

Formalization of the Data Encryption Standard¹

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Summary. In this article we formalize DES (the Data Encryption Standard), that was the most widely used symmetric cryptosystem in the world. DES is a block cipher which was selected by the National Bureau of Standards as an official Federal Information Processing Standard for the United States in 1976 [15].

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The papers [14], [5], [12], [1], [16], [4], [6], [18], [11], [7], [8], [17], [20], [2], [3], [9], [21], [22], [13], [19], and [10] provide the terminology and notation for this paper.

1. PRELIMINARIES

Let n be a natural number and let f be an n -element finite sequence. Note that $\text{Rev}(f)$ is n -element.

Let D be a non empty set, let n be a natural number, and let f be an element of D^n . Then $\text{Rev}(f)$ is an element of D^n .

Let n be a natural number and let f be a finite sequence. We introduce $\text{Op-Left}(f, n)$ as a synonym of $f \upharpoonright n$. We introduce $\text{Op-Right}(f, n)$ as a synonym of $f \downharpoonright n$.

Let D be a non empty set, let n be a natural number, and let f be a finite sequence of elements of D . Then $\text{Op-Left}(f, n)$ is a finite sequence of elements of D . Then $\text{Op-Right}(f, n)$ is a finite sequence of elements of D .

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Let D be a non empty set, let n be a natural number, and let s be an element of D^{2^n} . We introduce SP-Left s as a synonym of Op-Left(s, n). We introduce SP-Right s as a synonym of Op-Right(s, n).

Let D be a non empty set, let n be a natural number, and let s be an element of D^{2^n} . Then SP-Left s is an element of D^n .

One can prove the following propositions:

- (1) For all non empty elements m, n of \mathbb{N} and for every element s of D^n such that $m \leq n$ holds Op-Left(s, m) is an element of D^m .
- (2) Let m, n, l be non empty elements of \mathbb{N} and s be an element of D^n . If $m \leq n$ and $l = n - m$, then Op-Right(s, m) is an element of D^l .

Let D be a non empty set, let n be a non empty element of \mathbb{N} , and let s be an element of D^{2^n} . Then SP-Right s is an element of D^n .

Next we state the proposition

- (3) For every non empty element n of \mathbb{N} and for every element s of D^{2^n} holds (SP-Left s) \wedge SP-Right $s = s$.

Let s be a finite sequence. The functor Op-LeftShift s yielding a finite sequence is defined by:

(Def. 1) Op-LeftShift $s = (s_{|1}) \wedge \langle s(1) \rangle$.

Next we state three propositions:

- (4) For every finite sequence s such that $1 \leq \text{len } s$ holds $\text{len Op-LeftShift } s = \text{len } s$.
- (5) If $1 \leq \text{len } s$, then Op-LeftShift s is a finite sequence of elements of D and $\text{len Op-LeftShift } s = \text{len } s$.
- (6) For every non empty element n of \mathbb{N} and for every element s of D^n holds Op-LeftShift s is an element of D^n .

Let s be a finite sequence. The functor Op-RightShift s yields a finite sequence and is defined by:

(Def. 2) Op-RightShift $s = (\langle s(\text{len } s) \rangle \wedge s) \upharpoonright \text{len } s$.

One can prove the following three propositions:

- (7) For every finite sequence s holds $\text{len Op-RightShift } s = \text{len } s$.
- (8) If $1 \leq \text{len } s$, then Op-RightShift s is a finite sequence of elements of D and $\text{len Op-RightShift } s = \text{len } s$.
- (9) For every non empty element n of \mathbb{N} and for every element s of D^n holds Op-RightShift s is an element of D^n .

Let D be a non empty set, let s be a finite sequence of elements of D , and let n be an integer. Let us assume that $1 \leq \text{len } s$. The functor Op-Shift(s, n) yields a finite sequence of elements of D and is defined by:

(Def. 3) $\text{len Op-Shift}(s, n) = \text{len } s$ and for every natural number i such that $i \in \text{Seg len } s$ holds $(\text{Op-Shift}(s, n))(i) = s(((i - 1) + n) \bmod \text{len } s + 1)$.

The following propositions are true:

- (10) For all integers n, m such that $1 \leq \text{len } s$ holds $\text{Op-Shift}(\text{Op-Shift}(s, n), m) = \text{Op-Shift}(s, n + m)$.
- (11) If $1 \leq \text{len } s$, then $\text{Op-Shift}(s, 0) = s$.
- (12) If $1 \leq \text{len } s$, then $\text{Op-Shift}(s, \text{len } s) = s$.
- (13) If $1 \leq \text{len } s$, then $\text{Op-Shift}(s, -\text{len } s) = s$.
- (14) Let n be a non empty element of \mathbb{N} , m be an integer, and s be an element of D^n . Then $\text{Op-Shift}(s, m)$ is an element of D^n .
- (15) If $1 \leq \text{len } s$, then $\text{Op-Shift}(s, -1) = \text{Op-RightShift } s$.
- (16) If $1 \leq \text{len } s$, then $\text{Op-Shift}(s, 1) = \text{Op-LeftShift } s$.

Let x, y be elements of Boolean^{28} . Then $x \wedge y$ is an element of Boolean^{56} .

Let n be a non empty element of \mathbb{N} , let s be an element of Boolean^n , and let i be a natural number. Then $s(i)$ is an element of Boolean .

Let n be a non empty element of \mathbb{N} , let s be an element of \mathbb{N}^n , and let i be a natural number. Then $s(i)$ is an element of \mathbb{N} .

Let n be a natural number. Observe that every element of Boolean^n is boolean-valued.

Let n be an element of \mathbb{N} and let s, t be elements of Boolean^n . We introduce $\text{Op-XOR}(s, t)$ as a synonym of $s \oplus t$.

Let n be a non empty element of \mathbb{N} and let s, t be elements of Boolean^n . Then $\text{Op-XOR}(s, t)$ is an element of Boolean^n and it can be characterized by the condition:

- (Def. 4) For every natural number i such that $i \in \text{Seg } n$ holds $(\text{Op-XOR}(s, t))(i) = s(i) \oplus t(i)$.

Let us notice that the functor $\text{Op-XOR}(s, t)$ is commutative.

Let n, k be non empty elements of \mathbb{N} , let R_1 be an element of $(\text{Boolean}^n)^k$, and let i be an element of $\text{Seg } k$. Then $R_1(i)$ is an element of Boolean^n .

We now state the proposition

- (17) For every non empty element n of \mathbb{N} and for all elements s, t of Boolean^n holds $\text{Op-XOR}(\text{Op-XOR}(s, t), t) = s$.

Let m be a non empty element of \mathbb{N} , let D be a non empty set, let L be a sequence of D^m , and let i be a natural number. Then $L(i)$ is an element of D^m .

Let f be a function from 64 into 16 and let i be a set. Then $f(i)$ is an element of 16.

Next we state the proposition

- (18) For all natural numbers n, m such that $n + m \leq \text{len } s$ holds $(s \upharpoonright n) \wedge (s \upharpoonright_n \upharpoonright m) = s \upharpoonright (n + m)$.

The scheme *QuadChoiceRec* deals with non empty sets $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, an element \mathcal{E} of \mathcal{A} , an element \mathcal{F} of \mathcal{B} , an element \mathcal{G} of \mathcal{C} , an element \mathcal{H} of \mathcal{D} , and a 9-ary predicate \mathcal{P} , and states that:

There exists a function f from \mathbb{N} into \mathcal{A} and there exists a function g from \mathbb{N} into \mathcal{B} and there exists a function h from \mathbb{N} into \mathcal{C} and there exists a function i from \mathbb{N} into \mathcal{D} such that $f(0) = \mathcal{E}$ and $g(0) = \mathcal{F}$ and $h(0) = \mathcal{G}$ and $i(0) = \mathcal{H}$ and for every element n of \mathbb{N} holds $\mathcal{P}[n, f(n), g(n), h(n), i(n), f(n+1), g(n+1), h(n+1), i(n+1))]$ provided the following condition is satisfied:

- Let n be an element of \mathbb{N} , x be an element of \mathcal{A} , y be an element of \mathcal{B} , z be an element of \mathcal{C} , and w be an element of \mathcal{D} . Then there exists an element x_1 of \mathcal{A} and there exists an element y_1 of \mathcal{B} and there exists an element z_1 of \mathcal{C} and there exists an element w_1 of \mathcal{D} such that $\mathcal{P}[n, x, y, z, w, x_1, y_1, z_1, w_1]$.

Next we state a number of propositions:

- (19) Let x be a set. Suppose $x \in \text{Seg } 16$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$.
- (20) Let x be a set. Suppose $x \in \text{Seg } 32$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$.
- (21) Let x be a set. Suppose $x \in \text{Seg } 48$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$.
- (22) Let x be a set. Suppose $x \in \text{Seg } 56$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or $x = 54$ or $x = 55$ or $x = 56$.
- (23) Let x be a set. Suppose $x \in \text{Seg } 64$. Then $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or

$x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or
 $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or
 $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or
 $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or
 $x = 54$ or $x = 55$ or $x = 56$ or $x = 57$ or $x = 58$ or $x = 59$ or $x = 60$ or
 $x = 61$ or $x = 62$ or $x = 63$ or $x = 64$.

