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Cayley's Theorem

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Summary. The article formalizes the Cayley's theorem saying that every group G is isomorphic to a subgroup of the symmetric group on G.

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The notation and terminology used in this paper have been introduced in the following papers: [3], [6], [4], [5], [10], [11], [7], [2], [1], [9], and [8].

In this paper X, Y denote sets, G denotes a group, and n denotes a natural number.

Let us consider X. Note that $\emptyset_{X,\emptyset}$ is onto.

Let us observe that every set which is permutational is also functional.

Let us consider X. The functor permutations X is defined as follows:

(Def. 1) permutations $X = \{f : f$ ranges over permutations of $X\}$.

Next we state three propositions:

(1) For every set f such that $f \in$ permutations X holds f is a permutation of X.

(2) permutations $X \subseteq X^X$.

(3) permutations $\mathrm{Seg}\, n =$ the permutations of n.

Let us consider X. One can verify that permutations X is non empty and functional.

Let X be a finite set. One can verify that permutations X is finite.

Next we state the proposition

(4) permutations $\emptyset = 1$.

Let us consider X. The functor SymGroup X yields a strict constituted functions multiplicative magma and is defined by:

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(Def. 2) The carrier of SymGroup X = permutations X and for all elements x, y of SymGroup X holds $x \cdot y = (y$ **qua** function$)\ \cdot x$.

One can prove the following proposition

(5) Every element of SymGroup X is a permutation of X.

Let us consider X. Note that SymGroup X is non empty, associative, and group-like.

The following propositions are true:

(6) $\mathbf{1}_{\mathrm{SymGroup}\,X} = \mathrm{id}_X$.

(7) For every element x of SymGroup X holds $x^{-1} = (x$ **qua** function$)\ ^{-1}$.

Let us consider n. One can verify that A_n is constituted functions.

One can prove the following proposition

(8) SymGroup Seg $n = A_n$.

Let X be a finite set. Observe that SymGroup X is finite.

We now state the proposition

(9) SymGroup $\emptyset$ = Trivial-multMagma.

Let us note that SymGroup $\emptyset$ is trivial.

Let us consider X, Y and let p be a function from X into Y. Let us assume that $X \neq \emptyset$ and $Y \neq \emptyset$ and p is bijective. The functor SymGroupsIso p yielding a function from SymGroup X into SymGroup Y is defined by:

(Def. 3) For every element x of SymGroup X holds (SymGroupsIso p)$(x) = p \cdot x \cdot p^{-1}$.

We now state four propositions:

(10) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is multiplicative.

(11) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is one-to-one.

(12) For all non empty sets X, Y and for every function p from X into Y such that p is bijective holds SymGroupsIso p is onto.

(13) If $X \approx Y$, then SymGroup X and SymGroup Y are isomorphic.

Let us consider G. The functor CayleyIso G yields a function from G into SymGroup (the carrier of G) and is defined as follows:

(Def. 4) For every element g of G holds (CayleyIso G)$(g) = \cdot g$.

Let us consider G. One can verify that CayleyIso G is multiplicative.

Let us consider G. One can verify that CayleyIso G is one-to-one.

One can prove the following proposition

(14) G and Im CayleyIso G are isomorphic.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[2] Grzegorz Bancerek. Monoids. *Formalized Mathematics*, 3(**2**):213–225, 1992.
[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[5] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[6] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[7] Katarzyna Jankowska. Transpose matrices and groups of permutations. *Formalized Mathematics*, 2(**5**):711–717, 1991.
[8] Artur Korniłowicz. The definition and basic properties of topological groups. *Formalized Mathematics*, 7(**2**):217–225, 1998.
[9] Andrzej Trybulec. Classes of independent partitions. *Formalized Mathematics*, 9(**3**):623–625, 2001.
[10] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.
[11] Wojciech A. Trybulec and Michał J. Trybulec. Homomorphisms and isomorphisms of groups. Quotient group. *Formalized Mathematics*, 2(**4**):573–578, 1991.

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