

# Isomorphisms of Direct Products of Finite Commutative Groups<sup>1</sup>

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**Summary.** We have been working on the formalization of groups. In [1], we encoded some theorems concerning the product of cyclic groups. In this article, we present the generalized formalization of [1]. First, we show that every finite commutative group which order is composite number is isomorphic to a direct product of finite commutative groups which orders are relatively prime. Next, we describe finite direct products of finite commutative groups.

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The notation and terminology used in this paper have been introduced in the following articles: [2], [3], [19], [7], [13], [20], [8], [9], [10], [23], [24], [25], [26], [27], [14], [22], [17], [4], [5], [15], [16], [6], [11], [21], [18], [29], [28], and [12].

## 1. PRELIMINARIES

Now we state the propositions:

- (1) Let us consider sets  $A$ ,  $B$ ,  $A_1$ ,  $B_1$ . Suppose
  - (i)  $A$  misses  $B$ , and
  - (ii)  $A_1 \subseteq A$ , and
  - (iii)  $B_1 \subseteq B$ , and
  - (iv)  $A_1 \cup B_1 = A \cup B$ .

Then

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(v)  $A_1 = A$ , and

(vi)  $B_1 = B$ .

PROOF:  $A \subseteq A_1$ .  $B \subseteq B_1$ .  $\square$

(2) Let us consider non empty finite sets  $H, K$ . Then  $\overline{\prod \langle H, K \rangle} = \overline{H} \cdot \overline{K}$ .

Let us consider bags  $p_2, p_1, f$  of Prime and a natural number  $q$ . Now we state the propositions:

(3) If support  $p_2$  misses support  $p_1$  and  $f = p_2 + p_1$  and  $q \in \text{support } p_2$ , then  $p_2(q) = f(q)$ .

(4) If support  $p_2$  misses support  $p_1$  and  $f = p_2 + p_1$  and  $q \in \text{support } p_1$ , then  $p_1(q) = f(q)$ .

Now we state the propositions:

(5) Let us consider a non zero natural number  $h$  and a prime number  $q$ . If  $q$  and  $h$  are not relatively prime, then  $q \mid h$ .

(6) Let us consider non zero natural numbers  $h, s$ . Suppose a prime number  $q$ . Suppose  $q \in \text{support PrimeFactorization}(s)$ . Then  $q$  and  $h$  are not relatively prime. Then  $\text{support PrimeFactorization}(s) \subseteq \text{support PrimeFactorization}(h)$ . The theorem is a consequence of (5).

(7) Let us consider non zero natural numbers  $h, k, s, t$ . Suppose

(i)  $h$  and  $k$  are relatively prime, and

(ii)  $s \cdot t = h \cdot k$ , and

(iii) for every prime number  $q$  such that  $q \in \text{support PrimeFactorization}(s)$  holds  $q$  and  $h$  are not relatively prime, and

(iv) for every prime number  $q$  such that  $q \in \text{support PrimeFactorization}(t)$  holds  $q$  and  $k$  are not relatively prime.

Then

(v)  $s = h$ , and

(vi)  $t = k$ .

The theorem is a consequence of (6), (1), (3), and (4). PROOF: Set  $p_2 = \text{PrimeFactorization}(s)$ . Set  $p_1 = \text{PrimeFactorization}(t)$ . For every natural number  $p$  such that  $p \in \text{support PFEExp}(h)$  holds  $p_2(p) = p^{p\text{-count}(h)}$ . For every natural number  $p$  such that  $p \in \text{support PFEExp}(k)$  holds  $p_1(p) = p^{p\text{-count}(k)}$ .  $\square$

Let  $G$  be a non empty multiplicative magma,  $I$  be a finite set, and  $b$  be a (the carrier of  $G$ )-valued total  $I$ -defined function. The functor  $\prod b$  yielding an element of  $G$  is defined by

(Def. 1) There exists a finite sequence  $f$  of elements of  $G$  such that

(i)  $it = \prod f$ , and

(ii)  $f = b \cdot \text{CFS}(I)$ .