- (24) For every non empty natural number n holds $n = \{0\} \cup (\text{Seg } n \setminus \{n\})$.
- (25) For every non empty natural number n and for every set x such that $x \in n$ holds $x = 0$ or $x \in \text{Seg } n$ and $x \neq n$.
- (26) Let x be a set. Suppose $x \in 16$. Then $x = 0$ or $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$.
- (27) Let x be a set. Suppose $x \in 64$. Then $x = 0$ or $x = 1$ or $x = 2$ or $x = 3$ or $x = 4$ or $x = 5$ or $x = 6$ or $x = 7$ or $x = 8$ or $x = 9$ or $x = 10$ or $x = 11$ or $x = 12$ or $x = 13$ or $x = 14$ or $x = 15$ or $x = 16$ or $x = 17$ or $x = 18$ or $x = 19$ or $x = 20$ or $x = 21$ or $x = 22$ or $x = 23$ or $x = 24$ or $x = 25$ or $x = 26$ or $x = 27$ or $x = 28$ or $x = 29$ or $x = 30$ or $x = 31$ or $x = 32$ or $x = 33$ or $x = 34$ or $x = 35$ or $x = 36$ or $x = 37$ or $x = 38$ or $x = 39$ or $x = 40$ or $x = 41$ or $x = 42$ or $x = 43$ or $x = 44$ or $x = 45$ or $x = 46$ or $x = 47$ or $x = 48$ or $x = 49$ or $x = 50$ or $x = 51$ or $x = 52$ or $x = 53$ or $x = 54$ or $x = 55$ or $x = 56$ or $x = 57$ or $x = 58$ or $x = 59$ or $x = 60$ or $x = 61$ or $x = 62$ or $x = 63$.
- (28) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ be elements of S . Then there exists a finite sequence s of elements of S such that s is 8-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$.
- (29) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}$ be elements of S . Then there exists a finite sequence s of elements of S such that s is 16-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$.
- (30) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}$ be elements of S . Then there exists a finite sequence s of elements of S such that s is 32-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$

and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$
 and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$
 and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and
 $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$.

- (31) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}$ be elements of S . Then there exists a finite sequence s of elements of S such that

s is 48-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$
 and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$
 and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$
 and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$
 and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$
 and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$
 and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$
 and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$
 and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$
 and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$
 and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$ and
 $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$.

- (32) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}$ be elements of S . Then there exists a finite sequence s of elements of S such that

s is 56-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$
 and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$
 and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$
 and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$
 and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$
 and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$
 and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$
 and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$
 and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$
 and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$
 and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$
 and $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$ and $s(49) = x_{49}$
 and $s(50) = x_{50}$ and $s(51) = x_{51}$ and $s(52) = x_{52}$ and $s(53) = x_{53}$ and
 $s(54) = x_{54}$ and $s(55) = x_{55}$ and $s(56) = x_{56}$.

- (33) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11},$

$x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}, x_{57}, x_{58}, x_{59}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64}$ be elements of S . Then there exists a finite sequence s of elements of S such that

s is 64-element and $s(1) = x_1$ and $s(2) = x_2$ and $s(3) = x_3$ and $s(4) = x_4$ and $s(5) = x_5$ and $s(6) = x_6$ and $s(7) = x_7$ and $s(8) = x_8$ and $s(9) = x_9$ and $s(10) = x_{10}$ and $s(11) = x_{11}$ and $s(12) = x_{12}$ and $s(13) = x_{13}$ and $s(14) = x_{14}$ and $s(15) = x_{15}$ and $s(16) = x_{16}$ and $s(17) = x_{17}$ and $s(18) = x_{18}$ and $s(19) = x_{19}$ and $s(20) = x_{20}$ and $s(21) = x_{21}$ and $s(22) = x_{22}$ and $s(23) = x_{23}$ and $s(24) = x_{24}$ and $s(25) = x_{25}$ and $s(26) = x_{26}$ and $s(27) = x_{27}$ and $s(28) = x_{28}$ and $s(29) = x_{29}$ and $s(30) = x_{30}$ and $s(31) = x_{31}$ and $s(32) = x_{32}$ and $s(33) = x_{33}$ and $s(34) = x_{34}$ and $s(35) = x_{35}$ and $s(36) = x_{36}$ and $s(37) = x_{37}$ and $s(38) = x_{38}$ and $s(39) = x_{39}$ and $s(40) = x_{40}$ and $s(41) = x_{41}$ and $s(42) = x_{42}$ and $s(43) = x_{43}$ and $s(44) = x_{44}$ and $s(45) = x_{45}$ and $s(46) = x_{46}$ and $s(47) = x_{47}$ and $s(48) = x_{48}$ and $s(49) = x_{49}$ and $s(50) = x_{50}$ and $s(51) = x_{51}$ and $s(52) = x_{52}$ and $s(53) = x_{53}$ and $s(54) = x_{54}$ and $s(55) = x_{55}$ and $s(56) = x_{56}$ and $s(57) = x_{57}$ and $s(58) = x_{58}$ and $s(59) = x_{59}$ and $s(60) = x_{60}$ and $s(61) = x_{61}$ and $s(62) = x_{62}$ and $s(63) = x_{63}$ and $s(64) = x_{64}$.

Let n be a non empty natural number and let i be an element of n . We introduce $\text{ntoSeg } i$ as a synonym of $\text{succ } i$.

Let n be a non empty natural number and let i be an element of n . Then $\text{ntoSeg } i$ is an element of $\text{Seg } n$.

Let n be a non empty natural number and let f be a function from n into $\text{Seg } n$. We say that f is NtoSeg if and only if:

(Def. 5) For every element i of n holds $f(i) = \text{ntoSeg } i$.

Let n be a non empty natural number. One can check that there exists a function from n into $\text{Seg } n$ which is NtoSeg .

Let n be a non empty natural number. Observe that every function from n into $\text{Seg } n$ is bijective and NtoSeg .

We now state two propositions:

(34) Let n be a non empty natural number, f be an NtoSeg function from n into $\text{Seg } n$, and i be a natural number. If $i < n$, then $f(i) = i + 1$ and $i \in \text{dom } f$.

(35) Let S be a non empty set and $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}, x_{57}, x_{58}, x_{59}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64}$ be elements of S . Then there exists a function f

from 64 into S such that

$f(0) = x_1$ and $f(1) = x_2$ and $f(2) = x_3$ and $f(3) = x_4$ and $f(4) = x_5$ and
 $f(5) = x_6$ and $f(6) = x_7$ and $f(7) = x_8$ and $f(8) = x_9$ and $f(9) = x_{10}$
and $f(10) = x_{11}$ and $f(11) = x_{12}$ and $f(12) = x_{13}$ and $f(13) = x_{14}$
and $f(14) = x_{15}$ and $f(15) = x_{16}$ and $f(16) = x_{17}$ and $f(17) = x_{18}$
and $f(18) = x_{19}$ and $f(19) = x_{20}$ and $f(20) = x_{21}$ and $f(21) = x_{22}$
and $f(22) = x_{23}$ and $f(23) = x_{24}$ and $f(24) = x_{25}$ and $f(25) = x_{26}$
and $f(26) = x_{27}$ and $f(27) = x_{28}$ and $f(28) = x_{29}$ and $f(29) = x_{30}$
and $f(30) = x_{31}$ and $f(31) = x_{32}$ and $f(32) = x_{33}$ and $f(33) = x_{34}$
and $f(34) = x_{35}$ and $f(35) = x_{36}$ and $f(36) = x_{37}$ and $f(37) = x_{38}$
and $f(38) = x_{39}$ and $f(39) = x_{40}$ and $f(40) = x_{41}$ and $f(41) = x_{42}$
and $f(42) = x_{43}$ and $f(43) = x_{44}$ and $f(44) = x_{45}$ and $f(45) = x_{46}$
and $f(46) = x_{47}$ and $f(47) = x_{48}$ and $f(48) = x_{49}$ and $f(49) = x_{50}$
and $f(50) = x_{51}$ and $f(51) = x_{52}$ and $f(52) = x_{53}$ and $f(53) = x_{54}$
and $f(54) = x_{55}$ and $f(55) = x_{56}$ and $f(56) = x_{57}$ and $f(57) = x_{58}$
and $f(58) = x_{59}$ and $f(59) = x_{60}$ and $f(60) = x_{61}$ and $f(61) = x_{62}$ and
 $f(62) = x_{63}$ and $f(63) = x_{64}$.

2. S-BOXES

The function DES-SBOX1 from 64 into 16 is defined by the conditions (Def. 6).

(Def. 6) (DES-SBOX1)(0) = 14 and (DES-SBOX1)(1) = 4 and (DES-SBOX1)(2) = 13 and (DES-SBOX1)(3) = 1 and (DES-SBOX1)(4) = 2 and (DES-SBOX1)(5) = 15 and (DES-SBOX1)(6) = 11 and (DES-SBOX1)(7) = 8 and (DES-SBOX1)(8) = 3 and (DES-SBOX1)(9) = 10 and (DES-SBOX1)(10) = 6 and (DES-SBOX1)(11) = 12 and (DES-SBOX1)(12) = 5 and (DES-SBOX1)(13) = 9 and (DES-SBOX1)(14) = 0 and (DES-SBOX1)(15) = 7 and (DES-SBOX1)(16) = 0 and (DES-SBOX1)(17) = 15 and (DES-SBOX1)(18) = 7 and (DES-SBOX1)(19) = 4 and (DES-SBOX1)(20) = 14 and (DES-SBOX1)(21) = 2 and (DES-SBOX1)(22) = 13 and (DES-SBOX1)(23) = 1 and (DES-SBOX1)(24) = 10 and (DES-SBOX1)(25) = 6 and (DES-SBOX1)(26) = 12 and (DES-SBOX1)(27) = 11 and (DES-SBOX1)(28) = 9 and (DES-SBOX1)(29) = 5 and (DES-SBOX1)(30) = 3 and (DES-SBOX1)(31) = 8 and (DES-SBOX1)(32) = 4 and (DES-SBOX1)(33) = 1 and (DES-SBOX1)(34) = 14 and (DES-SBOX1)(35) = 8 and (DES-SBOX1)(36) = 13 and (DES-SBOX1)(37) = 6 and (DES-SBOX1)(38) = 2 and (DES-SBOX1)(39) = 11 and (DES-SBOX1)(40) = 15 and (DES-SBOX1)(41) = 12 and (DES-SBOX1)(42) = 9 and (DES-SBOX1)(43) = 7 and

(DES-SBOX1)(44) = 3 and (DES-SBOX1)(45) = 10 and (DES-SBOX1)(46) = 5 and (DES-SBOX1)(47) = 0 and (DES-SBOX1)(48) = 15 and (DES-SBOX1)(49) = 12 and (DES-SBOX1)(50) = 8 and (DES-SBOX1)(51) = 2 and (DES-SBOX1)(52) = 4 and (DES-SBOX1)(53) = 9 and (DES-SBOX1)(54) = 1 and (DES-SBOX1)(55) = 7 and (DES-SBOX1)(56) = 5 and (DES-SBOX1)(57) = 11 and (DES-SBOX1)(58) = 3 and (DES-SBOX1)(59) = 14 and (DES-SBOX1)(60) = 10 and (DES-SBOX1)(61) = 0 and (DES-SBOX1)(62) = 6 and (DES-SBOX1)(63) = 13.

