

Uniform Space

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Summary. In this article, we formalize in Mizar [1] the notion of uniform space introduced by André Weil using the concepts of entourages [2].

We present some results between uniform space and pseudo metric space. We introduce the concepts of left-uniformity and right-uniformity of a topological group.

Next, we define the concept of the partition topology. Following the Vlach's works [11, 10], we define the semi-uniform space induced by a tolerance and the uniform space induced by an equivalence relation.

Finally, using mostly Gehrke, Grigorieff and Pin [4] works, a Pervin uniform space defined from the sets of the form $((X \setminus A) \times (X \setminus A)) \cup (A \times A)$ is presented.

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1. PRELIMINARIES

From now on X denotes a set, D denotes a partition of X , T denotes a non empty topological group, and A denotes a subset of X .

Now we state the propositions:

- (1) $A \times A \cup (X \setminus A) \times (X \setminus A) \subseteq (X \setminus A) \times X \cup X \times A$.
- (2) $\{1, 2, 3\} \setminus \{1\} = \{2, 3\}$.
- (3) Suppose $X = \{1, 2, 3\}$ and $A = \{1\}$. Then
 - (i) $\langle 2, 1 \rangle \in (X \setminus A) \times X \cup X \times A$, and
 - (ii) $\langle 2, 1 \rangle \notin A \times A \cup (X \setminus A) \times (X \setminus A)$.

The theorem is a consequence of (2).

- (4) Let us consider a subset A of X . Then $(A \times A \cup (X \setminus A) \times (X \setminus A))^\smile = A \times A \cup (X \setminus A) \times (X \setminus A)$.
- (5) Let us consider subsets P_1, P_2 of D . If $\bigcup P_1 = \bigcup P_2$, then $P_1 = P_2$.
- (6) Let us consider a subset P of D . Then $\bigcup(D \setminus P) = \bigcup D \setminus \bigcup P$.
- (7) Let us consider an upper family S_1 of subsets of X , and an element S of S_1 . Then $\bigcap S_1 \subseteq S$.
- (8) Let us consider an additive group G , and subsets A, B, C, D of G . If $A \subseteq B$ and $C \subseteq D$, then $A + C \subseteq B + D$.

Let us consider an element e of T and a neighbourhood V of $\mathbf{1}_T$. Now we state the propositions:

- (9) $\{e\} \cdot V$ is a neighbourhood of e .
- (10) $V \cdot \{e\}$ is a neighbourhood of e .
- (11) Let us consider a neighbourhood V of $\mathbf{1}_T$. Then V^{-1} is a neighbourhood of $\mathbf{1}_T$.

2. UNIFORM SPACE

A uniform space is an upper, \cap -closed uniform space structure satisfying axiom U1, axiom U2, and axiom U3. From now on Q denotes a uniform space.

Now we state the propositions:

- (12) Q is a quasi-uniform space.
- (13) Q is a semi-uniform space.

Let X be a set and \mathcal{B} be a family of subsets of $X \times X$. We say that \mathcal{B} satisfies axiom UP2 if and only if

- (Def. 1) for every element B_1 of \mathcal{B} , there exists an element B_2 of \mathcal{B} such that $B_2 \subseteq B_1^\smile$.

Now we state the proposition:

- (14) Let us consider an empty set X . Then every empty family of subsets of $X \times X$ is quasi-basis and satisfies axiom UP1, axiom UP2, and axiom UP3.

One can verify that there exists a uniform space which is strict.

Now we state the proposition:

- (15) Let us consider a set X , and a family S_1 of subsets of $X \times X$. Suppose $X = \{\emptyset\}$ and $S_1 = \{X \times X\}$. Then $\langle X, S_1 \rangle$ is a uniform space.

Let us observe that there exists a strict uniform space which is non empty.

Now we state the proposition:

- (16) Let us consider a set X , and a family \mathcal{B} of subsets of $X \times X$. Suppose \mathcal{B} is quasi-basis and satisfies axiom UP1, axiom UP2, and axiom UP3. Then there exists a strict uniform space Q such that
- (i) the carrier of $Q = X$, and
 - (ii) the entourages $Q = [\mathcal{B}]$.

