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Definition of First Order Language with Arbitrary Alphabet. Syntax of Terms, Atomic Formulas and their Subterms[1]

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Summary. Second of a series of articles laying down the bases for classical first order model theory. A language is defined basically as a tuple made of an integer-valued function (adicity), a symbol of equality and a symbol for the NOR logical connective. The only requests for this tuple to be a language is that the value of the adicity in = is -2 and that its preimage (i.e. the variables set) in 0 is infinite. Existential quantification will be rendered (see [11]) by mere prefixing a formula with a letter. Then the hierarchy among symbols according to their adicity is introduced, taking advantage of attributes and clusters.

The strings of symbols of a language are depth-recursively classified as terms using the standard approach (see for example [16], definition 1.1.2); technically, this is done here by deploying the '-multiCat' functor and the 'unambiguous' attribute previously introduced in [10], and the set of atomic formulas is introduced. The set of all terms is shown to be unambiguous with respect to concatenation; we say that it is a prefix set. This fact is exploited to uniquely define the subterms both of a term and of an atomic formula without resorting to a parse tree.

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The papers [1], [3], [18], [5], [6], [12], [10], [7], [8], [9], [19], [14], [13], [2], [17], [4], [21], [22], [15], and [20] provide the terminology and notation for this paper.

We follow the rules: m, n are natural numbers, m_1, n_1 are elements of $\mathbb{N}$, and X, x, z are sets.

Let z be a zero integer number. One can check that $|z|$ is zero.

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Let us observe that there exists a real number which is negative and integer and every integer number which is positive is also natural.

Let S be a non degenerated zero-one structure. Observe that (the carrier of S) $\setminus$ {the one of S} is non empty.

We introduce languages-like which are extensions of zero-one structure and are systems

$\langle$ a carrier, a zero, a one, an adicity $\rangle$,

where the carrier is a set, the zero and the one are elements of the carrier, and the adicity is a function from the carrier $\setminus${the one} into $\mathbb{Z}$.

Let S be a language-like. The functor AllSymbolsOf S is defined by:

(Def. 1) AllSymbolsOf S = the carrier of S.

The functor LettersOf S is defined as follows:

(Def. 2) LettersOf S = (the adicity of $S)^{-1}(\{0\})$.

The functor OpSymbolsOf S is defined by:

(Def. 3) OpSymbolsOf S = (the adicity of $S)^{-1}(\mathbb{N} \setminus \{0\})$.

The functor RelSymbolsOf S is defined by:

(Def. 4) RelSymbolsOf S = (the adicity of $S)^{-1}(\mathbb{Z} \setminus \mathbb{N})$.

The functor TermSymbolsOf S is defined as follows:

(Def. 5) TermSymbolsOf S = (the adicity of $S)^{-1}(\mathbb{N})$.

The functor LowerCompoundersOf S is defined as follows:

(Def. 6) LowerCompoundersOf S = (the adicity of $S)^{-1}(\mathbb{Z} \setminus \{0\})$.

The functor TheEqSymbOf S is defined as follows:

(Def. 7) TheEqSymbOf S = the zero of S.

The functor TheNorSymbOf S is defined as follows:

(Def. 8) TheNorSymbOf S = the one of S.

The functor OwnSymbolsOf S is defined by:

(Def. 9) OwnSymbolsOf S = (the carrier of S) $\setminus$ {the zero of S, the one of S}.

Let S be a language-like. An element of S is an element of AllSymbolsOf S. The functor AtomicFormulaSymbolsOf S is defined by:

(Def. 10) AtomicFormulaSymbolsOf S = AllSymbolsOf S $\setminus$ {TheNorSymbOf S}.

The functor AtomicTermsOf S is defined by:

(Def. 11) AtomicTermsOf S = (LettersOf $S)^1$.

We say that S is operational if and only if:

(Def. 12) OpSymbolsOf S is non empty.

