\, K = \bigcup$ (the topology of K).

(17) For every subset S of K such that S is simplex-like holds $S \subseteq \mathrm{Vertices}\, K$.

(18) If K is finite-vertices, then the topology of K is finite.

(19) If the topology of K is finite and K is non finite-vertices, then K is non finite-membered.

(20) If K is subset-closed and the topology of K is finite, then K is finite-vertices.

Unauthenticated
Download Date | 5/25/17 5:15 AM

3. The Simplicial Complex Generated on the Set

Let us consider X and let Y be a family of subsets of X. The complex of Y yielding a strict simplicial complex structure of X is defined as follows:

(Def. 11) The complex of $Y = \langle X, \text{the subset-closure of } Y \rangle$.

Let us consider X and let Y be a family of subsets of X. One can verify that the complex of Y is total and subset-closed.

Let us consider X and let Y be a non empty family of subsets of X. Note that the complex of Y has empty element.

Let us consider X and let Y be a finite-membered family of subsets of X. Note that the complex of Y is finite-membered.

Let us consider X and let Y be a finite finite-membered family of subsets of X. Observe that the complex of Y is finite-vertices.

One can prove the following proposition

(21) If K is subset-closed, then the topological structure of $K =$ the complex of the topology of K.

Let us consider X. A simplicial complex of X is a finite-membered subset-closed simplicial complex structure of X.

Let K be a non void simplicial complex structure. A simplex of K is a simplex-like subset of K.

Let K be a simplicial complex structure with empty element. One can check that every subset of K which is empty is also simplex-like and there exists a simplex of K which is empty.

Let K be a non void finite-membered simplicial complex structure. Note that there exists a simplex of K which is finite.

4. The Degree of Simplicial Complexes

Let us consider K. The functor degree(K) yields an extended real number and is defined as follows:

(Def. 12)(i) For every finite subset S of K such that S is simplex-like holds $\overline{\overline{S}} \leq \text{degree}(K) + 1$ and there exists a subset S of K such that S is simplex-like and $\text{Card}\, S = \text{degree}(K) + 1$ if K is non void and finite-degree,

(ii) $\text{degree}(K) = -1$ if K is void,

(iii) $\text{degree}(K) = +\infty$, otherwise.

Let K be a finite-degree simplicial complex structure. Note that $\text{degree}(K) + 1$ is natural and degree(K) is integer.

The following propositions are true:

(22) $\text{degree}(K) = -1$ iff K is empty-membered.

(23) $-1 \leq \text{degree}(K)$.

Unauthenticated
Download Date | 5/25/17 5:15 AM

(24) For every finite subset S of K such that S is simplex-like holds $\overline{\overline{S}} \leq \mathrm{degree}(K) + 1$.

(25) Suppose K is non void or $i \geq -1$. Then $\mathrm{degree}(K) \leq i$ if and only if the following conditions are satisfied:

(i) K is finite-membered, and

(ii) for every finite subset S of K such that S is simplex-like holds $\overline{\overline{S}} \leq i+1$.

(26) For every finite subset A of X holds degree(the complex of $\{A\}$) $= \overline{\overline{A}}-1$.

5. Subcomplexes

Let us consider X and let K_1 be a simplicial complex structure of X. A simplicial complex of X is said to be a subsimplicial complex of K_1 if:

(Def. 13) $\Omega_{\mathrm{it}} \subseteq \Omega_{(K_1)}$ and the topology of it $\subseteq$ the topology of K_1.

In the sequel K_1 denotes a simplicial complex structure of X and S_1 denotes a subsimplicial complex of K_1.

Let us consider X, K_1. One can check that there exists a subsimplicial complex of K_1 which is empty, void, and strict.

Let us consider X and let K_1 be a void simplicial complex structure of X. Observe that every subsimplicial complex of K_1 is void.

Let us consider D and let K_2 be a non void subset-closed simplicial complex structure of D. Note that there exists a subsimplicial complex of K_2 which is non void.

Let us consider X and let K_1 be a finite-vertices simplicial complex structure of X. One can check that every subsimplicial complex of K_1 is finite-vertices.

