

The Sum and Product of Finite Sequences of Complex Numbers

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Summary. This article extends the [10]. We define the sum and the product of the sequence of complex numbers, and formalize these theorems. Our method refers to the [11].

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The notation and terminology used in this paper have been introduced in the following papers: [5], [7], [6], [4], [8], [13], [9], [2], [3], [15], [10], [12], and [14].

1. AUXILIARY THEOREMS

Let F be a complex-valued binary relation. Then $\text{rng } F$ is a subset of \mathbb{C} .

Let D be a non empty set, let F be a function from \mathbb{C} into D , and let F_1 be a complex-valued finite sequence. Note that $F \cdot F_1$ is finite sequence-like.

For simplicity, we adopt the following rules: i, j denote natural numbers, x, x_1 denote elements of \mathbb{C} , c denotes a complex number, F, F_1, F_2 denote complex-valued finite sequences, and R, R_1 denote i -element finite sequences of elements of \mathbb{C} .

The unary operation sqrcomplex on \mathbb{C} is defined as follows:

(Def. 1) For every c holds $(\text{sqrcomplex})(c) = c^2$.

Next we state two propositions:

- (1) sqrcomplex is distributive w.r.t. $\cdot_{\mathbb{C}}$.
- (2) $\cdot_{\mathbb{C}}$ is distributive w.r.t. $+_{\mathbb{C}}$.

2. SOME FUNCTORS ON THE i -TUPLES OF COMPLEX NUMBERS

Let us consider F_1, F_2 . Then $F_1 + F_2$ is a finite sequence of elements of \mathbb{C} and it can be characterized by the condition:

$$\text{(Def. 2)} \quad F_1 + F_2 = (+_{\mathbb{C}})^{\circ}(F_1, F_2).$$

Let us observe that the functor $F_1 + F_2$ is commutative.

Let us consider i, R_1, R_2 . Then $R_1 + R_2$ is an element of \mathbb{C}^i .

The following propositions are true:

$$(3) \quad (R_1 + R_2)(s) = R_1(s) + R_2(s).$$

$$(4) \quad \varepsilon_{\mathbb{C}} + F = \varepsilon_{\mathbb{C}}.$$

$$(5) \quad \langle c_1 \rangle + \langle c_2 \rangle = \langle c_1 + c_2 \rangle.$$

$$(6) \quad i \mapsto c_1 + i \mapsto c_2 = i \mapsto (c_1 + c_2).$$

Let us consider F . Then $-F$ is a finite sequence of elements of \mathbb{C} and it can be characterized by the condition:

$$\text{(Def. 3)} \quad -F = -_{\mathbb{C}} \cdot F.$$

Let us consider i, R . Then $-R$ is an element of \mathbb{C}^i .

The following propositions are true:

$$(7) \quad -\langle c \rangle = \langle -c \rangle.$$

$$(8) \quad -i \mapsto c = i \mapsto (-c).$$

$$(9) \quad \text{If } R_1 + R = R_2 + R, \text{ then } R_1 = R_2.$$

$$(10) \quad -(F_1 + F_2) = -F_1 + -F_2.$$

Let us consider F_1, F_2 . Then $F_1 - F_2$ is a finite sequence of elements of \mathbb{C} and it can be characterized by the condition:

$$\text{(Def. 4)} \quad F_1 - F_2 = (-_{\mathbb{C}})^{\circ}(F_1, F_2).$$

Let us consider i, R_1, R_2 . Then $R_1 - R_2$ is an element of \mathbb{C}^i .

The following propositions are true:

$$(11) \quad (R_1 - R_2)(s) = R_1(s) - R_2(s).$$

$$(12) \quad \varepsilon_{\mathbb{C}} - F = \varepsilon_{\mathbb{C}} \text{ and } F - \varepsilon_{\mathbb{C}} = \varepsilon_{\mathbb{C}}.$$

$$(13) \quad \langle c_1 \rangle - \langle c_2 \rangle = \langle c_1 - c_2 \rangle.$$

$$(14) \quad i \mapsto c_1 - i \mapsto c_2 = i \mapsto (c_1 - c_2).$$

$$(15) \quad R - i \mapsto 0_{\mathbb{C}} = R.$$

$$(16) \quad -(F_1 - F_2) = F_2 - F_1.$$

$$(17) \quad -(F_1 - F_2) = -F_1 + F_2.$$

$$(18) \quad \text{If } R_1 - R_2 = i \mapsto 0_{\mathbb{C}}, \text{ then } R_1 = R_2.$$

$$(19) \quad R_1 = (R_1 + R) - R.$$

$$(20) \quad R_1 = (R_1 - R) + R.$$

Let us consider F, c . We introduce $c \cdot F$ as a synonym of cF .