3. OPEN SET AND UNIFORM SPACE

Now we state the propositions:

- (17) Let us consider a non empty uniform space Q . Then
- (i) the carrier of the topological space induced by $Q =$ the carrier of Q , and
 - (ii) the topology of the topological space induced by $Q =$ the open set family of the FMTinduced by Q .
- (18) Let us consider a non empty uniform space Q , and a subset S of the FMTinduced by Q . Then S is open if and only if for every element x of Q such that $x \in S$ holds $S \in$ Neighborhood x .
- (19) Let us consider a non empty uniform space Q . Then the open set family of the FMTinduced by $Q =$ the set of all O where O is an open subset of the FMTinduced by Q .

Let us consider a non empty uniform space Q and a subset S of the FMTinduced by Q . Now we state the propositions:

- (20) S is open if and only if $S \in$ the open set family of the FMTinduced by Q .
- (21) $S \in$ the open set family of the FMTinduced by Q if and only if for every element x of Q such that $x \in S$ holds $S \in$ Neighborhood x .

4. PSEUDO METRIC SPACE AND UNIFORM SPACE

Let M be a non empty metric structure and r be a positive real number. The functor $\text{ent}(M, r)$ yielding a subset of (the carrier of M) \times (the carrier of M) is defined by the term

(Def. 2) $\{\langle x, y \rangle, \text{ where } x, y \text{ are elements of } M : \rho(x, y) \leq r\}$.

Let M be a non empty, reflexive metric structure. Let us observe that $\text{ent}(M, r)$ is non empty.

Let M be a non empty metric structure. The functor $\text{ENT}(M)$ yielding a non empty family of subsets of (the carrier of M) \times (the carrier of M) is defined by the term

(Def. 3) the set of all $\text{ent}(M, r)$ where r is a positive real number.

The uniformity induced by M yielding a uniform space structure is defined by the term

(Def. 4) $\langle \text{the carrier of } M, [\text{ENT}(M)] \rangle$.

Let M be a pseudo metric space. The uniformity induced by M yielding a non empty, strict uniform space is defined by the term

(Def. 5) $\langle \text{the carrier of } M, [\text{ENT}(M)] \rangle$.

Let us consider a pseudo metric space M . Now we state the propositions:

(22) The open set family of the FMT induced by the uniformity induced by $M =$ the open set family of M .

PROOF: Set $X =$ the open set family of the FMT induced by the uniformity induced by M . Set $Y =$ the open set family of M . $X \subseteq Y$ by (18), (20), [5, (11)]. Reconsider $t_1 = t$ as a subset of M . For every element x of the uniformity induced by M such that $x \in t_1$ holds $t_1 \in \text{Neighborhood } x$ by [5, (11)]. \square

(23) The topological space induced by the uniformity induced by $M = M_{\text{top}}$. The theorem is a consequence of (22).

5. UNIFORM SPACE AND TOPOLOGICAL GROUP

Let G be a topological group and Q be a neighbourhood of $\mathbf{1}_G$. The functor $\text{leftU}(Q)$ yielding a subset of $(\text{the carrier of } G) \times (\text{the carrier of } G)$ is defined by the term

(Def. 6) $\{ \langle x, y \rangle, \text{ where } x \text{ is an element of } G, y \text{ is an element of } G : x^{-1} \cdot y \in Q \}$.

Let T be a non empty topological group. The functor $\text{SleftU}(T)$ yielding a non empty family of subsets of $(\text{the carrier of } T) \times (\text{the carrier of } T)$ is defined by the term

(Def. 7) the set of all $\text{leftU}(Q)$ where Q is a neighbourhood of $\mathbf{1}_T$.

The left-uniformity T yielding a non empty uniform space is defined by the term

(Def. 8) $\langle \text{the carrier of } T, [\text{SleftU}(T)] \rangle$.

Let G be a topological group and Q be a neighbourhood of $\mathbf{1}_G$. The functor $\text{rightU}(Q)$ yielding a subset of $(\text{the carrier of } G) \times (\text{the carrier of } G)$ is defined by the term

(Def. 9) $\{ \langle x, y \rangle, \text{ where } x \text{ is an element of } G, y \text{ is an element of } G : y \cdot x^{-1} \in Q \}$.

Let T be a non empty topological group. The functor $\text{SrightU}(T)$ yielding a non empty family of subsets of $(\text{the carrier of } T) \times (\text{the carrier of } T)$ is defined by the term

(Def. 10) the set of all $\text{rightU}(Q)$ where Q is a neighbourhood of $\mathbf{1}_T$.