We say that S is relational if and only if:

(Def. 13) RelSymbolsOf S $\setminus$ {TheEqSymbOf S} is non empty.

Let S be a language-like and let s be an element of S. We say that s is literal if and only if:

(Def. 14) $s \in$ LettersOf S.

We say that s is low-compounding if and only if:

(Def. 15) $s \in$ LowerCompoundersOf S.

We say that s is operational if and only if:

(Def. 16) $s \in$ OpSymbolsOf S.

We say that s is relational if and only if:

(Def. 17) $s \in$ RelSymbolsOf S.

We say that s is termal if and only if:

(Def. 18) $s \in$ TermSymbolsOf S.

We say that s is own if and only if:

(Def. 19) $s \in$ OwnSymbolsOf S.

We say that s is of-atomic-formula if and only if:

(Def. 20) $s \in$ AtomicFormulaSymbolsOf S.

Let S be a zero-one structure and let s be an element of (the carrier of S) $\setminus$ {the one of S}. The functor TrivialArity s yields an integer number and is defined by:

(Def. 21) $\text{TrivialArity}\, s = \begin{cases} -2, & \text{if } s = \text{the zero of } S, \\ 0, & \text{otherwise.} \end{cases}$

Let S be a zero-one structure and let s be an element of (the carrier of S) $\setminus$ {the one of S}. Then TrivialArity s is an element of $\mathbb{Z}$.

Let S be a non degenerated zero-one structure. The functor S TrivialArity yielding a function from (the carrier of S) $\setminus$ {the one of S} into $\mathbb{Z}$ is defined by:

(Def. 22) For every element s of (the carrier of S) $\setminus$ {the one of S} holds (S TrivialArity)(s) = TrivialArity s.

Let us observe that there exists a non degenerated zero-one structure which is infinite.

Let S be an infinite non degenerated zero-one structure.

Observe that $(S\,\text{TrivialArity})^{-1}(\{0\})$ is infinite.

Let S be a language-like. We say that S is eligible if and only if:

(Def. 23) LettersOf S is infinite and (the adicity of S)(TheEqSymbOf S) $= -2$.

One can check that there exists a language-like which is non degenerated.

One can check that there exists a non degenerated language-like which is eligible.

A language is an eligible non degenerated language-like.

We follow the rules: S, S_1, S_2 are languages and s, s_1, s_2 are elements of S.

Let S be a non empty language-like. Then AllSymbolsOf S is a non empty set.

Let S be an eligible language-like. Note that LettersOf S is infinite.

Let S be a language.

Then LettersOf S is a non empty subset of AllSymbolsOf S. Note that TheEqSymbOf S is relational.

Let S be a non degenerated language-like. Then AtomicFormulaSymbolsOf S is a non empty subset of AllSymbolsOf S.

Let S be a non degenerated language-like. Then TheEqSymbOf S is an element of AtomicFormulaSymbolsOf S.

We now state the proposition

(1) Let S be a language. Then LettersOf $S \cap$ OpSymbolsOf $S = \emptyset$ and TermSymbolsOf $S \cap$ LowerCompoundersOf $S =$ OpSymbolsOf S and RelSymbolsOf $S \setminus$ OwnSymbolsOf $S = \{$TheEqSymbOf $S\}$ and OwnSymbolsOf $S \subseteq$ AtomicFormulaSymbolsOf S and RelSymbolsOf $S \subseteq$ LowerCompoundersOf S and OpSymbolsOf $S \subseteq$ TermSymbolsOf S and LettersOf $S \subseteq$ TermSymbolsOf $S \subseteq$ OwnSymbolsOf S and OpSymbolsOf $S \subseteq$ LowerCompoundersOf $S \subseteq$ AtomicFormulaSymbolsOf S.

Let S be a language. One can verify the following observations:

* TermSymbolsOf S is non empty,
* every element of S which is own is also of-atomic-formula,
* every element of S which is relational is also low-compounding,
* every element of S which is operational is also termal,
* every element of S which is literal is also termal,
* every element of S which is termal is also own,
* every element of S which is operational is also low-compounding,
* every element of S which is low-compounding is also of-atomic-formula,
* every element of S which is termal is also non relational,
* every element of S which is literal is also non relational, and
* every element of S which is literal is also non operational.