Now we state the propositions:

- (8) Let us consider a commutative group  $G$ , non empty finite sets  $A, B$ , a (the carrier of  $G$ )-valued total  $A$ -defined function  $F_3$ , a (the carrier of  $G$ )-valued total  $B$ -defined function  $F_2$ , and a (the carrier of  $G$ )-valued total  $A \cup B$ -defined function  $F_1$ . Suppose

(i)  $A$  misses  $B$ , and

(ii)  $F_1 = F_3 + F_2$ .

Then  $\prod F_1 = \prod F_3 \cdot \prod F_2$ .

- (9) Let us consider a non empty multiplicative magma  $G$ , a set  $q$ , an element  $z$  of  $G$ , and a (the carrier of  $G$ )-valued total  $\{q\}$ -defined function  $f$ . If  $f = q \mapsto z$ , then  $\prod f = z$ .

## 2. DIRECT PRODUCT OF FINITE COMMUTATIVE GROUPS

Now we state the propositions:

- (10) Let us consider non empty multiplicative magmas  $X, Y$ . Then the carrier of  $\prod \langle X, Y \rangle = \prod \langle \text{the carrier of } X, \text{the carrier of } Y \rangle$ . PROOF: Set  $\text{Carr}X = \text{the carrier of } X$ . Set  $\text{Carr}Y = \text{the carrier of } Y$ . For every element  $a$  such that  $a \in \text{dom the support of } \langle X, Y \rangle$  holds (the support of  $\langle X, Y \rangle$ )( $a$ ) =  $\langle \text{the carrier of } X, \text{the carrier of } Y \rangle(a)$ .  $\square$

- (11) Let us consider a group  $G$  and normal subgroups  $A, B$  of  $G$ . Suppose  $(\text{the carrier of } A) \cap (\text{the carrier of } B) = \{1_G\}$ . Let us consider elements  $a, b$  of  $G$ . If  $a \in A$  and  $b \in B$ , then  $a \cdot b = b \cdot a$ .

- (12) Let us consider a group  $G$  and normal subgroups  $A, B$  of  $G$ . Suppose
- (i) for every element  $x$  of  $G$ , there exist elements  $a, b$  of  $G$  such that  $a \in A$  and  $b \in B$  and  $x = a \cdot b$ , and
- (ii)  $(\text{the carrier of } A) \cap (\text{the carrier of } B) = \{1_G\}$ .

Then there exists a homomorphism  $h$  from  $\prod \langle A, B \rangle$  to  $G$  such that

(iii)  $h$  is bijective, and

(iv) for every elements  $a, b$  of  $G$  such that  $a \in A$  and  $b \in B$  holds  $h(\langle a, b \rangle) = a \cdot b$ .

The theorem is a consequence of (11). PROOF: Define  $\mathcal{P}[\text{set}, \text{set}] \equiv$  there exists an element  $x$  of  $G$  and there exists an element  $y$  of  $G$  such that  $x \in A$  and  $y \in B$  and  $\$1 = \langle x, y \rangle$  and  $\$2 = x \cdot y$ . For every element  $z$  of  $\prod \langle A, B \rangle$ , there exists an element  $w$  of  $G$  such that  $\mathcal{P}[z, w]$ . Consider  $h$  being a function from  $\prod \langle A, B \rangle$  into  $G$  such that for every element  $z$  of  $\prod \langle A, B \rangle$ ,  $\mathcal{P}[z, h(z)]$ . For every elements  $a, b$  of  $G$  such that  $a \in A$  and  $b \in B$  holds

$h(\langle a, b \rangle) = a \cdot b$ . For every elements  $z, w$  of  $\prod \langle A, B \rangle$ ,  $h(z \cdot w) = h(z) \cdot h(w)$ .  
□

Let us consider a finite commutative group  $G$ , a natural number  $m$ , and a subset  $A$  of  $G$ . Now we state the propositions:

- (13) Suppose  $A = \{x \text{ where } x \text{ is an element of } G : x^m = \mathbf{1}_G\}$ . Then
- (i)  $A \neq \emptyset$ , and
  - (ii) for every elements  $g_1, g_2$  of  $G$  such that  $g_1, g_2 \in A$  holds  $g_1 \cdot g_2 \in A$ , and
  - (iii) for every element  $g$  of  $G$  such that  $g \in A$  holds  $g^{-1} \in A$ .
- (14) Suppose  $A = \{x \text{ where } x \text{ is an element of } G : x^m = \mathbf{1}_G\}$ . Then there exists a strict finite subgroup  $H$  of  $G$  such that
- (i) the carrier of  $H = A$ , and
  - (ii)  $H$  is commutative and normal.

Now we state the propositions:

- (15) Let us consider a finite commutative group  $G$ , a natural number  $m$ , and a finite subgroup  $H$  of  $G$ . Suppose the carrier of  $H = \{x \text{ where } x \text{ is an element of } G : x^m = \mathbf{1}_G\}$ . Let us consider a prime number  $q$ . Suppose  $q \in \text{support PrimeFactorization}(\overline{H})$ . Then  $q$  and  $m$  are not relatively prime.
- (16) Let us consider a finite commutative group  $G$  and natural numbers  $h, k$ . Suppose
- (i)  $\overline{G} = h \cdot k$ , and
  - (ii)  $h$  and  $k$  are relatively prime.

Then there exist strict finite subgroups  $H, K$  of  $G$  such that

- (iii) the carrier of  $H = \{x \text{ where } x \text{ is an element of } G : x^h = \mathbf{1}_G\}$ , and
- (iv) the carrier of  $K = \{x \text{ where } x \text{ is an element of } G : x^k = \mathbf{1}_G\}$ , and
- (v)  $H$  is normal, and
- (vi)  $K$  is normal, and
- (vii) for every element  $x$  of  $G$ , there exist elements  $a, b$  of  $G$  such that  $a \in H$  and  $b \in K$  and  $x = a \cdot b$ , and
- (viii)  $(\text{the carrier of } H) \cap (\text{the carrier of } K) = \{\mathbf{1}_G\}$ .

The theorem is a consequence of (14). PROOF: Set  $A = \{x \text{ where } x \text{ is an element of } G : x^h = \mathbf{1}_G\}$ . Set  $B = \{x \text{ where } x \text{ is an element of } G : x^k = \mathbf{1}_G\}$ .  $A \subseteq$  the carrier of  $G$ .  $B \subseteq$  the carrier of  $G$ . Consider  $H$  being a strict finite subgroup of  $G$  such that the carrier of  $H = A$  and  $H$  is commutative and  $H$  is normal. Consider  $K$  being a strict finite subgroup of  $G$  such that the carrier of  $K = B$  and  $K$  is commutative and  $K$  is

normal. Consider  $a, b$  being integers such that  $a \cdot h + b \cdot k = 1$ . (The carrier of  $H$ )  $\cap$  (the carrier of  $K$ )  $\subseteq \{1_G\}$ . For every element  $x$  of  $G$ , there exist elements  $s, t$  of  $G$  such that  $s \in H$  and  $t \in K$  and  $x = s \cdot t$ .  $\square$

(17) Let us consider finite groups  $H, K$ . Then  $\overline{\prod\langle H, K \rangle} = \overline{H} \cdot \overline{K}$ . The theorem is a consequence of (10) and (2).

(18) Let us consider a finite commutative group  $G$  and non zero natural numbers  $h, k$ . Suppose

- (i)  $\overline{G} = h \cdot k$ , and
- (ii)  $h$  and  $k$  are relatively prime.