The function DES-SBOX2 from 64 into 16 is defined by the conditions (Def. 7).

(Def. 7) (DES-SBOX2)(0) = 15 and (DES-SBOX2)(1) = 1 and (DES-SBOX2)(2) = 8 and (DES-SBOX2)(3) = 14 and (DES-SBOX2)(4) = 6 and (DES-SBOX2)(5) = 11 and (DES-SBOX2)(6) = 3 and (DES-SBOX2)(7) = 4 and (DES-SBOX2)(8) = 9 and (DES-SBOX2)(9) = 7 and (DES-SBOX2)(10) = 2 and (DES-SBOX2)(11) = 13 and (DES-SBOX2)(12) = 12 and (DES-SBOX2)(13) = 0 and (DES-SBOX2)(14) = 5 and (DES-SBOX2)(15) = 10 and (DES-SBOX2)(16) = 3 and (DES-SBOX2)(17) = 13 and (DES-SBOX2)(18) = 4 and (DES-SBOX2)(19) = 7 and (DES-SBOX2)(20) = 15 and (DES-SBOX2)(21) = 2 and (DES-SBOX2)(22) = 8 and (DES-SBOX2)(23) = 14 and (DES-SBOX2)(24) = 12 and (DES-SBOX2)(25) = 0 and (DES-SBOX2)(26) = 1 and (DES-SBOX2)(27) = 10 and (DES-SBOX2)(28) = 6 and (DES-SBOX2)(29) = 9 and (DES-SBOX2)(30) = 11 and (DES-SBOX2)(31) = 5 and (DES-SBOX2)(32) = 0 and (DES-SBOX2)(33) = 14 and (DES-SBOX2)(34) = 7 and (DES-SBOX2)(35) = 11 and (DES-SBOX2)(36) = 10 and (DES-SBOX2)(37) = 4 and (DES-SBOX2)(38) = 13 and (DES-SBOX2)(39) = 1 and (DES-SBOX2)(40) = 5 and (DES-SBOX2)(41) = 8 and (DES-SBOX2)(42) = 12 and (DES-SBOX2)(43) = 6 and (DES-SBOX2)(44) = 9 and (DES-SBOX2)(45) = 3 and (DES-SBOX2)(46) = 2 and (DES-SBOX2)(47) = 15 and (DES-SBOX2)(48) = 13 and (DES-SBOX2)(49) = 8 and (DES-SBOX2)(50) = 10 and (DES-SBOX2)(51) = 1 and (DES-SBOX2)(52) = 3 and (DES-SBOX2)(53) = 15 and (DES-SBOX2)(54) = 4 and (DES-SBOX2)(55) = 2 and (DES-SBOX2)(56) = 11 and (DES-SBOX2)(57) = 6 and (DES-SBOX2)(58) = 7 and (DES-SBOX2)(59) = 12 and (DES-SBOX2)(60) = 0 and (DES-SBOX2)(61) = 5 and (DES-SBOX2)(62) = 14 and (DES-SBOX2)(63) = 9.

The function DES-SBOX3 from 64 into 16 is defined by the conditions (Def. 8).

(Def. 8) $(\text{DES-SBOX3})(0) = 10$ and $(\text{DES-SBOX3})(1) = 0$ and $(\text{DES-SBOX3})(2) = 9$ and $(\text{DES-SBOX3})(3) = 14$ and $(\text{DES-SBOX3})(4) = 6$ and $(\text{DES-SBOX3})(5) = 3$ and $(\text{DES-SBOX3})(6) = 15$ and $(\text{DES-SBOX3})(7) = 5$ and $(\text{DES-SBOX3})(8) = 1$ and $(\text{DES-SBOX3})(9) = 13$ and $(\text{DES-SBOX3})(10) = 12$ and $(\text{DES-SBOX3})(11) = 7$ and $(\text{DES-SBOX3})(12) = 11$ and $(\text{DES-SBOX3})(13) = 4$ and $(\text{DES-SBOX3})(14) = 2$ and $(\text{DES-SBOX3})(15) = 8$ and $(\text{DES-SBOX3})(16) = 13$ and $(\text{DES-SBOX3})(17) = 7$ and $(\text{DES-SBOX3})(18) = 0$ and $(\text{DES-SBOX3})(19) = 9$ and $(\text{DES-SBOX3})(20) = 3$ and $(\text{DES-SBOX3})(21) = 4$ and $(\text{DES-SBOX3})(22) = 6$ and $(\text{DES-SBOX3})(23) = 10$ and $(\text{DES-SBOX3})(24) = 2$ and $(\text{DES-SBOX3})(25) = 8$ and $(\text{DES-SBOX3})(26) = 5$ and $(\text{DES-SBOX3})(27) = 14$ and $(\text{DES-SBOX3})(28) = 12$ and $(\text{DES-SBOX3})(29) = 11$ and $(\text{DES-SBOX3})(30) = 15$ and $(\text{DES-SBOX3})(31) = 1$ and $(\text{DES-SBOX3})(32) = 13$ and $(\text{DES-SBOX3})(33) = 6$ and $(\text{DES-SBOX3})(34) = 4$ and $(\text{DES-SBOX3})(35) = 9$ and $(\text{DES-SBOX3})(36) = 8$ and $(\text{DES-SBOX3})(37) = 15$ and $(\text{DES-SBOX3})(38) = 3$ and $(\text{DES-SBOX3})(39) = 0$ and $(\text{DES-SBOX3})(40) = 11$ and $(\text{DES-SBOX3})(41) = 1$ and $(\text{DES-SBOX3})(42) = 2$ and $(\text{DES-SBOX3})(43) = 12$ and $(\text{DES-SBOX3})(44) = 5$ and $(\text{DES-SBOX3})(45) = 10$ and $(\text{DES-SBOX3})(46) = 14$ and $(\text{DES-SBOX3})(47) = 7$ and $(\text{DES-SBOX3})(48) = 1$ and $(\text{DES-SBOX3})(49) = 10$ and $(\text{DES-SBOX3})(50) = 13$ and $(\text{DES-SBOX3})(51) = 0$ and $(\text{DES-SBOX3})(52) = 6$ and $(\text{DES-SBOX3})(53) = 9$ and $(\text{DES-SBOX3})(54) = 8$ and $(\text{DES-SBOX3})(55) = 7$ and $(\text{DES-SBOX3})(56) = 4$ and $(\text{DES-SBOX3})(57) = 15$ and $(\text{DES-SBOX3})(58) = 14$ and $(\text{DES-SBOX3})(59) = 3$ and $(\text{DES-SBOX3})(60) = 11$ and $(\text{DES-SBOX3})(61) = 5$ and $(\text{DES-SBOX3})(62) = 2$ and $(\text{DES-SBOX3})(63) = 12$.

The function DES-SBOX4 from 64 into 16 is defined by the conditions (Def. 9).

(Def. 9) $(\text{DES-SBOX4})(0) = 7$ and $(\text{DES-SBOX4})(1) = 13$ and $(\text{DES-SBOX4})(2) = 14$ and $(\text{DES-SBOX4})(3) = 3$ and $(\text{DES-SBOX4})(4) = 0$ and $(\text{DES-SBOX4})(5) = 6$ and $(\text{DES-SBOX4})(6) = 9$ and $(\text{DES-SBOX4})(7) = 10$ and $(\text{DES-SBOX4})(8) = 1$ and $(\text{DES-SBOX4})(9) = 2$ and $(\text{DES-SBOX4})(10) = 8$ and $(\text{DES-SBOX4})(11) = 5$ and $(\text{DES-SBOX4})(12) = 11$ and $(\text{DES-SBOX4})(13) = 12$ and $(\text{DES-SBOX4})(14) = 4$ and $(\text{DES-SBOX4})(15) = 15$ and $(\text{DES-SBOX4})(16) = 13$ and $(\text{DES-SBOX4})(17) = 8$ and $(\text{DES-SBOX4})(18) = 11$ and $(\text{DES-SBOX4})(19) = 5$ and $(\text{DES-SBOX4})(20) = 6$ and $(\text{DES-SBOX4})(21) = 15$ and $(\text{DES-SBOX4})(22) = 0$ and $(\text{DES-SBOX4})(23) = 3$ and $(\text{DES-SBOX4})(24) = 4$ and $(\text{DES-SBOX4})(25) = 7$

and $(\text{DES-SBOX4})(26) = 2$ and $(\text{DES-SBOX4})(27) = 12$ and
 $(\text{DES-SBOX4})(28) = 1$ and $(\text{DES-SBOX4})(29) = 10$ and $(\text{DES-SBOX4})(30) =$
 14 and $(\text{DES-SBOX4})(31) = 9$ and $(\text{DES-SBOX4})(32) = 10$
and $(\text{DES-SBOX4})(33) = 6$ and $(\text{DES-SBOX4})(34) = 9$ and
 $(\text{DES-SBOX4})(35) = 0$ and $(\text{DES-SBOX4})(36) = 12$ and $(\text{DES-SBOX4})(37) =$
 11 and $(\text{DES-SBOX4})(38) = 7$ and $(\text{DES-SBOX4})(39) = 13$
and $(\text{DES-SBOX4})(40) = 15$ and $(\text{DES-SBOX4})(41) = 1$ and
 $(\text{DES-SBOX4})(42) = 3$ and $(\text{DES-SBOX4})(43) = 14$ and $(\text{DES-SBOX4})(44) =$
 5 and $(\text{DES-SBOX4})(45) = 2$ and $(\text{DES-SBOX4})(46) = 8$
and $(\text{DES-SBOX4})(47) = 4$ and $(\text{DES-SBOX4})(48) = 3$ and
 $(\text{DES-SBOX4})(49) = 15$ and $(\text{DES-SBOX4})(50) = 0$ and $(\text{DES-SBOX4})(51) =$
 6 and $(\text{DES-SBOX4})(52) = 10$ and $(\text{DES-SBOX4})(53) = 1$
and $(\text{DES-SBOX4})(54) = 13$ and $(\text{DES-SBOX4})(55) = 8$ and
 $(\text{DES-SBOX4})(56) = 9$ and $(\text{DES-SBOX4})(57) = 4$ and $(\text{DES-SBOX4})(58) =$
 5 and $(\text{DES-SBOX4})(59) = 11$ and $(\text{DES-SBOX4})(60) = 12$
and $(\text{DES-SBOX4})(61) = 7$ and $(\text{DES-SBOX4})(62) = 2$ and
 $(\text{DES-SBOX4})(63) = 14$.