Let us consider X and let K_1 be a finite-degree simplicial complex structure of X. Note that every subsimplicial complex of K_1 is finite-degree.

Next we state several propositions:

(27) Every subsimplicial complex of S_1 is a subsimplicial complex of K_1.

(28) Let A be a subset of K_1 and S be a finite-membered family of subsets of A. Suppose the subset-closure of $S \subseteq$ the topology of K_1. Then the complex of S is a strict subsimplicial complex of K_1.

(29) Let K_1 be a subset-closed simplicial complex structure of X, A be a subset of K_1, and S be a finite-membered family of subsets of A. Suppose $S \subseteq$ the topology of K_1. Then the complex of S is a strict subsimplicial complex of K_1.

(30) Let Y_1, Y_2 be families of subsets of X. Suppose Y_1 is finite-membered and finer than Y_2. Then the complex of Y_1 is a subsimplicial complex of the complex of Y_2.

(31) $\mathrm{Vertices}\, S_1 \subseteq \mathrm{Vertices}\, K_1$.

(32) $\mathrm{degree}(S_1) \leq \mathrm{degree}(K_1)$.

Unauthenticated
Download Date | 5/25/17 5:15 AM

Let us consider X, K_1, S_1. We say that S_1 is maximal if and only if:

(Def. 14) For every subset A of S_1 such that $A \in$ the topology of K_1 holds A is simplex-like.

We now state the proposition

(33) S_1 is maximal iff $2^{\Omega_{(S_1)}} \cap$ the topology of $K_1 \subseteq$ the topology of S_1.

Let us consider X, K_1. Note that there exists a subsimplicial complex of K_1 which is maximal and strict.

We now state three propositions:

(34) Let S_2 be a subsimplicial complex of S_1. Suppose S_1 is maximal and S_2 is maximal. Then S_2 is a maximal subsimplicial complex of K_1.

(35) Let S_2 be a subsimplicial complex of S_1. If S_2 is a maximal subsimplicial complex of K_1, then S_2 is maximal.

(36) Let K_3, K_4 be maximal subsimplicial complexes of K_1. Suppose $\Omega_{(K_3)} = \Omega_{(K_4)}$. Then the topological structure of $K_3 =$ the topological structure of K_4.

Let us consider X, let K_1 be a subset-closed simplicial complex structure of X, and let A be a subset of K_1. Let us assume that $2^A \cap$ the topology of K_1 is finite-membered. The functor $K_1 \restriction A$ yields a maximal strict subsimplicial complex of K_1 and is defined as follows:

(Def. 15) $\Omega_{K_1 \restriction A} = A$.

In the sequel S_3 denotes a simplicial complex of X.

Let us consider X, S_3 and let A be a subset of S_3. Then $S_3 \restriction A$ is a maximal strict subsimplicial complex of S_3 and it can be characterized by the condition:

(Def. 16) $\Omega_{S_3 \restriction A} = A$.

The following four propositions are true:

(37) For every subset A of S_3 holds the topology of $S_3 \restriction A = 2^A \cap$ the topology of S_3.

(38) For all subsets A, B of S_3 and for every subset B' of $S_3 \restriction A$ such that $B' = B$ holds $S_3 \restriction A \restriction B' = S_3 \restriction B$.

(39) $S_3 \restriction \Omega_{(S_3)} =$ the topological structure of S_3.

(40) For all subsets A, B of S_3 such that $A \subseteq B$ holds $S_3 \restriction A$ is a subsimplicial complex of $S_3 \restriction B$.

Let us observe that every integer is finite.

6. The Skeleton of a Simplicial Complex

Let us consider X, K_1 and let i be a real number. The skeleton of K_1 and i yielding a simplicial complex structure of X is defined by the condition (Def. 17).

Unauthenticated
Download Date | 5/25/17 5:15 AM

(Def. 17) The skeleton of K_1 and i = the complex of the subsets of the topology of K_1 with cardinality limited by $i + 1$.

Let us consider X, K_1. Observe that the skeleton of K_1 and -1 is empty-membered. Let us consider i. Note that the skeleton of K_1 and i is finite-degree.

Let us consider X, let K_1 be an empty-membered simplicial complex structure of X, and let us consider i. One can check that the skeleton of K_1 and i is empty-membered.