Then there exist strict finite subgroups  $H, K$  of  $G$  such that

- (iii)  $\overline{H} = h$ , and
- (iv)  $\overline{K} = k$ , and
- (v) (the carrier of  $H$ )  $\cap$  (the carrier of  $K$ ) =  $\{1_G\}$ , and
- (vi) there exists a homomorphism  $F$  from  $\prod\langle H, K \rangle$  to  $G$  such that  $F$  is bijective and for every elements  $a, b$  of  $G$  such that  $a \in H$  and  $b \in K$  holds  $F(\langle a, b \rangle) = a \cdot b$ .

The theorem is a consequence of (16), (12), (17), (15), and (7).

### 3. FINITE DIRECT PRODUCTS OF FINITE COMMUTATIVE GROUPS

Let us consider a group  $G$ , a set  $q$ , an associative group-like multiplicative magma family  $F$  of  $\{q\}$ , and a function  $f$  from  $G$  into  $\prod F$ . Now we state the propositions:

- (19) If  $F = q \dashrightarrow G$  and for every element  $x$  of  $G$ ,  $f(x) = q \dashrightarrow x$ , then  $f$  is a homomorphism from  $G$  to  $\prod F$ .
- (20) If  $F = q \dashrightarrow G$  and for every element  $x$  of  $G$ ,  $f(x) = q \dashrightarrow x$ , then  $f$  is bijective.

Now we state the propositions:

(21) Let us consider a set  $q$ , an associative group-like multiplicative magma family  $F$  of  $\{q\}$ , and a group  $G$ . Suppose  $F = q \dashrightarrow G$ . Then there exists a homomorphism  $I$  from  $G$  to  $\prod F$  such that

- (i)  $I$  is bijective, and
- (ii) for every element  $x$  of  $G$ ,  $I(x) = q \dashrightarrow x$ .

The theorem is a consequence of (19) and (20). PROOF: Define  $\mathcal{P}[\text{set}, \text{set}] \equiv \mathcal{S}_2 = q \dashrightarrow \mathcal{S}_1$ . For every element  $z$  of  $G$ , there exists an element  $w$  of  $\prod F$  such that  $\mathcal{P}[z, w]$ . Consider  $I$  being a function from  $G$  into  $\prod F$  such that for every element  $x$  of  $G$ ,  $\mathcal{P}[x, I(x)]$ .  $\square$

(22) Let us consider non empty finite sets  $I_0, I$ , an associative group-like multiplicative magma family  $F_0$  of  $I_0$ , an associative group-like multiplicative magma family  $F$  of  $I$ , groups  $H, K$ , an element  $q$  of  $I$ , an element  $k$  of  $K$ , and a function  $g$ . Suppose

- (i)  $g \in$  the carrier of  $\prod F_0$ , and
- (ii)  $q \notin I_0$ , and
- (iii)  $I = I_0 \cup \{q\}$ , and
- (iv)  $F = F_0 + \cdot (q \dashrightarrow K)$ .

Then  $g + \cdot (q \dashrightarrow k) \in$  the carrier of  $\prod F$ . PROOF: Set  $HK = \langle H, K \rangle$ . Set  $w = g + \cdot (q \dashrightarrow k)$ . For every element  $x$  such that  $x \in$  dom the support of  $F$  holds  $w(x) \in$  (the support of  $F$ )( $x$ ).  $\square$

Let us consider non empty finite sets  $I_0, I$ , an associative group-like multiplicative magma family  $F_0$  of  $I_0$ , an associative group-like multiplicative magma family  $F$  of  $I$ , groups  $H, K$ , an element  $q$  of  $I$ , a function  $G_0$  from  $H$  into  $\prod F_0$ , and a function  $G$  from  $\prod \langle H, K \rangle$  into  $\prod F$ . Now we state the propositions:

- (23) Suppose  $G_0$  is a homomorphism from  $H$  to  $\prod F_0$  and  $G_0$  is bijective and  $q \notin I_0$  and  $I = I_0 \cup \{q\}$  and  $F = F_0 + \cdot (q \dashrightarrow K)$ . Then suppose for every element  $h$  of  $H$  and for every element  $k$  of  $K$ , there exists a function  $g$  such that  $g = G_0(h)$  and  $G(\langle h, k \rangle) = g + \cdot (q \dashrightarrow k)$ . Then  $G$  is a homomorphism from  $\prod \langle H, K \rangle$  to  $\prod F$ .
- (24) Suppose  $G_0$  is a homomorphism from  $H$  to  $\prod F_0$  and  $G_0$  is bijective and  $q \notin I_0$  and  $I = I_0 \cup \{q\}$  and  $F = F_0 + \cdot (q \dashrightarrow K)$ . Then suppose for every element  $h$  of  $H$  and for every element  $k$  of  $K$ , there exists a function  $g$  such that  $g = G_0(h)$  and  $G(\langle h, k \rangle) = g + \cdot (q \dashrightarrow k)$ . Then  $G$  is bijective.

Now we state the propositions:

- (25) Let us consider a set  $q$ , a multiplicative magma family  $F$  of  $\{q\}$ , and a non empty multiplicative magma  $G$ . Suppose  $F = q \dashrightarrow G$ . Let us consider a (the carrier of  $G$ )-valued total  $\{q\}$ -defined function  $y$ . Then
  - (i)  $y \in$  the carrier of  $\prod F$ , and
  - (ii)  $y(q) \in$  the carrier of  $G$ , and
  - (iii)  $y = q \dashrightarrow y(q)$ .
- (26) Let us consider a set  $q$ , an associative group-like multiplicative magma family  $F$  of  $\{q\}$ , and a group  $G$ . Suppose  $F = q \dashrightarrow G$ . Then there exists a homomorphism  $H_0$  from  $\prod F$  to  $G$  such that
  - (i)  $H_0$  is bijective, and
  - (ii) for every (the carrier of  $G$ )-valued total  $\{q\}$ -defined function  $x$ ,  $H_0(x) = \prod x$ .

The theorem is a consequence of (21), (25), and (9). PROOF: Consider  $I$  being a homomorphism from  $G$  to  $\prod F$  such that  $I$  is bijective and for every element  $x$  of  $G$ ,  $I(x) = q \cdot x$ . Set  $H_0 = I^{-1}$ . For every (the carrier of  $G$ )-valued total  $\{q\}$ -defined function  $y$ ,  $H_0(y) = \prod y$ .  $\square$

- (27) Let us consider non empty finite sets  $I_0, I$ , an associative group-like multiplicative magma family  $F_0$  of  $I_0$ , an associative group-like multiplicative magma family  $F$  of  $I$ , groups  $H, K$ , an element  $q$  of  $I$ , and a homomorphism  $G_0$  from  $H$  to  $\prod F_0$ . Suppose

- (i)  $q \notin I_0$ , and
- (ii)  $I = I_0 \cup \{q\}$ , and
- (iii)  $F = F_0 + \cdot (q \cdot K)$ , and
- (iv)  $G_0$  is bijective.

Then there exists a homomorphism  $G$  from  $\prod \langle H, K \rangle$  to  $\prod F$  such that

- (v)  $G$  is bijective, and
- (vi) for every element  $h$  of  $H$  and for every element  $k$  of  $K$ , there exists a function  $g$  such that  $g = G_0(h)$  and  $G(\langle h, k \rangle) = g + \cdot (q \cdot k)$ .

The theorem is a consequence of (22), (23), and (24). PROOF: Set  $HK = \langle H, K \rangle$ . Define  $\mathcal{P}[\text{set}, \text{set}] \equiv$  there exists an element  $h$  of  $H$  and there exists an element  $k$  of  $K$  and there exists a function  $g$  such that  $\$1 = \langle h, k \rangle$  and  $g = G_0(h)$  and  $\$2 = g + \cdot (q \cdot k)$ . For every element  $z$  of  $\prod \langle H, K \rangle$ , there exists an element  $w$  of the carrier of  $\prod F$  such that  $\mathcal{P}[z, w]$ . Consider  $G$  being a function from  $\prod \langle H, K \rangle$  into  $\prod F$  such that for every element  $x$  of  $\prod \langle H, K \rangle$ ,  $\mathcal{P}[x, G(x)]$ . For every element  $h$  of  $H$  and for every element  $k$  of  $K$ , there exists a function  $g$  such that  $g = G_0(h)$  and  $G(\langle h, k \rangle) = g + \cdot (q \cdot k)$ .  $\square$