Let us consider F, c . Then $c \cdot F$ is a finite sequence of elements of \mathbb{C} and it can be characterized by the condition:

(Def. 5) $c \cdot F = \cdot_c \cdot F$.

Let us consider i, R, c . Then $c \cdot R$ is an element of \mathbb{C}^i .

One can prove the following four propositions:

(21) $c \cdot \langle c_1 \rangle = \langle c \cdot c_1 \rangle$.

(22) $c_1 \cdot (i \mapsto c_2) = i \mapsto (c_1 \cdot c_2)$.

(23) $(c_1 + c_2) \cdot F = c_1 \cdot F + c_2 \cdot F$.

(24) $0_{\mathbb{C}} \cdot R = i \mapsto 0_{\mathbb{C}}$.

Let us consider F_1, F_2 . We introduce $F_1 \bullet F_2$ as a synonym of $F_1 F_2$.

Let us consider F_1, F_2 . Then $F_1 \bullet F_2$ is a finite sequence of elements of \mathbb{C} and it can be characterized by the condition:

(Def. 6) $F_1 \bullet F_2 = (\cdot_{\mathbb{C}})^{\circ}(F_1, F_2)$.

Let us note that the functor $F_1 \bullet F_2$ is commutative.

Let us consider i, R_1, R_2 . Then $R_1 \bullet R_2$ is an element of \mathbb{C}^i .

Next we state four propositions:

(25) $\varepsilon_{\mathbb{C}} \bullet F = \varepsilon_{\mathbb{C}}$.

(26) $\langle c_1 \rangle \bullet \langle c_2 \rangle = \langle c_1 \cdot c_2 \rangle$.

(27) $i \mapsto c \bullet R = c \cdot R$.

(28) $i \mapsto c_1 \bullet i \mapsto c_2 = i \mapsto (c_1 \cdot c_2)$.

3. FINITE SUM OF FINITE SEQUENCE OF COMPLEX NUMBERS

One can prove the following propositions:

(29) $\sum(\varepsilon_{\mathbb{C}}) = 0_{\mathbb{C}}$.

(30) $\sum\langle c \rangle = c$.

(31) $\sum(F \wedge \langle c \rangle) = \sum F + c$.

(32) $\sum(F_1 \wedge F_2) = \sum F_1 + \sum F_2$.

(33) $\sum(\langle c \rangle \wedge F) = c + \sum F$.

(34) $\sum\langle c_1, c_2 \rangle = c_1 + c_2$.

(35) $\sum\langle c_1, c_2, c_3 \rangle = c_1 + c_2 + c_3$.

(36) $\sum(i \mapsto c) = i \cdot c$.

(37) $\sum(i \mapsto 0_{\mathbb{C}}) = 0_{\mathbb{C}}$.

(38) $\sum(c \cdot F) = c \cdot \sum F$.

(39) $\sum(-F) = -\sum F$.

(40) $\sum(R_1 + R_2) = \sum R_1 + \sum R_2$.

(41) $\sum(R_1 - R_2) = \sum R_1 - \sum R_2$.

4. THE PRODUCT OF FINITE SEQUENCES OF COMPLEX NUMBERS

One can prove the following propositions:

- (42) $\prod(\varepsilon_{\mathbb{C}}) = 1.$
- (43) $\prod(\langle c \rangle \wedge F) = c \cdot \prod F.$
- (44) For every element R of \mathbb{C}^0 holds $\prod R = 1.$
- (45) $\prod((i + j) \mapsto c) = \prod(i \mapsto c) \cdot \prod(j \mapsto c).$
- (46) $\prod((i \cdot j) \mapsto c) = \prod(j \mapsto \prod(i \mapsto c)).$
- (47) $\prod(i \mapsto (c_1 \cdot c_2)) = \prod(i \mapsto c_1) \cdot \prod(i \mapsto c_2).$
- (48) $\prod(R_1 \bullet R_2) = \prod R_1 \cdot \prod R_2.$
- (49) $\prod(c \cdot R) = \prod(i \mapsto c) \cdot \prod R.$

5. MODIFIED PART OF [1]

We now state several propositions:

- (50) For every complex-valued finite sequence x holds $\text{len}(-x) = \text{len } x.$
- (51) For all complex-valued finite sequences x_1, x_2 such that $\text{len } x_1 = \text{len } x_2$ holds $\text{len}(x_1 + x_2) = \text{len } x_1.$
- (52) For all complex-valued finite sequences x_1, x_2 such that $\text{len } x_1 = \text{len } x_2$ holds $\text{len}(x_1 - x_2) = \text{len } x_1.$
- (53) For every real number a and for every complex-valued finite sequence x holds $\text{len}(a \cdot x) = \text{len } x.$
- (54) For all complex-valued finite sequences x, y, z such that $\text{len } x = \text{len } y = \text{len } z$ holds $(x + y) \bullet z = x \bullet z + y \bullet z.$

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