Let S be a language. Note that there exists an element of S which is relational and there exists an element of S which is literal. Observe that every low-compounding element of S which is termal is also operational. One can check that there exists an element of S which is of-atomic-formula.

Let s be an of-atomic-formula element of S. The functor ar s yielding an element of $\mathbb{Z}$ is defined by:

(Def. 24) ar $s =$ (the adicity of S)(s).

Let S be a language and let s be a literal element of S. Note that ar s is zero. The functor S-cons yielding a binary operation on (AllSymbolsOf $S)^*$ is defined as follows:

(Def. 25) S-cons $=$ the concatenation of AllSymbolsOf S.

Let S be a language.

The functor S-multiCat yields a function from $((\mathrm{AllSymbolsOf}\, S)^*)^*$ into $(\mathrm{AllSymbolsOf}\, S)^*$ and is defined by:

(Def. 26) S-multiCat = (AllSymbolsOf S)-multiCat .

Let S be a language. The functor S-firstChar yielding a function from $(\mathrm{AllSymbolsOf}\, S)^* \setminus \{\emptyset\}$ into AllSymbolsOf S is defined as follows:

(Def. 27) S-firstChar = (AllSymbolsOf S)-firstChar .

Let S be a language and let X be a set. We say that X is S-prefix if and only if:

(Def. 28) X is AllSymbolsOf S-prefix.

Let S be a language. Note that every set which is S-prefix is also AllSymbolsOf S-prefix and every set which is AllSymbolsOf S-prefix is also S-prefix. A string of S is an element of $(\mathrm{AllSymbolsOf}\, S)^* \setminus \{\emptyset\}$.

Let us consider S. One can check that $(\mathrm{AllSymbolsOf}\, S)^* \setminus \{\emptyset\}$ is non empty. Note that every string of S is non empty.

Let us note that every language is infinite. Observe that AllSymbolsOf S is infinite.

Let s be an of-atomic-formula element of S, and let S_3 be a set. The functor Compound(s, S_3) is defined by:

(Def. 29) $\mathrm{Compound}(s, S_3) = \{\langle s\rangle \frown S\text{-multiCat}(S_4); S_4$ ranges over elements of $((\mathrm{AllSymbolsOf}\, S)^*)^*$: $\mathrm{rng}\, S_4 \subseteq S_3 \ \wedge\ S_4$ is $|\mathrm{ar}\, s|$-element$\}$.

Let S be a language, let s be an of-atomic-formula element of S, and let S_3 be a set. Then Compound(s, S_3) is an element of $2^{(\mathrm{AllSymbolsOf}\, S)^* \setminus \{\emptyset\}}$. The functor S-termsOfMaxDepth yields a function and is defined by the conditions (Def. 30).

(Def. 30)(i) dom(S-termsOfMaxDepth) = $\mathbb{N}$,

(ii) S-termsOfMaxDepth(0) = AtomicTermsOf S, and

(iii) for every natural number n holds S-termsOfMaxDepth$(n + 1) = \bigcup\{\mathrm{Compound}(s, S\text{-termsOfMaxDepth}(n)); s$ ranges over of-atomic-formula elements of S: s is operational$\} \cup S$-termsOfMaxDepth(n).

Let us consider S. Then AtomicTermsOf S is a subset of $(\mathrm{AllSymbolsOf}\, S)^*$.

Let S be a language. The functor AllTermsOf S is defined as follows:

(Def. 31) AllTermsOf $S = \bigcup \mathrm{rng}(S$-termsOfMaxDepth).

One can prove the following proposition

(2) S-termsOfMaxDepth$(m_1) \subseteq$ AllTermsOf S.