The function DES-SBOX5 from 64 into 16 is defined by the conditions
(Def. 10).

(Def. 10) $(\text{DES-SBOX5})(0) = 2$ and $(\text{DES-SBOX5})(1) = 12$ and $(\text{DES-SBOX5})(2) =$
 4 and $(\text{DES-SBOX5})(3) = 1$ and $(\text{DES-SBOX5})(4) = 7$ and
 $(\text{DES-SBOX5})(5) = 10$ and $(\text{DES-SBOX5})(6) = 11$ and $(\text{DES-SBOX5})(7) =$
 6 and $(\text{DES-SBOX5})(8) = 8$ and $(\text{DES-SBOX5})(9) = 5$ and
 $(\text{DES-SBOX5})(10) = 3$ and $(\text{DES-SBOX5})(11) = 15$ and $(\text{DES-SBOX5})(12) =$
 13 and $(\text{DES-SBOX5})(13) = 0$ and $(\text{DES-SBOX5})(14) = 14$
and $(\text{DES-SBOX5})(15) = 9$ and $(\text{DES-SBOX5})(16) = 14$ and
 $(\text{DES-SBOX5})(17) = 11$ and $(\text{DES-SBOX5})(18) = 2$ and $(\text{DES-SBOX5})(19) =$
 12 and $(\text{DES-SBOX5})(20) = 4$ and $(\text{DES-SBOX5})(21) = 7$
and $(\text{DES-SBOX5})(22) = 13$ and $(\text{DES-SBOX5})(23) = 1$ and
 $(\text{DES-SBOX5})(24) = 5$ and $(\text{DES-SBOX5})(25) = 0$ and $(\text{DES-SBOX5})(26) =$
 15 and $(\text{DES-SBOX5})(27) = 10$ and $(\text{DES-SBOX5})(28) = 3$
and $(\text{DES-SBOX5})(29) = 9$ and $(\text{DES-SBOX5})(30) = 8$ and
 $(\text{DES-SBOX5})(31) = 6$ and $(\text{DES-SBOX5})(32) = 4$ and $(\text{DES-SBOX5})(33) =$
 2 and $(\text{DES-SBOX5})(34) = 1$ and $(\text{DES-SBOX5})(35) = 11$
and $(\text{DES-SBOX5})(36) = 10$ and $(\text{DES-SBOX5})(37) = 13$
and $(\text{DES-SBOX5})(38) = 7$ and $(\text{DES-SBOX5})(39) = 8$ and
 $(\text{DES-SBOX5})(40) = 15$ and $(\text{DES-SBOX5})(41) = 9$ and $(\text{DES-SBOX5})(42) =$
 12 and $(\text{DES-SBOX5})(43) = 5$ and $(\text{DES-SBOX5})(44) = 6$
and $(\text{DES-SBOX5})(45) = 3$ and $(\text{DES-SBOX5})(46) = 0$ and
 $(\text{DES-SBOX5})(47) = 14$ and $(\text{DES-SBOX5})(48) = 11$ and
 $(\text{DES-SBOX5})(49) = 8$ and $(\text{DES-SBOX5})(50) = 12$ and $(\text{DES-SBOX5})(51) =$

7 and $(\text{DES-SBOX5})(52) = 1$ and $(\text{DES-SBOX5})(53) = 14$
 and $(\text{DES-SBOX5})(54) = 2$ and $(\text{DES-SBOX5})(55) = 13$ and
 $(\text{DES-SBOX5})(56) = 6$ and $(\text{DES-SBOX5})(57) = 15$ and $(\text{DES-SBOX5})(58) =$
 0 and $(\text{DES-SBOX5})(59) = 9$ and $(\text{DES-SBOX5})(60) = 10$
 and $(\text{DES-SBOX5})(61) = 4$ and $(\text{DES-SBOX5})(62) = 5$ and
 $(\text{DES-SBOX5})(63) = 3$.

The function DES-SBOX6 from 64 into 16 is defined by the conditions
 (Def. 11).

(Def. 11) $(\text{DES-SBOX6})(0) = 12$ and $(\text{DES-SBOX6})(1) = 1$ and $(\text{DES-SBOX6})(2) =$
 10 and $(\text{DES-SBOX6})(3) = 15$ and $(\text{DES-SBOX6})(4) = 9$
 and $(\text{DES-SBOX6})(5) = 2$ and $(\text{DES-SBOX6})(6) = 6$ and
 $(\text{DES-SBOX6})(7) = 8$ and $(\text{DES-SBOX6})(8) = 0$ and $(\text{DES-SBOX6})(9) =$
 13 and $(\text{DES-SBOX6})(10) = 3$ and $(\text{DES-SBOX6})(11) = 4$
 and $(\text{DES-SBOX6})(12) = 14$ and $(\text{DES-SBOX6})(13) = 7$ and
 $(\text{DES-SBOX6})(14) = 5$ and $(\text{DES-SBOX6})(15) = 11$ and $(\text{DES-SBOX6})(16) =$
 10 and $(\text{DES-SBOX6})(17) = 15$ and $(\text{DES-SBOX6})(18) = 4$
 and $(\text{DES-SBOX6})(19) = 2$ and $(\text{DES-SBOX6})(20) = 7$ and
 $(\text{DES-SBOX6})(21) = 12$ and $(\text{DES-SBOX6})(22) = 9$ and $(\text{DES-SBOX6})(23) =$
 5 and $(\text{DES-SBOX6})(24) = 6$ and $(\text{DES-SBOX6})(25) = 1$
 and $(\text{DES-SBOX6})(26) = 13$ and $(\text{DES-SBOX6})(27) = 14$
 and $(\text{DES-SBOX6})(28) = 0$ and $(\text{DES-SBOX6})(29) = 11$ and
 $(\text{DES-SBOX6})(30) = 3$ and $(\text{DES-SBOX6})(31) = 8$ and $(\text{DES-SBOX6})(32) =$
 9 and $(\text{DES-SBOX6})(33) = 14$ and $(\text{DES-SBOX6})(34) = 15$
 and $(\text{DES-SBOX6})(35) = 5$ and $(\text{DES-SBOX6})(36) = 2$ and
 $(\text{DES-SBOX6})(37) = 8$ and $(\text{DES-SBOX6})(38) = 12$ and $(\text{DES-SBOX6})(39) =$
 3 and $(\text{DES-SBOX6})(40) = 7$ and $(\text{DES-SBOX6})(41) = 0$
 and $(\text{DES-SBOX6})(42) = 4$ and $(\text{DES-SBOX6})(43) = 10$ and
 $(\text{DES-SBOX6})(44) = 1$ and $(\text{DES-SBOX6})(45) = 13$ and $(\text{DES-SBOX6})(46) =$
 11 and $(\text{DES-SBOX6})(47) = 6$ and $(\text{DES-SBOX6})(48) = 4$
 and $(\text{DES-SBOX6})(49) = 3$ and $(\text{DES-SBOX6})(50) = 2$ and
 $(\text{DES-SBOX6})(51) = 12$ and $(\text{DES-SBOX6})(52) = 9$ and $(\text{DES-SBOX6})(53) =$
 5 and $(\text{DES-SBOX6})(54) = 15$ and $(\text{DES-SBOX6})(55) = 10$
 and $(\text{DES-SBOX6})(56) = 11$ and $(\text{DES-SBOX6})(57) = 14$
 and $(\text{DES-SBOX6})(58) = 1$ and $(\text{DES-SBOX6})(59) = 7$ and
 $(\text{DES-SBOX6})(60) = 6$ and $(\text{DES-SBOX6})(61) = 0$ and $(\text{DES-SBOX6})(62) =$
 8 and $(\text{DES-SBOX6})(63) = 13$.

The function DES-SBOX7 from 64 into 16 is defined by the conditions
 (Def. 12).

(Def. 12) $(\text{DES-SBOX7})(0) = 4$ and $(\text{DES-SBOX7})(1) = 11$ and $(\text{DES-SBOX7})(2) =$
 2 and $(\text{DES-SBOX7})(3) = 14$ and $(\text{DES-SBOX7})(4) = 15$ and
 $(\text{DES-SBOX7})(5) = 0$ and $(\text{DES-SBOX7})(6) = 8$ and $(\text{DES-SBOX7})(7) =$

13 and (DES-SBOX7)(8) = 3 and (DES-SBOX7)(9) = 12
 and (DES-SBOX7)(10) = 9 and (DES-SBOX7)(11) = 7 and
 (DES-SBOX7)(12) = 5 and (DES-SBOX7)(13) = 10 and (DES-SBOX7)(14) =
 6 and (DES-SBOX7)(15) = 1 and (DES-SBOX7)(16) = 13
 and (DES-SBOX7)(17) = 0 and (DES-SBOX7)(18) = 11 and
 (DES-SBOX7)(19) = 7 and (DES-SBOX7)(20) = 4 and (DES-SBOX7)(21) =
 9 and (DES-SBOX7)(22) = 1 and (DES-SBOX7)(23) = 10
 and (DES-SBOX7)(24) = 14 and (DES-SBOX7)(25) = 3 and
 (DES-SBOX7)(26) = 5 and (DES-SBOX7)(27) = 12 and (DES-SBOX7)(28) =
 2 and (DES-SBOX7)(29) = 15 and (DES-SBOX7)(30) = 8
 and (DES-SBOX7)(31) = 6 and (DES-SBOX7)(32) = 1 and
 (DES-SBOX7)(33) = 4 and (DES-SBOX7)(34) = 11 and (DES-SBOX7)(35) =
 13 and (DES-SBOX7)(36) = 12 and (DES-SBOX7)(37) = 3
 and (DES-SBOX7)(38) = 7 and (DES-SBOX7)(39) = 14 and
 (DES-SBOX7)(40) = 10 and (DES-SBOX7)(41) = 15 and
 (DES-SBOX7)(42) = 6 and (DES-SBOX7)(43) = 8 and (DES-SBOX7)(44) =
 0 and (DES-SBOX7)(45) = 5 and (DES-SBOX7)(46) = 9
 and (DES-SBOX7)(47) = 2 and (DES-SBOX7)(48) = 6 and
 (DES-SBOX7)(49) = 11 and (DES-SBOX7)(50) = 13 and
 (DES-SBOX7)(51) = 8 and (DES-SBOX7)(52) = 1 and (DES-SBOX7)(53) =
 4 and (DES-SBOX7)(54) = 10 and (DES-SBOX7)(55) = 7
 and (DES-SBOX7)(56) = 9 and (DES-SBOX7)(57) = 5 and
 (DES-SBOX7)(58) = 0 and (DES-SBOX7)(59) = 15 and (DES-SBOX7)(60) =
 14 and (DES-SBOX7)(61) = 2 and (DES-SBOX7)(62) = 3 and
 (DES-SBOX7)(63) = 12.