The right-uniformity T yielding a non empty uniform space is defined by the term

(Def. 11) $\langle \text{the carrier of } T, [\text{SrightU}(T)] \rangle$.

Now we state the propositions:

(24) Let us consider a non empty, commutative topological group T , and a neighbourhood Q of $\mathbf{1}_T$. Then $\text{leftU}(Q) = \text{rightU}(Q)$.

(25) Let us consider a non empty, commutative topological group T . Then the left-uniformity $T =$ the right-uniformity T . The theorem is a consequence of (24).

Let G be a semi additive topological group and Q be a neighbourhood of 0_G . The functor $\text{leftU}(Q)$ yielding a subset of $(\text{the carrier of } G) \times (\text{the carrier of } G)$ is defined by the term

(Def. 12) $\{ \langle x, y \rangle, \text{ where } x \text{ is an element of } G, y \text{ is an element of } G : -x + y \in Q \}$.

Let T be a non empty semi additive topological group. The functor $\text{SleftU}(T)$ yielding a non empty family of subsets of $(\text{the carrier of } T) \times (\text{the carrier of } T)$ is defined by the term

(Def. 13) the set of all $\text{leftU}(Q)$ where Q is a neighbourhood of 0_T .

Let T be a non empty topological additive group. The left-uniformity T yielding a non empty uniform space is defined by the term

(Def. 14) $\langle \text{the carrier of } T, [\text{SleftU}(T)] \rangle$.

Let G be a semi additive topological group and Q be a neighbourhood of 0_G . The functor $\text{rightU}(Q)$ yielding a subset of $(\text{the carrier of } G) \times (\text{the carrier of } G)$ is defined by the term

(Def. 15) $\{ \langle x, y \rangle, \text{ where } x \text{ is an element of } G, y \text{ is an element of } G : y + -x \in Q \}$.

Let T be a non empty semi additive topological group. The functor $\text{SrightU}(T)$ yielding a non empty family of subsets of $(\text{the carrier of } T) \times (\text{the carrier of } T)$ is defined by the term

(Def. 16) the set of all $\text{rightU}(Q)$ where Q is a neighbourhood of 0_T .

Let T be a non empty topological additive group. The right-uniformity T yielding a non empty uniform space is defined by the term

(Def. 17) $\langle \text{the carrier of } T, [\text{SrightU}(T)] \rangle$.

Now we state the propositions:

(26) Let us consider an Abelian semi additive topological group T , and a neighbourhood Q of 0_T . Then $\text{leftU}(Q) = \text{rightU}(Q)$.

(27) Let us consider a non empty topological additive group T . Suppose T

is Abelian. Then the left-uniformity $T =$ the right-uniformity T . The theorem is a consequence of (26).

- (28) The topology of the topological space induced by the left-uniformity $T =$ the topology of T .

PROOF: Set $X =$ the topology of FMT2TopSpace (the FMT induced by the left-uniformity T). Set $Y =$ the topology of T . $X \subseteq Y$ by (9), [6, (7)]. $Y \subseteq X$ by [9, (3)], [6, (6)], [8, (6)]. \square

- (29) The topology of the topological space induced by the right-uniformity $T =$ the topology of T .

PROOF: Set $X =$ the topology of FMT2TopSpace (the FMT induced by the right-uniformity T). Set $Y =$ the topology of T . $X \subseteq Y$ by (10), [6, (7)]. $Y \subseteq X$ by [9, (3)], [6, (6)], [8, (6)]. \square

6. FUNCTION UNIFORMLY CONTINUOUS

Let Q_1, Q_2 be uniform space structures and f be a function from Q_1 into Q_2 . We say that f is uniformly continuous if and only if

- (Def. 18) for every element V of the entourages Q_2 , there exists an element Q of the entourages Q_1 such that for every objects x, y such that $\langle x, y \rangle \in Q$ holds $\langle f(x), f(y) \rangle \in V$.

Let Q_1, Q_2 be non empty uniform space structures satisfying axiom U1. One can check that there exists a function from Q_1 into Q_2 which is uniformly continuous.

7. PARTITION TOPOLOGY

Now we state the propositions:

- (30) the set of all $\cup P$ where P is a subset of $D = \text{UniCl}(D)$.
 (31) $X \in \text{UniCl}(D)$. The theorem is a consequence of (30).
 (32) If $D = \emptyset$, then X is empty and $\text{UniCl}(D) = \{\emptyset\}$.