Let S be a language and let w be a string of S. We say that w is termal if and only if:

(Def. 32) $w \in$ AllTermsOf S.

Let m be a natural number, let S be a language, and let w be a string of S. We say that w is m-termal if and only if:

(Def. 33) $w \in S$-termsOfMaxDepth(m).

Let m be a natural number and let S be a language. Note that every string of S which is m-termal is also termal.

Let us consider S. Then S-termsOfMaxDepth is a function from $\mathbb{N}$ into $2^{(\text{AllSymbolsOf}\,S)^*}$. Then AllTermsOf S is a non empty subset of (AllSymbolsOf $S)^*$. Note that AllTermsOf S is non empty.

Let us consider m. One can verify that S-termsOfMaxDepth(m) is non empty. Observe that every element of S-termsOfMaxDepth(m) is non empty. Observe that every element of AllTermsOf S is non empty.

Let m be a natural number and let S be a language. Note that there exists a string of S which is m-termal. Observe that every string of S which is 0-termal is also 1-element.

Let S be a language and let w be a 0-termal string of S. Observe that S-firstChar(w) is literal.

Let us consider S and let w be a termal string of S. Note that S-firstChar(w) is termal.

Let us consider S and let t be a termal string of S. The functor ar t yielding an element of $\mathbb{Z}$ is defined as follows:

(Def. 34) $\operatorname{ar} t = \operatorname{ar} S$-firstChar$(t)$.

Next we state the proposition

(3) For every $m_1 + 1$-termal string w of S there exists an element T of S-termsOfMaxDepth$(m_1)^*$ such that T is $|\operatorname{ar} S$-firstChar$(w)|$-element and $w = \langle S$-firstChar$(w)\rangle \frown S$-multiCat(T).

Let us consider S, m. Note that S-termsOfMaxDepth(m) is S-prefix.

Let us consider S and let V be an element of (AllTermsOf $S)^*$. Observe that S-multiCat(V) is relation-like.

Let us consider S and let V be an element of (AllTermsOf $S)^*$. One can verify that S-multiCat(V) is function-like.

Let us consider S and let p_1 be a string of S. We say that p_1 is 0-w.f.f. if and only if:

(Def. 35) There exists a relational element s of S and there exists an $|\operatorname{ar} s|$-element element V of (AllTermsOf $S)^*$ such that $p_1 = \langle s\rangle \frown S$-multiCat$(V)$.

Let us consider S. Note that there exists a string of S which is 0-w.f.f..

Let p_1 be a 0-w.f.f. string of S. Observe that S-firstChar(p_1) is relational. The functor AtomicFormulasOf S is defined as follows:

(Def. 36) AtomicFormulasOf $S = \{p_1; p_1$ ranges over strings of S: p_1 is 0-w.f.f.$\}$.

Let us consider S. Then AtomicFormulasOf S is a subset of (AllSymbolsOf $S)^* \setminus \{\emptyset\}$. Note that AtomicFormulasOf S is non empty. Observe that every element of AtomicFormulasOf S is 0-w.f.f.. Observe that AllTermsOf S is S-prefix.

Let us consider S and let t be a termal string of S. The functor SubTerms t yields an element of (AllTermsOf S)* and is defined by:

(Def. 37) SubTerms t is $|$ar S-firstChar$(t)|$-element and $t = \langle S\text{-firstChar}(t)\rangle \frown S$-multiCat(SubTerms t).

Let us consider S and let t be a termal string of S. One can verify that SubTerms t is $|$ar $t|$-element.

Let t_0 be a 0-termal string of S. Note that SubTerms t_0 is empty.

Let us consider m_1, S and let t be an $m_1 + 1$-termal string of S. One can verify that SubTerms t is S-termsOfMaxDepth(m_1)-valued.

Let us consider S and let p_1 be a 0-w.f.f. string of S. The functor SubTerms p_1 yields an $|$ar S-firstChar$(p_1)|$-element element of (AllTermsOf S)* and is defined as follows:

(Def. 38) $p_1 = \langle S\text{-firstChar}(p_1)\rangle \frown S$-multiCat(SubTerms p_1).