Let us consider D, let K_2 be a non void subset-closed simplicial complex structure of D, and let us consider i. One can check that the skeleton of K_2 and i is non void.

One can prove the following proposition

(41) If $-1 \leq i_1 \leq i_2$, then the skeleton of K_1 and i_1 is a subsimplicial complex of the skeleton of K_1 and i_2.

Let us consider X, let K_1 be a subset-closed simplicial complex structure of X, and let us consider i. Then the skeleton of K_1 and i is a subsimplicial complex of K_1.

We now state several propositions:

(42) If K_1 is subset-closed and the skeleton of K_1 and i is empty-membered, then K_1 is empty-membered or $i = -1$.

(43) degree(the skeleton of K_1 and i) $\leq$ degree(K_1).

(44) If $-1 \leq i$, then degree(the skeleton of K_1 and i) $\leq i$.

(45) If $-1 \leq i$ and the skeleton of K_1 and i = the topological structure of K_1, then degree(K_1) $\leq i$.

(46) If K_1 is subset-closed and degree(K_1) $\leq i$, then the skeleton of K_1 and i = the topological structure of K_1.

In the sequel K is a non void subset-closed simplicial complex structure.

Let us consider K and let i be a real number. Let us assume that i is integer. A finite simplex of K is said to be a simplex of i and K if:

(Def. 18)(i) $\overline{\overline{\text{it}}} = i + 1$ if $-1 \leq i \leq$ degree(K),

(ii) it is empty, otherwise.

Let us consider K. Note that every simplex of -1 and K is empty.

The following three propositions are true:

(47) For every simplex S of i and K such that S is non empty holds i is natural.

(48) Every finite simplex S of K is a simplex of $\overline{\overline{S}} - 1$ and K.

(49) Let K be a non void subset-closed simplicial complex structure of D, S be a non void subsimplicial complex of K, i be an integer, and A be a simplex of i and S. If A is non empty or $i \leq$ degree(S) or degree(S) = degree(K), then A is a simplex of i and K.

Unauthenticated
Download Date | 5/25/17 5:15 AM

Let us consider K and let i be a real number. Let us assume that i is integer and $i \leq \operatorname{degree}(K)$. Let S be a simplex of i and K. A simplex of $\max(i-1,-1)$ and K is said to be a face of S if:

(Def. 19) It $\subseteq S$.

One can prove the following proposition

(50) Let S be a simplex of n and K. Suppose $n \leq \operatorname{degree}(K)$. Then X is a face of S if and only if there exists x such that $x \in S$ and $S \setminus \{x\} = X$.

7. The Subdivision of a Simplicial Complex

In the sequel P is a function.

Let us consider X, K_1, P. The functor subdivision(P, K_1) yields a strict simplicial complex structure of X and is defined by the conditions (Def. 20).

(Def. 20)(i) $\Omega_{\mathrm{subdivision}(P,K_1)} = \Omega_{(K_1)}$, and

(ii) for every subset A of subdivision(P, K_1) holds A is simplex-like iff there exists a $\subseteq$-linear finite simplex-like family S of subsets of K_1 such that $A = P^{\circ}S$.

Let us consider X, K_1, P. One can verify that subdivision(P, K_1) is subset-closed and finite-membered and has empty element.

Let us consider X, let K_1 be a void simplicial complex structure of X, and let us consider P. Observe that subdivision(P, K_1) is empty-membered.

The following propositions are true:

(51) $\operatorname{degree}(\mathrm{subdivision}(P, K_1)) \leq \operatorname{degree}(K_1) + 1$.

(52) If $\operatorname{dom} P$ has non empty elements, then $\operatorname{degree}(\mathrm{subdivision}(P, K_1)) \leq \operatorname{degree}(K_1)$.

Let us consider X, let K_1 be a finite-degree simplicial complex structure of X, and let us consider P. Note that subdivision(P, K_1) is finite-degree.

Let us consider X, let K_1 be a finite-vertices simplicial complex structure of X, and let us consider P. One can check that subdivision(P, K_1) is finite-vertices.