The function DES-SBOX8 from 64 into 16 is defined by the conditions
 (Def. 13).

(Def. 13) (DES-SBOX8)(0) = 13 and (DES-SBOX8)(1) = 2 and (DES-SBOX8)(2) =
 8 and (DES-SBOX8)(3) = 4 and (DES-SBOX8)(4) = 6 and
 (DES-SBOX8)(5) = 15 and (DES-SBOX8)(6) = 11 and (DES-SBOX8)(7) =
 1 and (DES-SBOX8)(8) = 10 and (DES-SBOX8)(9) = 9
 and (DES-SBOX8)(10) = 3 and (DES-SBOX8)(11) = 14 and
 (DES-SBOX8)(12) = 5 and (DES-SBOX8)(13) = 0 and (DES-SBOX8)(14) =
 12 and (DES-SBOX8)(15) = 7 and (DES-SBOX8)(16) = 1
 and (DES-SBOX8)(17) = 15 and (DES-SBOX8)(18) = 13
 and (DES-SBOX8)(19) = 8 and (DES-SBOX8)(20) = 10 and
 (DES-SBOX8)(21) = 3 and (DES-SBOX8)(22) = 7 and (DES-SBOX8)(23) =
 4 and (DES-SBOX8)(24) = 12 and (DES-SBOX8)(25) = 5
 and (DES-SBOX8)(26) = 5 and (DES-SBOX8)(27) = 11 and
 (DES-SBOX8)(28) = 0 and (DES-SBOX8)(29) = 14 and (DES-SBOX8)(30) =
 9 and (DES-SBOX8)(31) = 2 and (DES-SBOX8)(32) = 7

and $(\text{DES-SBOX8})(33) = 11$ and $(\text{DES-SBOX8})(34) = 4$ and
 $(\text{DES-SBOX8})(35) = 1$ and $(\text{DES-SBOX8})(36) = 9$ and $(\text{DES-SBOX8})(37) =$
 12 and $(\text{DES-SBOX8})(38) = 14$ and $(\text{DES-SBOX8})(39) = 2$
and $(\text{DES-SBOX8})(40) = 0$ and $(\text{DES-SBOX8})(41) = 6$ and
 $(\text{DES-SBOX8})(42) = 10$ and $(\text{DES-SBOX8})(43) = 13$ and
 $(\text{DES-SBOX8})(44) = 15$ and $(\text{DES-SBOX8})(45) = 3$ and $(\text{DES-SBOX8})(46) =$
 5 and $(\text{DES-SBOX8})(47) = 8$ and $(\text{DES-SBOX8})(48) = 2$
and $(\text{DES-SBOX8})(49) = 1$ and $(\text{DES-SBOX8})(50) = 14$ and
 $(\text{DES-SBOX8})(51) = 7$ and $(\text{DES-SBOX8})(52) = 4$ and $(\text{DES-SBOX8})(53) =$
 10 and $(\text{DES-SBOX8})(54) = 8$ and $(\text{DES-SBOX8})(55) = 13$
and $(\text{DES-SBOX8})(56) = 15$ and $(\text{DES-SBOX8})(57) = 12$
and $(\text{DES-SBOX8})(58) = 9$ and $(\text{DES-SBOX8})(59) = 0$ and
 $(\text{DES-SBOX8})(60) = 3$ and $(\text{DES-SBOX8})(61) = 5$ and $(\text{DES-SBOX8})(62) =$
 6 and $(\text{DES-SBOX8})(63) = 11$.

3. INITIAL PERMUTATION

Let r be an element of Boolean^{64} . The functor $\text{DES-IP } r$ yields an element of Boolean^{64} and is defined by the conditions (Def. 14).

(Def. 14) $(\text{DES-IP } r)(1) = r(58)$ and $(\text{DES-IP } r)(2) = r(50)$ and $(\text{DES-IP } r)(3) =$
 $r(42)$ and $(\text{DES-IP } r)(4) = r(34)$ and $(\text{DES-IP } r)(5) = r(26)$
and $(\text{DES-IP } r)(6) = r(18)$ and $(\text{DES-IP } r)(7) = r(10)$ and
 $(\text{DES-IP } r)(8) = r(2)$ and $(\text{DES-IP } r)(9) = r(60)$ and $(\text{DES-IP } r)(10) =$
 $r(52)$ and $(\text{DES-IP } r)(11) = r(44)$ and $(\text{DES-IP } r)(12) = r(36)$
and $(\text{DES-IP } r)(13) = r(28)$ and $(\text{DES-IP } r)(14) = r(20)$ and
 $(\text{DES-IP } r)(15) = r(12)$ and $(\text{DES-IP } r)(16) = r(4)$ and $(\text{DES-IP } r)(17) =$
 $r(62)$ and $(\text{DES-IP } r)(18) = r(54)$ and $(\text{DES-IP } r)(19) = r(46)$
and $(\text{DES-IP } r)(20) = r(38)$ and $(\text{DES-IP } r)(21) = r(30)$ and
 $(\text{DES-IP } r)(22) = r(22)$ and $(\text{DES-IP } r)(23) = r(14)$ and $(\text{DES-IP } r)(24) =$
 $r(6)$ and $(\text{DES-IP } r)(25) = r(64)$ and $(\text{DES-IP } r)(26) = r(56)$
and $(\text{DES-IP } r)(27) = r(48)$ and $(\text{DES-IP } r)(28) = r(40)$ and
 $(\text{DES-IP } r)(29) = r(32)$ and $(\text{DES-IP } r)(30) = r(24)$ and $(\text{DES-IP } r)(31) =$
 $r(16)$ and $(\text{DES-IP } r)(32) = r(8)$ and $(\text{DES-IP } r)(33) = r(57)$
and $(\text{DES-IP } r)(34) = r(49)$ and $(\text{DES-IP } r)(35) = r(41)$ and
 $(\text{DES-IP } r)(36) = r(33)$ and $(\text{DES-IP } r)(37) = r(25)$ and $(\text{DES-IP } r)(38) =$
 $r(17)$ and $(\text{DES-IP } r)(39) = r(9)$ and $(\text{DES-IP } r)(40) = r(1)$
and $(\text{DES-IP } r)(41) = r(59)$ and $(\text{DES-IP } r)(42) = r(51)$ and
 $(\text{DES-IP } r)(43) = r(43)$ and $(\text{DES-IP } r)(44) = r(35)$ and $(\text{DES-IP } r)(45) =$
 $r(27)$ and $(\text{DES-IP } r)(46) = r(19)$ and $(\text{DES-IP } r)(47) = r(11)$ and
 $(\text{DES-IP } r)(48) = r(3)$ and $(\text{DES-IP } r)(49) = r(61)$ and $(\text{DES-IP } r)(50) =$
 $r(53)$ and $(\text{DES-IP } r)(51) = r(45)$ and $(\text{DES-IP } r)(52) = r(37)$

and $(\text{DES-IP } r)(53) = r(29)$ and $(\text{DES-IP } r)(54) = r(21)$ and
 $(\text{DES-IP } r)(55) = r(13)$ and $(\text{DES-IP } r)(56) = r(5)$ and $(\text{DES-IP } r)(57) =$
 $r(63)$ and $(\text{DES-IP } r)(58) = r(55)$ and $(\text{DES-IP } r)(59) = r(47)$
and $(\text{DES-IP } r)(60) = r(39)$ and $(\text{DES-IP } r)(61) = r(31)$ and
 $(\text{DES-IP } r)(62) = r(23)$ and $(\text{DES-IP } r)(63) = r(15)$ and $(\text{DES-IP } r)(64) =$
 $r(7)$.

The function DES-PIP from Boolean^{64} into Boolean^{64} is defined by:

(Def. 15) For every element i of Boolean^{64} holds $(\text{DES-PIP})(i) = \text{DES-IP } i$.

Let r be an element of Boolean^{64} . The functor DES-IPINV r yields an element of Boolean^{64} and is defined by the conditions (Def. 16).