Let us consider S and let p_1 be a 0-w.f.f. string of S. Note that SubTerms p_1 is $|$ar S-firstChar$(p_1)|$-element.

Then AllTermsOf S is an element of $2^{(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}}$. Note that every element of AllTermsOf S is termal. The functor S-subTerms yielding a function from AllTermsOf S into (AllTermsOf S)* is defined by:

(Def. 39) For every element t of AllTermsOf S holds S-subTerms(t) = SubTerms t.

We now state several propositions:

(4) S-termsOfMaxDepth$(m) \subseteq S$-termsOfMaxDepth$(m + n)$.

(5) If $x \in$ AllTermsOf S, then there exists n_1 such that $x \in S$-termsOfMaxDepth(n_1).

(6) AllTermsOf $S \subseteq$ (AllSymbolsOf S)$^* \setminus \{\emptyset\}$.

(7) AllTermsOf S is S-prefix.

(8) If $x \in$ AllTermsOf S, then x is a string of S.

(9) AtomicFormulaSymbolsOf $S \setminus$ OwnSymbolsOf S = {TheEqSymbOf S}.

(10) TermSymbolsOf $S \setminus$ LettersOf S = OpSymbolsOf S.

(11) AtomicFormulaSymbolsOf $S \setminus$ RelSymbolsOf S = TermSymbolsOf S.

Let us consider S. Observe that every of-atomic-formula element of S which is non relational is also termal.

Then OwnSymbolsOf S is a subset of AllSymbolsOf S. Observe that every termal element of S which is non literal is also operational.

Next we state three propositions:

(12) x is a string of S iff x is a non empty element of (AllSymbolsOf S)*.

(13) x is a string of S iff x is a non empty finite sequence of elements of AllSymbolsOf S.

(14) S-termsOfMaxDepth is a function from $\mathbb{N}$ into $2^{(\text{AllSymbolsOf } S)^*}$.

Let us consider S. Note that every element of LettersOf S is literal. One can check that TheNorSymbOf S is non low-compounding.

Observe that TheNorSymbOf S is non own.

Next we state the proposition

(15) If $s \neq$ TheNorSymbOf S and $s \neq$ TheEqSymbOf S, then $s \in$ OwnSymbolsOf S.

For simplicity, we use the following convention: l, l_1, l_2 denote literal elements of S, a denotes an of-atomic-formula element of S, r denotes a relational element of S, w, w_1 denote strings of S, and t_2 denotes an element of AllTermsOf S.

Let us consider S, t. The functor Depth t yielding a natural number is defined by:

(Def. 40) t is Depth t-termal and for every n such that t is n-termal holds Depth $t \leq n$.

Let us consider S, let m_0 be a zero number, and let t be an m_0-termal string of S. Note that Depth t is zero.

Let us consider S and let s be a low-compounding element of S. Note that ar s is non zero.

Let us consider S and let s be a termal element of S. Observe that ar s is non negative and extended real.

Let us consider S and let s be a relational element of S. Note that ar s is negative and extended real.

Next we state the proposition

(16) If t is non 0-termal, then S-firstChar(t) is operational and SubTerms $t \neq \emptyset$.

Let us consider S. Observe that S-multiCat is finite sequence-yielding.

Let us consider S and let W be a non empty AllSymbolsOf $S^* \setminus \{\emptyset\}$-valued finite sequence. One can verify that S-multiCat(W) is non empty.

Let us consider S, l. Note that $\langle l \rangle$ is 0-termal.

Let us consider S, m, n. One can check that every string of S which is $m + 0 \cdot n$-termal is also $m + n$-termal.

Let us consider S. One can check that every own element of S which is non low-compounding is also literal.

Let us consider S, t. One can check that SubTerms t is rng t^*-valued.