Let X be a set and D be a partition of X . Let us note that $\text{UniCl}(D)$ is \cap -closed and $\text{UniCl}(D)$ is union-closed and every family of subsets of X which is union-closed is also \cup -closed.

Let D be a partition of X . Let us note that $\text{UniCl}(D)$ is closed for complement operator and $\text{UniCl}(D)$ is \cup -closed and \setminus -closed.

Now we state the proposition:

- (33) $\text{UniCl}(D)$ is a ring of sets. The theorem is a consequence of (30).

Let us consider X and D . One can verify that $\text{UniCl}(D)$ has the empty element.

Let X be a set and D be a partition of X . Let us observe that $\text{UniCl}(D)$ is non empty.

Now we state the proposition:

(34) $\text{UniCl}(D)$ is a field of subsets of X .

Let X be a set and D be a partition of X . Observe that $\text{UniCl}(D)$ is σ -additive and $\text{UniCl}(D)$ is σ -multiplicative.

Now we state the proposition:

(35) $\text{UniCl}(D)$ is a σ -field of subsets of X .

Let X be a set and D be a partition of X . Observe that $\text{UniCl}(D)$ is closed for countable unions and closed for countable meets.

Now we state the proposition:

(36) Let us consider a non empty set Ω , and a partition D of Ω . Then $\text{UniCl}(D)$ is a Dynkin system of Ω .

Let X be a set and D be a partition of X . The partition topology D yielding a topological space is defined by the term

(Def. 19) $\langle X, \text{UniCl}(D) \rangle$.

Now we state the propositions:

(37) Every open subset of the partition topology D is closed.

(38) Every closed subset of the partition topology D is open.

(39) Let us consider a subset S of the partition topology D . Then S is open if and only if S is closed.

Let X be a non empty set and D be a partition of X . Observe that the partition topology D is non empty.

Let us consider a non empty set X and a partition D of X . Now we state the propositions:

(40) $\text{LC}(\text{the partition topology } D) = \text{UniCl}(D)$. The theorem is a consequence of (38) and (31).

(41) $\text{OpenClosedSet}(\text{the partition topology } D) = \text{the topology of the partition topology } D$. The theorem is a consequence of (37).

8. UNIFORM SPACE AND PARTITION TOPOLOGY

In the sequel R denotes a binary relation on X .

Let X be a set and R be a binary relation on X . The functor $\rho(R)$ yielding a non empty family of subsets of $X \times X$ is defined by the term

(Def. 20) $\{S, \text{ where } S \text{ is a subset of } X \times X : R \subseteq S\}$.

Now we state the propositions:

$$(42) \quad [\rho(R)] = \rho(R).$$

$$(43) \quad [\{R\}] = \rho(R).$$

$$(44) \quad \rho(R) \text{ is upper and } \cap\text{-closed.}$$

Let us consider X and R . Observe that $\rho(R)$ is quasi-basis.

Now we state the propositions:

(45) Let us consider a total, reflexive binary relation R on X . Then $\rho(R)$ satisfies axiom UP1.

(46) Let us consider a symmetric binary relation R on X . Then $\rho(R)$ satisfies axiom UP2.

(47) Let us consider a total, transitive binary relation R on X . Then $\rho(R)$ satisfies axiom UP3.

Let X be a set and R be a binary relation on X . The uniformity induced by R yielding an upper, \cap -closed, strict uniform space structure is defined by the term

(Def. 21) $\langle X, \rho(R) \rangle$.

Now we state the propositions:

(48) Let us consider a set X , and a total, reflexive binary relation R on X . Then the uniformity induced by R satisfies axiom U1. The theorem is a consequence of (45).

(49) Let us consider a set X , and a symmetric binary relation R on X . Then the uniformity induced by R satisfies axiom U2. The theorem is a consequence of (46).

(50) Let us consider a set X , and a total, transitive binary relation R on X . Then the uniformity induced by R satisfies axiom U3. The theorem is a consequence of (47).

Let X be a set and R be a tolerance of X . Note that the uniformity induced by R yields a strict semi-uniform space. Now we state the proposition:

(51) Let us consider a set X , and an equivalence relation R of X . Then the uniformity induced by R is a uniform space.

Let X be a set and R be an equivalence relation of X . Observe that the uniformity induced by R yields a strict uniform space. Let X be a non empty set and R be a tolerance of X . Let us note that the uniformity induced by R is non empty and every non empty uniform space is topological.