(Def. 16) $(\text{DES-IPINV } r)(1) = r(40)$ and $(\text{DES-IPINV } r)(2) = r(8)$ and
 $(\text{DES-IPINV } r)(3) = r(48)$ and $(\text{DES-IPINV } r)(4) = r(16)$ and
 $(\text{DES-IPINV } r)(5) = r(56)$ and $(\text{DES-IPINV } r)(6) = r(24)$ and
 $(\text{DES-IPINV } r)(7) = r(64)$ and $(\text{DES-IPINV } r)(8) = r(32)$ and
 $(\text{DES-IPINV } r)(9) = r(39)$ and $(\text{DES-IPINV } r)(10) = r(7)$ and
 $(\text{DES-IPINV } r)(11) = r(47)$ and $(\text{DES-IPINV } r)(12) = r(15)$ and
 $(\text{DES-IPINV } r)(13) = r(55)$ and $(\text{DES-IPINV } r)(14) = r(23)$ and
 $(\text{DES-IPINV } r)(15) = r(63)$ and $(\text{DES-IPINV } r)(16) = r(31)$ and
 $(\text{DES-IPINV } r)(17) = r(38)$ and $(\text{DES-IPINV } r)(18) = r(6)$ and
 $(\text{DES-IPINV } r)(19) = r(46)$ and $(\text{DES-IPINV } r)(20) = r(14)$ and
 $(\text{DES-IPINV } r)(21) = r(54)$ and $(\text{DES-IPINV } r)(22) = r(22)$ and
 $(\text{DES-IPINV } r)(23) = r(62)$ and $(\text{DES-IPINV } r)(24) = r(30)$ and
 $(\text{DES-IPINV } r)(25) = r(37)$ and $(\text{DES-IPINV } r)(26) = r(5)$ and
 $(\text{DES-IPINV } r)(27) = r(45)$ and $(\text{DES-IPINV } r)(28) = r(13)$ and
 $(\text{DES-IPINV } r)(29) = r(53)$ and $(\text{DES-IPINV } r)(30) = r(21)$ and
 $(\text{DES-IPINV } r)(31) = r(61)$ and $(\text{DES-IPINV } r)(32) = r(29)$ and
 $(\text{DES-IPINV } r)(33) = r(36)$ and $(\text{DES-IPINV } r)(34) = r(4)$ and
 $(\text{DES-IPINV } r)(35) = r(44)$ and $(\text{DES-IPINV } r)(36) = r(12)$ and
 $(\text{DES-IPINV } r)(37) = r(52)$ and $(\text{DES-IPINV } r)(38) = r(20)$ and
 $(\text{DES-IPINV } r)(39) = r(60)$ and $(\text{DES-IPINV } r)(40) = r(28)$ and
 $(\text{DES-IPINV } r)(41) = r(35)$ and $(\text{DES-IPINV } r)(42) = r(3)$ and
 $(\text{DES-IPINV } r)(43) = r(43)$ and $(\text{DES-IPINV } r)(44) = r(11)$ and
 $(\text{DES-IPINV } r)(45) = r(51)$ and $(\text{DES-IPINV } r)(46) = r(19)$ and
 $(\text{DES-IPINV } r)(47) = r(59)$ and $(\text{DES-IPINV } r)(48) = r(27)$ and
 $(\text{DES-IPINV } r)(49) = r(34)$ and $(\text{DES-IPINV } r)(50) = r(2)$ and
 $(\text{DES-IPINV } r)(51) = r(42)$ and $(\text{DES-IPINV } r)(52) = r(10)$ and
 $(\text{DES-IPINV } r)(53) = r(50)$ and $(\text{DES-IPINV } r)(54) = r(18)$ and
 $(\text{DES-IPINV } r)(55) = r(58)$ and $(\text{DES-IPINV } r)(56) = r(26)$ and
 $(\text{DES-IPINV } r)(57) = r(33)$ and $(\text{DES-IPINV } r)(58) = r(1)$ and
 $(\text{DES-IPINV } r)(59) = r(41)$ and $(\text{DES-IPINV } r)(60) = r(9)$ and
 $(\text{DES-IPINV } r)(61) = r(49)$ and $(\text{DES-IPINV } r)(62) = r(17)$ and

$$(\text{DES-IPINV } r)(63) = r(57) \text{ and } (\text{DES-IPINV } r)(64) = r(25).$$

The function DES-PIPINV from Boolean^{64} into Boolean^{64} is defined by:

(Def. 17) For every element i of Boolean^{64} holds $(\text{DES-PIPINV})(i) = \text{DES-IPINV } i$.

Let us note that DES-PIP is bijective.

Let us note that DES-PIPINV is bijective.

The following proposition is true

$$(36) \quad \text{DES-PIPINV} = (\text{DES-PIP})^{-1}.$$

4. FEISTEL FUNCTION

Let r be an element of Boolean^{32} . The functor DES- Er yielding an element of Boolean^{48} is defined by the conditions (Def. 18).

(Def. 18) $(\text{DES-}Er)(1) = r(32)$ and $(\text{DES-}Er)(2) = r(1)$ and $(\text{DES-}Er)(3) = r(2)$ and $(\text{DES-}Er)(4) = r(3)$ and $(\text{DES-}Er)(5) = r(4)$ and $(\text{DES-}Er)(6) = r(5)$ and $(\text{DES-}Er)(7) = r(4)$ and $(\text{DES-}Er)(8) = r(5)$ and $(\text{DES-}Er)(9) = r(6)$ and $(\text{DES-}Er)(10) = r(7)$ and $(\text{DES-}Er)(11) = r(8)$ and $(\text{DES-}Er)(12) = r(9)$ and $(\text{DES-}Er)(13) = r(8)$ and $(\text{DES-}Er)(14) = r(9)$ and $(\text{DES-}Er)(15) = r(10)$ and $(\text{DES-}Er)(16) = r(11)$ and $(\text{DES-}Er)(17) = r(12)$ and $(\text{DES-}Er)(18) = r(13)$ and $(\text{DES-}Er)(19) = r(12)$ and $(\text{DES-}Er)(20) = r(13)$ and $(\text{DES-}Er)(21) = r(14)$ and $(\text{DES-}Er)(22) = r(15)$ and $(\text{DES-}Er)(23) = r(16)$ and $(\text{DES-}Er)(24) = r(17)$ and $(\text{DES-}Er)(25) = r(16)$ and $(\text{DES-}Er)(26) = r(17)$ and $(\text{DES-}Er)(27) = r(18)$ and $(\text{DES-}Er)(28) = r(19)$ and $(\text{DES-}Er)(29) = r(20)$ and $(\text{DES-}Er)(30) = r(21)$ and $(\text{DES-}Er)(31) = r(20)$ and $(\text{DES-}Er)(32) = r(21)$ and $(\text{DES-}Er)(33) = r(22)$ and $(\text{DES-}Er)(34) = r(23)$ and $(\text{DES-}Er)(35) = r(24)$ and $(\text{DES-}Er)(36) = r(25)$ and $(\text{DES-}Er)(37) = r(24)$ and $(\text{DES-}Er)(38) = r(25)$ and $(\text{DES-}Er)(39) = r(26)$ and $(\text{DES-}Er)(40) = r(27)$ and $(\text{DES-}Er)(41) = r(28)$ and $(\text{DES-}Er)(42) = r(29)$ and $(\text{DES-}Er)(43) = r(28)$ and $(\text{DES-}Er)(44) = r(29)$ and $(\text{DES-}Er)(45) = r(30)$ and $(\text{DES-}Er)(46) = r(31)$ and $(\text{DES-}Er)(47) = r(32)$ and $(\text{DES-}Er)(48) = r(1)$.

Let r be an element of Boolean^{32} . The functor DES- Pr yielding an element of Boolean^{32} is defined by the conditions (Def. 19).

(Def. 19) $(\text{DES-}Pr)(1) = r(16)$ and $(\text{DES-}Pr)(2) = r(7)$ and $(\text{DES-}Pr)(3) = r(20)$ and $(\text{DES-}Pr)(4) = r(21)$ and $(\text{DES-}Pr)(5) = r(29)$ and $(\text{DES-}Pr)(6) = r(12)$ and $(\text{DES-}Pr)(7) = r(28)$ and $(\text{DES-}Pr)(8) = r(17)$ and $(\text{DES-}Pr)(9) = r(1)$ and $(\text{DES-}Pr)(10) = r(15)$ and $(\text{DES-}Pr)(11) = r(23)$ and $(\text{DES-}Pr)(12) = r(26)$ and $(\text{DES-}Pr)(13) = r(5)$ and $(\text{DES-}Pr)(14) = r(18)$ and $(\text{DES-}Pr)(15) = r(31)$ and

(DES-Pr)(16) = $r(10)$ and (DES-Pr)(17) = $r(2)$ and (DES-Pr)(18) = $r(8)$ and (DES-Pr)(19) = $r(24)$ and (DES-Pr)(20) = $r(14)$ and (DES-Pr)(21) = $r(32)$ and (DES-Pr)(22) = $r(27)$ and (DES-Pr)(23) = $r(3)$ and (DES-Pr)(24) = $r(9)$ and (DES-Pr)(25) = $r(19)$ and (DES-Pr)(26) = $r(13)$ and (DES-Pr)(27) = $r(30)$ and (DES-Pr)(28) = $r(6)$ and (DES-Pr)(29) = $r(22)$ and (DES-Pr)(30) = $r(11)$ and (DES-Pr)(31) = $r(4)$ and (DES-Pr)(32) = $r(25)$.

Let r be an element of $Boolean^{48}$. The functor DES-DIV8 r yielding an element of $(Boolean^6)^8$ is defined by the conditions (Def. 20).

- (Def. 20) (DES-DIV8 r)(1) = Op-Left(r , 6) and (DES-DIV8 r)(2) = Op-Left(Op-Right(r , 6), 6) and (DES-DIV8 r)(3) = Op-Left(Op-Right(r , 12), 6) and (DES-DIV8 r)(4) = Op-Left(Op-Right(r , 18), 6) and (DES-DIV8 r)(5) = Op-Left(Op-Right(r , 24), 6) and (DES-DIV8 r)(6) = Op-Left(Op-Right(r , 30), 6) and (DES-DIV8 r)(7) = Op-Left(Op-Right(r , 36), 6) and (DES-DIV8 r)(8) = Op-Right(r , 42).

Next we state the proposition

- (37) Let r be an element of $Boolean^{48}$. Then there exist elements $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8$ of $Boolean^6$ such that $s_1 =$ (DES-DIV8 r)(1) and $s_2 =$ (DES-DIV8 r)(2) and $s_3 =$ (DES-DIV8 r)(3) and $s_4 =$ (DES-DIV8 r)(4) and $s_5 =$ (DES-DIV8 r)(5) and $s_6 =$ (DES-DIV8 r)(6) and $s_7 =$ (DES-DIV8 r)(7) and $s_8 =$ (DES-DIV8 r)(8) and $r = s_1 \wedge s_2 \wedge s_3 \wedge s_4 \wedge s_5 \wedge s_6 \wedge s_7 \wedge s_8$.