- (28) Let us consider non empty finite sets  $I_0, I$ , an associative group-like multiplicative magma family  $F_0$  of  $I_0$ , an associative group-like multiplicative magma family  $F$  of  $I$ , groups  $H, K$ , an element  $q$  of  $I$ , and a homomorphism  $G_0$  from  $\prod F_0$  to  $H$ . Suppose

- (i)  $q \notin I_0$ , and
- (ii)  $I = I_0 \cup \{q\}$ , and
- (iii)  $F = F_0 + \cdot (q \cdot K)$ , and
- (iv)  $G_0$  is bijective.

Then there exists a homomorphism  $G$  from  $\prod F$  to  $\prod \langle H, K \rangle$  such that

- (v)  $G$  is bijective, and
- (vi) for every function  $x_0$  and for every element  $k$  of  $K$  and for every element  $h$  of  $H$  such that  $h = G_0(x_0)$  and  $x_0 \in \prod F_0$  holds  $G(x_0 + \cdot (q \cdot k)) = \langle h, k \rangle$ .

The theorem is a consequence of (27). PROOF: Set  $L0 = G_0^{-1}$ . Consider  $L$  being a homomorphism from  $\prod\langle H, K \rangle$  to  $\prod F$  such that  $L$  is bijective and for every element  $h$  of  $H$  and for every element  $k$  of  $K$ , there exists a function  $g$  such that  $g = L0(h)$  and  $L(\langle h, k \rangle) = g + \cdot (q \mapsto k)$ . Set  $G = L^{-1}$ . For every function  $x_0$  and for every element  $k$  of  $K$  and for every element  $h$  of  $H$  such that  $h = G_0(x_0)$  and  $x_0 \in \prod F_0$  holds  $G(x_0 + \cdot (q \mapsto k)) = \langle h, k \rangle$ .  $\square$

(29) Let us consider a non empty finite set  $I$ , an associative group-like multiplicative magma family  $F$  of  $I$ , and a total  $I$ -defined function  $x$ . Suppose an element  $p$  of  $I$ . Then  $x(p) \in F(p)$ . Then  $x \in$  the carrier of  $\prod F$ .

(30) Let us consider non empty finite sets  $I_0, I$ , an associative group-like multiplicative magma family  $F_0$  of  $I_0$ , an associative group-like multiplicative magma family  $F$  of  $I$ , a group  $K$ , an element  $q$  of  $I$ , and an element  $x$  of  $\prod F$ . Suppose

- (i)  $q \notin I_0$ , and
- (ii)  $I = I_0 \cup \{q\}$ , and
- (iii)  $F = F_0 + \cdot (q \mapsto K)$ .

Then there exists a total  $I_0$ -defined function  $x_0$  and there exists an element  $k$  of  $K$  such that  $x_0 \in \prod F_0$  and  $x = x_0 + \cdot (q \mapsto k)$  and for every element  $p$  of  $I_0$ ,  $x_0(p) \in F_0(p)$ . PROOF: Reconsider  $y = x$  as a total  $I$ -defined function. Reconsider  $k = y(q)$  as an element of  $K$ . Reconsider  $y_0 = y|_{I_0}$  as an  $I_0$ -defined function. For every element  $i$  of  $I_0$ ,  $y_0(i) \in$  (the support of  $F_0$ )( $i$ ) and  $y_0(i) \in F_0(i)$ .  $\square$