Let Q be a non empty uniform space. The functor ${}^{\textcircled{Q}}$ yielding a topological, non empty uniform space structure satisfying axiom U1 is defined by the term

(Def. 22) Q .

Now we state the proposition:

- (52) Let us consider a non empty set X , and an equivalence relation R of X . Then the topological space induced by \textcircled{Q} (the uniformity induced by R) = the partition topology Classes R . The theorem is a consequence of (30) and (18).

9. UNIFORMITY INDUCED BY A TOLERANCE OR BY AN EQUIVALENCE

Now we state the proposition:

- (53) Let us consider an upper uniform space structure Q . Suppose \bigcap (the entourages Q) \in the entourages Q . Then there exists a binary relation R on the carrier of Q such that
- (i) \bigcap (the entourages Q) = R , and
 - (ii) the entourages $Q = \rho(R)$.

PROOF: Reconsider $R = \bigcap$ (the entourages Q) as a binary relation on the carrier of Q . $\rho(R) \subseteq$ the entourages Q . The entourages $Q \subseteq \rho(R)$ by [7, (3)]. \square

Let Q be a uniform space structure. The functor $\text{Uniformity2InternalRel}(Q)$ yielding a binary relation on the carrier of Q is defined by the term

(Def. 23) \bigcap (the entourages Q).

The functor $\text{UniformSpaceStr2RelStr}(Q)$ yielding a relational structure is defined by the term

(Def. 24) \langle the carrier of $Q, \text{Uniformity2InternalRel}(Q)\rangle$.

Let R_1 be a relational structure. The functor $\text{InternalRel2Uniformity}(R_1)$ yielding a family of subsets of (the carrier of R_1) \times (the carrier of R_1) is defined by the term

(Def. 25) $\{R, \text{ where } R \text{ is a binary relation on the carrier of } R_1 : \text{ the internal relation of } R_1 \subseteq R\}$.

The functor $\text{RelStr2UniformSpaceStr}(R_1)$ yielding a strict uniform space structure is defined by the term

(Def. 26) \langle the carrier of $R_1, \text{InternalRel2Uniformity}(R_1)\rangle$.

The functor $\text{InternalRel2Element}(R_1)$ yielding an element of the entourages $\text{RelStr2UniformSpaceStr}(R_1)$ is defined by the term

(Def. 27) the internal relation of R_1 .

Now we state the propositions:

- (54) Let us consider a binary relation R on X . Then $\bigcap \rho(R) = R$.

- (55) Let us consider a strict relational structure R_1 . Then $\text{UniformSpaceStr2-RelStr}(\text{RelStr2UniformSpaceStr}(R_1)) = R_1$. The theorem is a consequence of (54).
- (56) Let us consider a uniform space structure Q . Then
- (i) the carrier of $\text{RelStr2UniformSpaceStr}(\text{UniformSpaceStr2RelStr}(Q)) =$ the carrier of Q , and
 - (ii) the entourages $\text{RelStr2UniformSpaceStr}(\text{UniformSpaceStr2RelStr}(Q)) = \rho(\cap(\text{the entourages } Q))$.
- (57) Let us consider a family S_1 of subsets of $X \times X$, and a binary relation R on X . If $S_1 = \rho(R)$, then $S_1 \subseteq \rho(\cap S_1)$.
- (58) Let us consider an upper family S_1 of subsets of $X \times X$. If $\cap S_1 \in S_1$, then $\rho(\cap S_1) \subseteq S_1$.
- (59) Let us consider an upper family S_1 of subsets of $X \times X$, and a binary relation R on X . Suppose $R \in S_1$ and $S_1 = \rho(R)$ and $\cap S_1 \in S_1$. Then $\rho(\cap S_1) = S_1$.
- (60) Let us consider an upper uniform space structure Q . Suppose there exists a binary relation R on the carrier of Q such that the entourages $Q = \rho(R)$ and $\cap(\text{the entourages } Q) \in$ the entourages Q . Then the entourages $Q = \rho(\cap(\text{the entourages } Q))$. The theorem is a consequence of (57) and (58).
- (61) Let us consider an upper uniform space structure Q , and a binary relation R on the carrier of Q . Suppose the entourages $Q = \rho(R)$ and $\cap(\text{the entourages } Q) \in$ the entourages Q . Then the entourages $Q = \rho(\cap(\text{the entourages } Q))$.