Let t be an element of $Boolean^6$. The functor B6toN64 t yielding an element of 64 is defined by:

- (Def. 21) B6toN64 $t = 32 \cdot t(1) + 16 \cdot t(6) + 8 \cdot t(2) + 4 \cdot t(3) + 2 \cdot t(4) + 1 \cdot t(5)$.

The function N16toB4 from 16 into $Boolean^4$ is defined by the conditions (Def. 22).

- (Def. 22) (N16toB4)(0) = $\langle 0, 0, 0, 0 \rangle$ and (N16toB4)(1) = $\langle 0, 0, 0, 1 \rangle$ and (N16toB4)(2) = $\langle 0, 0, 1, 0 \rangle$ and (N16toB4)(3) = $\langle 0, 0, 1, 1 \rangle$ and (N16toB4)(4) = $\langle 0, 1, 0, 0 \rangle$ and (N16toB4)(5) = $\langle 0, 1, 0, 1 \rangle$ and (N16toB4)(6) = $\langle 0, 1, 1, 0 \rangle$ and (N16toB4)(7) = $\langle 0, 1, 1, 1 \rangle$ and (N16toB4)(8) = $\langle 1, 0, 0, 0 \rangle$ and (N16toB4)(9) = $\langle 1, 0, 0, 1 \rangle$ and (N16toB4)(10) = $\langle 1, 0, 1, 0 \rangle$ and (N16toB4)(11) = $\langle 1, 0, 1, 1 \rangle$ and (N16toB4)(12) = $\langle 1, 1, 0, 0 \rangle$ and (N16toB4)(13) = $\langle 1, 1, 0, 1 \rangle$ and (N16toB4)(14) = $\langle 1, 1, 1, 0 \rangle$ and (N16toB4)(15) = $\langle 1, 1, 1, 1 \rangle$.

Let R be an element of $Boolean^{32}$ and let R_2 be an element of $Boolean^{48}$. The functor DES-F(R, R_2) yields an element of $Boolean^{32}$ and is defined by the condition (Def. 23).

- (Def. 23) There exist elements $D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8$ of $Boolean^6$ and

there exist elements $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ of $Boolean^4$ and there exists an element C_{32} of $Boolean^{32}$ such that

$$\begin{aligned}
D_1 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(1) \text{ and} \\
D_2 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(2) \text{ and} \\
D_3 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(3) \text{ and} \\
D_4 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(4) \text{ and} \\
D_5 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(5) \text{ and} \\
D_6 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(6) \text{ and} \\
D_7 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(7) \text{ and} \\
D_8 &= (\text{DES-DIV8 Op-XOR}(\text{DES-E } R, R_2))(8) \text{ and} \\
\text{Op-XOR}(\text{DES-E } R, R_2) &= D_1 \wedge D_2 \wedge D_3 \wedge D_4 \wedge D_5 \wedge D_6 \wedge D_7 \wedge D_8 \text{ and} \\
x_1 &= (\text{N16toB4})((\text{DES-SBOX1})(\text{B6toN64 } D_1)) \text{ and} \\
x_2 &= (\text{N16toB4})((\text{DES-SBOX2})(\text{B6toN64 } D_2)) \text{ and} \\
x_3 &= (\text{N16toB4})((\text{DES-SBOX3})(\text{B6toN64 } D_3)) \text{ and} \\
x_4 &= (\text{N16toB4})((\text{DES-SBOX4})(\text{B6toN64 } D_4)) \text{ and} \\
x_5 &= (\text{N16toB4})((\text{DES-SBOX5})(\text{B6toN64 } D_5)) \text{ and} \\
x_6 &= (\text{N16toB4})((\text{DES-SBOX6})(\text{B6toN64 } D_6)) \text{ and} \\
x_7 &= (\text{N16toB4})((\text{DES-SBOX7})(\text{B6toN64 } D_7)) \text{ and} \\
x_8 &= (\text{N16toB4})((\text{DES-SBOX8})(\text{B6toN64 } D_8)) \text{ and} \\
C_{32} &= x_1 \wedge x_2 \wedge x_3 \wedge x_4 \wedge x_5 \wedge x_6 \wedge x_7 \wedge x_8 \text{ and} \\
\text{DES-F}(R, R_2) &= \text{DES-P } C_{32}.
\end{aligned}$$

The function DES-FFUNC from $Boolean^{32} \times Boolean^{48}$ into $Boolean^{32}$ is defined as follows:

(Def. 24) For every element z of $Boolean^{32} \times Boolean^{48}$ holds $(\text{DES-FFUNC})(z) = \text{DES-F}(z_1, z_2)$.

5. KEY SCHEDULE

Let r be an element of $Boolean^{64}$. The functor $\text{DES-PC1 } r$ yields an element of $Boolean^{56}$ and is defined by the conditions (Def. 25).

(Def. 25) $(\text{DES-PC1 } r)(1) = r(57)$ and $(\text{DES-PC1 } r)(2) = r(49)$ and
 $(\text{DES-PC1 } r)(3) = r(41)$ and $(\text{DES-PC1 } r)(4) = r(33)$ and
 $(\text{DES-PC1 } r)(5) = r(25)$ and $(\text{DES-PC1 } r)(6) = r(17)$ and
 $(\text{DES-PC1 } r)(7) = r(9)$ and $(\text{DES-PC1 } r)(8) = r(1)$ and $(\text{DES-PC1 } r)(9) = r(58)$ and
 $(\text{DES-PC1 } r)(10) = r(50)$ and $(\text{DES-PC1 } r)(11) = r(42)$ and
 $(\text{DES-PC1 } r)(12) = r(34)$ and $(\text{DES-PC1 } r)(13) = r(26)$ and
 $(\text{DES-PC1 } r)(14) = r(18)$ and $(\text{DES-PC1 } r)(15) = r(10)$ and
 $(\text{DES-PC1 } r)(16) = r(2)$ and $(\text{DES-PC1 } r)(17) = r(59)$ and
 $(\text{DES-PC1 } r)(18) = r(51)$ and $(\text{DES-PC1 } r)(19) = r(43)$ and
 $(\text{DES-PC1 } r)(20) = r(35)$ and $(\text{DES-PC1 } r)(21) = r(27)$ and
 $(\text{DES-PC1 } r)(22) = r(19)$ and $(\text{DES-PC1 } r)(23) = r(11)$ and
 $(\text{DES-PC1 } r)(24) = r(3)$ and $(\text{DES-PC1 } r)(25) = r(60)$ and

$$\begin{aligned}
 (\text{DES-PC1 } r)(26) &= r(52) \text{ and } (\text{DES-PC1 } r)(27) = r(44) \text{ and} \\
 (\text{DES-PC1 } r)(28) &= r(36) \text{ and } (\text{DES-PC1 } r)(29) = r(63) \text{ and} \\
 (\text{DES-PC1 } r)(30) &= r(55) \text{ and } (\text{DES-PC1 } r)(31) = r(47) \text{ and} \\
 (\text{DES-PC1 } r)(32) &= r(39) \text{ and } (\text{DES-PC1 } r)(33) = r(31) \text{ and} \\
 (\text{DES-PC1 } r)(34) &= r(23) \text{ and } (\text{DES-PC1 } r)(35) = r(15) \text{ and} \\
 (\text{DES-PC1 } r)(36) &= r(7) \text{ and } (\text{DES-PC1 } r)(37) = r(62) \text{ and} \\
 (\text{DES-PC1 } r)(38) &= r(54) \text{ and } (\text{DES-PC1 } r)(39) = r(46) \text{ and} \\
 (\text{DES-PC1 } r)(40) &= r(38) \text{ and } (\text{DES-PC1 } r)(41) = r(30) \text{ and} \\
 (\text{DES-PC1 } r)(42) &= r(22) \text{ and } (\text{DES-PC1 } r)(43) = r(14) \text{ and} \\
 (\text{DES-PC1 } r)(44) &= r(6) \text{ and } (\text{DES-PC1 } r)(45) = r(61) \text{ and} \\
 (\text{DES-PC1 } r)(46) &= r(53) \text{ and } (\text{DES-PC1 } r)(47) = r(45) \text{ and} \\
 (\text{DES-PC1 } r)(48) &= r(37) \text{ and } (\text{DES-PC1 } r)(49) = r(29) \text{ and} \\
 (\text{DES-PC1 } r)(50) &= r(21) \text{ and } (\text{DES-PC1 } r)(51) = r(13) \text{ and} \\
 (\text{DES-PC1 } r)(52) &= r(5) \text{ and } (\text{DES-PC1 } r)(53) = r(28) \text{ and} \\
 (\text{DES-PC1 } r)(54) &= r(20) \text{ and } (\text{DES-PC1 } r)(55) = r(12) \text{ and} \\
 (\text{DES-PC1 } r)(56) &= r(4).
 \end{aligned}$$

Let r be an element of Boolean^{56} . The functor $\text{DES-PC2 } r$ yielding an element of Boolean^{48} is defined by the conditions (Def. 26).