(31) Let us consider a group  $G$ , a subgroup  $H$  of  $G$ , a finite sequence  $f$  of elements of  $G$ , and a finite sequence  $g$  of elements of  $H$ . If  $f = g$ , then  $\prod f = \prod g$ . PROOF: Define  $\mathcal{P}$ [natural number]  $\equiv$  for every finite sequence  $f$  of elements of  $G$  for every finite sequence  $g$  of elements of  $H$  such that  $\$1 = \text{len } f$  and  $f = g$  holds  $\prod f = \prod g$ .  $\mathcal{P}[0]$ . For every natural number  $k$  such that  $\mathcal{P}[k]$  holds  $\mathcal{P}[k + 1]$ .  $\square$

(32) Let us consider a non empty finite set  $I$ , a group  $G$ , a subgroup  $H$  of  $G$ , a (the carrier of  $G$ )-valued total  $I$ -defined function  $x$ , and a (the carrier of  $H$ )-valued total  $I$ -defined function  $x_0$ . If  $x = x_0$ , then  $\prod x = \prod x_0$ . The theorem is a consequence of (31).

(33) Let us consider a commutative group  $G$ , non empty finite sets  $I_0, I$ , an element  $q$  of  $I$ , a (the carrier of  $G$ )-valued total  $I$ -defined function  $x$ , a (the carrier of  $G$ )-valued total  $I_0$ -defined function  $x_0$ , and an element  $k$  of  $G$ . Suppose

- (i)  $q \notin I_0$ , and
- (ii)  $I = I_0 \cup \{q\}$ , and



(iii)  $x = x_0 + \cdot (q \mapsto k)$ .

Then  $\prod x = \prod x_0 \cdot k$ . The theorem is a consequence of (8) and (9). PROOF: Reconsider  $y = q \mapsto k$  as a (the carrier of  $G$ )-valued total  $\{q\}$ -defined function.  $I_0$  misses  $\{q\}$ .  $\square$

Let us consider a finite commutative group  $G$ . Now we state the propositions:

- (34) Suppose  $\overline{G} > 1$ . Then there exists a non empty finite set  $I$  and there exists an associative group-like commutative multiplicative magma family  $F$  of  $I$  and there exists a homomorphism  $H_0$  from  $\prod F$  to  $G$  such that  $I = \text{support PrimeFactorization}(\overline{G})$  and for every element  $p$  of  $I$ ,  $F(p)$  is a subgroup of  $G$  and  $\overline{F(p)} = (\text{PrimeFactorization}(\overline{G}))(p)$  and for every elements  $p, q$  of  $I$  such that  $p \neq q$  holds  $(\text{the carrier of } F(p)) \cap (\text{the carrier of } F(q)) = \{1_G\}$  and  $H_0$  is bijective and for every (the carrier of  $G$ )-valued total  $I$ -defined function  $x$  such that for every element  $p$  of  $I$ ,  $x(p) \in F(p)$  holds  $x \in \prod F$  and  $H_0(x) = \prod x$ .
- (35) Suppose  $\overline{G} > 1$ . Then there exists a non empty finite set  $I$  and there exists an associative group-like commutative multiplicative magma family  $F$  of  $I$  such that  $I = \text{support PrimeFactorization}(\overline{G})$  and for every element  $p$  of  $I$ ,  $F(p)$  is a subgroup of  $G$  and  $\overline{F(p)} = (\text{PrimeFactorization}(\overline{G}))(p)$  and for every elements  $p, q$  of  $I$  such that  $p \neq q$  holds  $(\text{the carrier of } F(p)) \cap (\text{the carrier of } F(q)) = \{1_G\}$  and for every element  $y$  of  $G$ , there exists a (the carrier of  $G$ )-valued total  $I$ -defined function  $x$  such that for every element  $p$  of  $I$ ,  $x(p) \in F(p)$  and  $y = \prod x$  and for every (the carrier of  $G$ )-valued total  $I$ -defined functions  $x_1, x_2$  such that for every element  $p$  of  $I$ ,  $x_1(p) \in F(p)$  and for every element  $p$  of  $I$ ,  $x_2(p) \in F(p)$  and  $\prod x_1 = \prod x_2$  holds  $x_1 = x_2$ .

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