Let us consider a tolerance R of X . Now we state the propositions:

- (62) (i) the uniformity induced by R is a semi-uniform space, and
- (ii) the entourages the uniformity induced by $R = \rho(R)$, and
 - (iii) $\cap(\text{the entourages the uniformity induced by } R) = R$.
- (63) $\text{RelStr2UniformSpaceStr}(\text{UniformSpaceStr2RelStr}(\text{the uniformity induced by } R)) =$ the uniformity induced by R . The theorem is a consequence of (54).
- (64) Let us consider an equivalence relation R of X . Then $\text{RelStr2UniformSpaceStr}(\text{UniformSpaceStr2RelStr}(\text{the uniformity induced by } R)) =$ the uniformity induced by R . The theorem is a consequence of (54).

10. UNIFORM PERVIN SPACE

Let X be a set, S_1 be a family of subsets of X , and A be an element of S_1 . The functor $\text{Block}(A)$ yielding a subset of $X \times X$ is defined by the term

(Def. 28) $(X \setminus A) \times (X \setminus A) \cup A \times A$.

From now on S_1 denotes a family of subsets of X and A denotes an element of S_1 .

Now we state the propositions:

(65) If $A = \emptyset$, then $\text{Block}(A) = X \times X$.

(66) Suppose X is not empty. Then $\text{Block}(A) = \{\langle x, y \rangle, \text{ where } x, y \text{ are elements of } X : x \in A \text{ iff } y \in A\}$.

PROOF: Set $S = \{\langle x, y \rangle, \text{ where } x, y \text{ are elements of } X : x \in A \text{ iff } y \in A\}$. $\text{Block}(A) \subseteq S$ by [3, (87)]. $S \subseteq \text{Block}(A)$ by [3, (87)]. \square

(67) (i) $\text{id}_X \subseteq \text{Block}(A)$, and

(ii) $\text{Block}(A) \cdot \text{Block}(A) \subseteq \text{Block}(A)$.

Let X be a set and S_1 be a family of subsets of X . The functor $\text{Blocks}(S_1)$ yielding a family of subsets of $X \times X$ is defined by the term

(Def. 29) the set of all $\text{Block}(A)$ where A is an element of S_1 .

Let us observe that $\text{Blocks}(S_1)$ is non empty.

The functor $\text{FMCBLOCKS}(S_1)$ yielding a family of subsets of $X \times X$ is defined by the term

(Def. 30) $\text{FinMeetCl}(\text{Blocks}(S_1))$.

Now we state the propositions:

(68) $\text{FMCBLOCKS}(S_1)$ is \cap -closed.

(69) $\text{FMCBLOCKS}(S_1)$ is quasi-basis. The theorem is a consequence of (68).

(70) $\text{FMCBLOCKS}(S_1)$ satisfies axiom UP1.

(71) Let us consider an element A of S_1 , and a binary relation R on X . If $R = \text{Block}(A)$, then $R^\sim = \text{Block}(A)$. The theorem is a consequence of (65) and (4).

(72) Let us consider a binary relation R on X . Suppose R is an element of $\text{Blocks}(S_1)$. Then R^\sim is an element of $\text{Blocks}(S_1)$. The theorem is a consequence of (71).

Let us consider a non empty family Y of subsets of $X \times X$. Now we state the propositions:

(73) If $Y \subseteq \text{Blocks}(S_1)$, then $Y[\sim] = Y$. The theorem is a consequence of (71).

- (74) If $Y \subseteq \text{Blocks}(S_1)$, then $(\bigcap Y)^\smile = \bigcap Y [\sim]$. The theorem is a consequence of (73) and (71).
- (75) If $Y \subseteq \text{Blocks}(S_1)$, then $\bigcap Y = (\bigcap Y)^\smile$. The theorem is a consequence of (73) and (74).
- (76) $\text{FMCBlocks}(S_1)$ satisfies axiom UP2. The theorem is a consequence of (73) and (75).
- (77) $\text{FMCBlocks}(S_1)$ satisfies axiom UP3. The theorem is a consequence of (67).

Let X be a set and S_1 be a family of subsets of X . The Pervin uniform space of S_1 yielding a strict uniform space is defined by the term

(Def. 31) $\langle X, [\text{FMCBlocks}(S_1)] \rangle$.

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