$$\begin{aligned}
 (\text{Def. 26}) \quad (\text{DES-PC2 } r)(1) &= r(14) \text{ and } (\text{DES-PC2 } r)(2) = r(17) \text{ and} \\
 (\text{DES-PC2 } r)(3) &= r(11) \text{ and } (\text{DES-PC2 } r)(4) = r(24) \text{ and} \\
 (\text{DES-PC2 } r)(5) &= r(1) \text{ and } (\text{DES-PC2 } r)(6) = r(5) \text{ and } (\text{DES-PC2 } r)(7) = \\
 &r(3) \text{ and } (\text{DES-PC2 } r)(8) = r(28) \text{ and } (\text{DES-PC2 } r)(9) = r(15) \\
 \text{and } (\text{DES-PC2 } r)(10) &= r(6) \text{ and } (\text{DES-PC2 } r)(11) = r(21) \text{ and} \\
 (\text{DES-PC2 } r)(12) &= r(10) \text{ and } (\text{DES-PC2 } r)(13) = r(23) \text{ and} \\
 (\text{DES-PC2 } r)(14) &= r(19) \text{ and } (\text{DES-PC2 } r)(15) = r(12) \text{ and} \\
 (\text{DES-PC2 } r)(16) &= r(4) \text{ and } (\text{DES-PC2 } r)(17) = r(26) \text{ and} \\
 (\text{DES-PC2 } r)(18) &= r(8) \text{ and } (\text{DES-PC2 } r)(19) = r(16) \text{ and} \\
 (\text{DES-PC2 } r)(20) &= r(7) \text{ and } (\text{DES-PC2 } r)(21) = r(27) \text{ and} \\
 (\text{DES-PC2 } r)(22) &= r(20) \text{ and } (\text{DES-PC2 } r)(23) = r(13) \text{ and} \\
 (\text{DES-PC2 } r)(24) &= r(2) \text{ and } (\text{DES-PC2 } r)(25) = r(41) \text{ and} \\
 (\text{DES-PC2 } r)(26) &= r(52) \text{ and } (\text{DES-PC2 } r)(27) = r(31) \text{ and} \\
 (\text{DES-PC2 } r)(28) &= r(37) \text{ and } (\text{DES-PC2 } r)(29) = r(47) \text{ and} \\
 (\text{DES-PC2 } r)(30) &= r(55) \text{ and } (\text{DES-PC2 } r)(31) = r(30) \text{ and} \\
 (\text{DES-PC2 } r)(32) &= r(40) \text{ and } (\text{DES-PC2 } r)(33) = r(51) \text{ and} \\
 (\text{DES-PC2 } r)(34) &= r(45) \text{ and } (\text{DES-PC2 } r)(35) = r(33) \text{ and} \\
 (\text{DES-PC2 } r)(36) &= r(48) \text{ and } (\text{DES-PC2 } r)(37) = r(44) \text{ and} \\
 (\text{DES-PC2 } r)(38) &= r(49) \text{ and } (\text{DES-PC2 } r)(39) = r(39) \text{ and} \\
 (\text{DES-PC2 } r)(40) &= r(56) \text{ and } (\text{DES-PC2 } r)(41) = r(34) \text{ and} \\
 (\text{DES-PC2 } r)(42) &= r(53) \text{ and } (\text{DES-PC2 } r)(43) = r(46) \text{ and} \\
 (\text{DES-PC2 } r)(44) &= r(42) \text{ and } (\text{DES-PC2 } r)(45) = r(50) \text{ and} \\
 (\text{DES-PC2 } r)(46) &= r(36) \text{ and } (\text{DES-PC2 } r)(47) = r(29) \text{ and}
 \end{aligned}$$

$$(\text{DES-PC2 } r)(48) = r(32).$$

The finite sequence $\text{bitshift}_{\text{DES}}$ of elements of \mathbb{N} is defined by the conditions (Def. 27).

- (Def. 27) $\text{bitshift}_{\text{DES}}$ is 16-element and $(\text{bitshift}_{\text{DES}})(1) = 1$ and $(\text{bitshift}_{\text{DES}})(2) = 1$ and $(\text{bitshift}_{\text{DES}})(3) = 2$ and $(\text{bitshift}_{\text{DES}})(4) = 2$ and $(\text{bitshift}_{\text{DES}})(5) = 2$ and $(\text{bitshift}_{\text{DES}})(6) = 2$ and $(\text{bitshift}_{\text{DES}})(7) = 2$ and $(\text{bitshift}_{\text{DES}})(8) = 2$ and $(\text{bitshift}_{\text{DES}})(9) = 1$ and $(\text{bitshift}_{\text{DES}})(10) = 2$ and $(\text{bitshift}_{\text{DES}})(11) = 2$ and $(\text{bitshift}_{\text{DES}})(12) = 2$ and $(\text{bitshift}_{\text{DES}})(13) = 2$ and $(\text{bitshift}_{\text{DES}})(14) = 2$ and $(\text{bitshift}_{\text{DES}})(15) = 2$ and $(\text{bitshift}_{\text{DES}})(16) = 1$.

Let K_1 be an element of Boolean^{64} . The functor $\text{DES-KS } K_1$ yielding an element of $(\text{Boolean}^{48})^{16}$ is defined by the condition (Def. 28).

- (Def. 28) There exist sequences C, D of Boolean^{28} such that
- (i) $C(0) = \text{Op-Left}(\text{DES-PC1 } K_1, 28)$,
 - (ii) $D(0) = \text{Op-Right}(\text{DES-PC1 } K_1, 28)$, and
 - (iii) for every element i of \mathbb{N} such that $0 \leq i \leq 15$ holds $(\text{DES-KS } K_1)(i+1) = \text{DES-PC2}(C(i+1) \wedge D(i+1))$ and $C(i+1) = \text{Op-Shift}(C(i), (\text{bitshift}_{\text{DES}})(i))$ and $D(i+1) = \text{Op-Shift}(D(i), (\text{bitshift}_{\text{DES}})(i))$.

6. ENCRYPTION AND DECRYPTION

Let n, m, k be non empty elements of \mathbb{N} , let R_1 be an element of $(\text{Boolean}^m)^k$, let F be a function from $\text{Boolean}^n \times \text{Boolean}^m$ into Boolean^n , let I_1 be a permutation of $\text{Boolean}^{2 \cdot n}$, and let M be an element of $\text{Boolean}^{2 \cdot n}$. The functor $\text{DES-like-CoDec}(M, F, I_1, R_1)$ yields an element of $\text{Boolean}^{2 \cdot n}$ and is defined by the condition (Def. 29).

- (Def. 29) There exist sequences L, R of Boolean^n such that
- (i) $L(0) = \text{SP-Left } I_1(M)$,
 - (ii) $R(0) = \text{SP-Right } I_1(M)$,
 - (iii) for every element i of \mathbb{N} such that $0 \leq i \leq k-1$ holds $L(i+1) = R(i)$ and $R(i+1) = \text{Op-XOR}(L(i), F(R(i), (R_1)_{i+1}))$, and
 - (iv) $\text{DES-like-CoDec}(M, F, I_1, R_1) = I_1^{-1}(R(k) \wedge L(k))$.

The following proposition is true

- (38) Let n, m, k be non empty elements of \mathbb{N} , R_1 be an element of $(\text{Boolean}^m)^k$, F be a function from $\text{Boolean}^n \times \text{Boolean}^m$ into Boolean^n , I_1 be a permutation of $\text{Boolean}^{2 \cdot n}$, and M be an element of $\text{Boolean}^{2 \cdot n}$. Then $\text{DES-like-CoDec}(\text{DES-like-CoDec}(M, F, I_1, R_1), F, I_1, \text{Rev}(R_1)) = M$.

Let R_1 be an element of $(\text{Boolean}^{48})^{16}$, let F be a function from $\text{Boolean}^{32} \times \text{Boolean}^{48}$ into Boolean^{32} , let I_1 be a permutation of Boolean^{64} , and let M be an

element of $Boolean^{64}$. The functor $DES-CoDec(M, F, I_1, R_1)$ yielding an element of $Boolean^{64}$ is defined by:

- (Def. 30) There exists a permutation I_2 of $Boolean^{2 \cdot 32}$ and there exists an element M_1 of $Boolean^{2 \cdot 32}$ such that $I_2 = I_1$ and $M_1 = M$ and $DES-CoDec(M, F, I_1, R_1) = DES-like-CoDec(M_1, F, I_2, R_1)$.

The following proposition is true

- (39) Let R_1 be an element of $(Boolean^{48})^{16}$, F be a function from $Boolean^{32} \times Boolean^{48}$ into $Boolean^{32}$, I_1 be a permutation of $Boolean^{64}$, and M be an element of $Boolean^{64}$.

Then $DES-CoDec(DES-CoDec(M, F, I_1, R_1), F, I_1, Rev(R_1)) = M$.

Let p_1, s_9 be elements of $Boolean^{64}$. The functor $DES-ENC(p_1, s_9)$ yields an element of $Boolean^{64}$ and is defined by:

- (Def. 31) $DES-ENC(p_1, s_9) = DES-CoDec(p_1, DES-FFUNC, DES-PIP, DES-KS s_9)$.

Let c_1, s_9 be elements of $Boolean^{64}$. The functor $DES-DEC(c_1, s_9)$ yields an element of $Boolean^{64}$ and is defined as follows:

- (Def. 32) $DES-DEC(c_1, s_9) =$
 $DES-CoDec(c_1, DES-FFUNC, DES-PIP, Rev(DES-KS s_9))$.

The following proposition is true

- (40) For all elements m_1, s_9 of $Boolean^{64}$ holds
 $DES-DEC(DES-ENC(m_1, s_9), s_9) = m_1$.

REFERENCES

- [1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(2):377–382, 1990.
- [2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(1):41–46, 1990.
- [3] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(1):91–96, 1990.
- [4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(1):107–114, 1990.
- [5] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(1):175–180, 1990.
- [6] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(3):529–536, 1990.
- [7] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(1):55–65, 1990.
- [8] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(1):153–164, 1990.
- [9] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(2):357–367, 1990.
- [10] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(1):47–53, 1990.
- [11] Czesław Byliński. Some properties of restrictions of finite sequences. *Formalized Mathematics*, 5(2):241–245, 1996.
- [12] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(2):249–254, 1998.
- [13] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Formalized Mathematics*, 3(2):275–278, 1992.
- [14] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(1):83–86, 1993.

- [15] U.S. Department of Commerce/National Institute of Standards and Technology. Fips pub 46-3, data encryption standard (DES). <http://csrc.nist.gov/publications/fips/fips46-3/fips46-3.pdf>. *Federal Information Processing Standards Publication*, 1999.
- [16] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(1):115–122, 1990.
- [17] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(3):501–505, 1990.
- [18] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(3):575–579, 1990.
- [19] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(1):67–71, 1990.
- [20] Edmund Woronowicz. Many argument relations. *Formalized Mathematics*, 1(4):733–737, 1990.
- [21] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(1):73–83, 1990.
- [22] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(1):181–186, 1990.

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