One can prove the following propositions:

(53) Let K_1 be a subset-closed simplicial complex structure of X and given P. Suppose that

(i) $\operatorname{dom} P$ has non empty elements, and

(ii) for every n such that $n \leq \operatorname{degree}(K_1)$ there exists a subset S of K_1 such that S is simplex-like and $\operatorname{Card} S = n+1$ and $2^S_+ \subseteq \operatorname{dom} P$ and $P^{\circ}2^S_+$ is a subset of K_1 and $P{\restriction}2^S_+$ is one-to-one.

Then $\operatorname{degree}(\mathrm{subdivision}(P, K_1)) = \operatorname{degree}(K_1)$.

(54) If $Y \subseteq Z$, then subdivision$(P{\restriction}Y, K_1)$ is a subsimplicial complex of subdivision$(P{\restriction}Z, K_1)$.

Unauthenticated
Download Date | 5/25/17 5:15 AM

(55) If $\operatorname{dom} P \cap$ the topology of $K_1 \subseteq Y$, then subdivision$(P{\restriction}Y, K_1) =$ subdivision(P, K_1).

(56) If $Y \subseteq Z$, then subdivision$(Y{\restriction}P, K_1)$ is a subsimplicial complex of subdivision$(Z{\restriction}P, K_1)$.

(57) If P°(the topology of K_1) $\subseteq Y$, then subdivision$(Y{\restriction}P, K_1) =$ subdivision(P, K_1).

(58) subdivision(P, S_1) is a subsimplicial complex of subdivision(P, K_1).

(59) For every subset A of subdivision(P, K_1) such that $\operatorname{dom} P \subseteq$ the topology of S_1 and $A = \Omega_{(S_1)}$ holds subdivision$(P, S_1) =$ subdivision$(P, K_1){\restriction}A$.

(60) Let K_3, K_4 be simplicial complex structures of X. Suppose the topological structure of K_3 = the topological structure of K_4. Then subdivision$(P, K_3) =$ subdivision(P, K_4).

Let us consider X, K_1, P, n. The functor subdivision(n, P, K_1) yielding a simplicial complex structure of X is defined by the condition (Def. 21).

(Def. 21) There exists a function F such that

(i) $F(0) = K_1$,

(ii) $F(n) =$ subdivision(n, P, K_1),

(iii) $\operatorname{dom} F = \mathbb{N}$, and

(iv) for every k and for every simplicial complex structure K_1' of X such that $K_1' = F(k)$ holds $F(k+1) =$ subdivision(P, K_1').

Next we state several propositions:

(61) subdivision$(0, P, K_1) = K_1$.

(62) subdivision$(1, P, K_1) =$ subdivision(P, K_1).

(63) For every natural number n_1 such that $n_1 = n + k$ holds subdivision$(n_1, P, K_1) =$ subdivision$(n, P,$ subdivision$(k, P, K_1))$.

(64) $\Omega_{\text{subdivision}(n,P,K_1)} = \Omega_{(K_1)}$.

(65) subdivision(n, P, S_1) is a subsimplicial complex of subdivision(n, P, K_1).

REFERENCES

[1] Broderick Arneson and Piotr Rudnicki. Recognizing chordal graphs: Lex BFS and MCS. *Formalized Mathematics*, 14(**4**):187–205, 2006, doi:10.2478/v10037-006-0022-z.

[2] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[3] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[4] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[5] Grzegorz Bancerek. Tarski's classes and ranks. *Formalized Mathematics*, 1(**3**):563–567, 1990.

[6] Grzegorz Bancerek. Continuous, stable, and linear maps of coherence spaces. *Formalized Mathematics*, 5(**3**):381–393, 1996.

[7] Grzegorz Bancerek and Yasunari Shidama. Introduction to matroids. *Formalized Mathematics*, 16(**4**):325–332, 2008, doi:10.2478/v10037-008-0040-0.

[8] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

Unauthenticated
Download Date | 5/25/17 5:15 AM

[9] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[10] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[11] Adam Naumowicz. On Segre's product of partial line spaces. *Formalized Mathematics*, 9(**2**):383–390, 2001.
[12] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.
[13] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[14] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[15] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[16] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received December 18, 2009

Unauthenticated
Download Date | 5/25/17 5:15 AM