Let p_0 be a 0-w.f.f. string of S. Observe that SubTerms p_0 is rng ${p_0}^*$-valued. Then S-termsOfMaxDepth is a function from $\mathbb{N}$ into $2^{(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}}$.

Let us consider S, m_1. Observe that S-termsOfMaxDepth(m_1) has non empty elements.

Let us consider S, m and let t be a termal string of S. One can verify that t null m is Depth $t + m$-termal. One can check that every string of S which is termal is also TermSymbolsOf S-valued. Observe that AllTermsOf $S \setminus$ (TermSymbolsOf $S)^*$ is empty.

Let p_0 be a 0-w.f.f. string of S. Observe that SubTerms p_0 is TermSymbolsOf S^*-valued. One can verify that every string of S which is 0-w.f.f. is also

AtomicFormulaSymbolsOf S-valued. One can check that OwnSymbolsOf S is non empty.

In the sequel p_0 is a 0-w.f.f. string of S.

The following proposition is true

(17) If S-firstChar$(p_0) \neq$ TheEqSymbOf S, then p_0 is OwnSymbolsOf S-valued.

Let us observe that there exists a language-like which is strict and non empty.

Let S_1, S_2 be languages-like. We say that S_2 is S_1-extending if and only if:

(Def. 41) The adicity of $S_1 \subseteq$ the adicity of S_2 and TheEqSymbOf S_1 = TheEqSymbOf S_2 and TheNorSymbOf S_1 = TheNorSymbOf S_2.

Let us consider S. One can verify that S null is S-extending. Observe that there exists a language which is S-extending.

Let us consider S_1 and let S_2 be an S_1-extending language. Observe that OwnSymbolsOf $S_1 \setminus$ OwnSymbolsOf S_2 is empty.

Let f be a $\mathbb{Z}$-valued function and let L be a non empty language-like. The functor L extendWith f yields a strict non empty language-like and is defined by the conditions (Def. 42).

(Def. 42)(i) The adicity of L extendWith $f = f{\restriction}(\mathrm{dom}\, f \setminus \{$the one of $L\})+\cdot$the adicity of L,

(ii) the zero of L extendWith f = the zero of L, and

(iii) the one of L extendWith f = the one of L.

Let S be a non empty language-like and let f be a $\mathbb{Z}$-valued function. Note that S extendWith f is S-extending.

Let S be a non degenerated language-like. Observe that every language-like which is S-extending is also non degenerated.

Let S be an eligible language-like. One can check that every language-like which is S-extending is also eligible.

Let E be an empty binary relation and let us consider X. Note that $X{\restriction}E$ is empty.

Let us consider X and let m be an integer number. Note that $X \longmapsto m$ is $\mathbb{Z}$-valued.

Let us consider S and let X be a functional set.

The functor S addLettersNotIn X yields an S-extending language and is defined as follows:

(Def. 43) S addLettersNotIn X = S extendWith((AllSymbolsOf S $\cup$ SymbolsOf X)-freeCountableSet $\longmapsto$ 0 **qua** $\mathbb{Z}$-valued function).

Let us consider S_1 and let X be a functional set.

Note that $\mathrm{LettersOf}(S_1 \,\mathrm{addLettersNotIn}\, X) \setminus \mathrm{SymbolsOf}\, X$ is infinite.

Let us note that there exists a language which is countable.

Let S be a countable language. Observe that $\mathrm{AllSymbolsOf}\, S$ is countable. One can verify that $(\mathrm{AllSymbolsOf}\, S)^* \setminus \{\emptyset\}$ is countable.

Let L be a non empty language-like and let f be a $\mathbb{Z}$-valued function. Note that $\mathrm{AllSymbolsOf}(L \,\mathrm{extendWith}\, f) \dot{-} (\mathrm{dom}\, f \cup \mathrm{AllSymbolsOf}\, L)$ is empty.

Let S be a countable language and let X be a functional set. One can check that $S \,\mathrm{addLettersNotIn}\, X$ is countable.

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