Cent. Eur. J. Geosci. • 3(3) • 2011 • 231-234
DOI: 10.2478/s13533-011-0023-5

VERSITA

Central European Journal of **Geosciences**

A Commentary on the Shape of Loess Particles assuming a Spatial Exponential Distribution for the Cracks in Quartz

Commentary

John James Howarth*

Giotto Loess Research Group, 55 Eastry Road, Erith, Kent DA8 1NW UK

Received 21 April 2011; accepted 21 July 2011

Abstract: An important property of loess is a tendency to collapse on loading and wetting (hydroconsolidation) which can have serious consequences worldwide for civil engineering projects. Randomly generated particles are classified according to Zingg shape categories: disc, sphere, blade and rod. This paper differs from the previous by the same author [8] in that a uniform distribution is no longer assumed for the underlying spatial distribution. Randomly placed faults in the quartz mother-rock lead naturally to an exponential distribution for the linear dimension of the basic particle. Monte Carlo processes and analytical formulae are used to calculate the average dimensions for particles in the blade category, into which most loess has been shown to fall.

Keywords: loess particles • exponential distribution probability • Monte Carlo • Zingg shape • blade • hydroconsolidation • comminution • aeolean abrasion • aeolean transport • Moss defects

1. Introduction

Classical loess particles [1, 2] are the wind-borne product of comminuted quartz, having a mode size of approximately 30 μm. For loess to occur, there must be a source of particles [3] (usually a desert or sub-desert existing, or having once existed, nearby); a prevailing wind; a mechanism for deposition and sedimentation. Loess is found in most parts of the world, in particular China and central Asia. Smaller deposits occur in Europe, North America, New Zealand and Libya. An overview of its formation and geographical properties is given by Smalley [4]. In the right climatic conditions loess deposits give rise to a friable and fertile soil, often supporting high population densities. An important property of loess is a tendency to collapse on loading and wetting (hydroconsolidation) [5] which has serious consequences worldwide for civil engineering projects. In order to explain and predict such properties one needs to know the general shape of the basic particle, so that, for example, wet and dry packing densities can be derived. In 1993 Rogers and Smalley [6] proposed a model for the distribution of the linear dimensions of Loess particles. Using a modified Zingg [7] classification, they demonstrated using a Monte Carlo process that approximately 72% of particles could be expected to be tabular (blade shaped), 1% would be approximately spherical and the remaining 27% would be either disc or rod shaped. Howarth [8] recomputed these proportions using Monte Carlo techniques and confirmed

*E-mail: johnhowart@aol.com

Table 1. Theoretical Proportions Pi of Particle lying in each shape Category.

r	P_1 (class Im disc)	P_2 (class IIm sphere)	P_3 (class IIIm blade)	P_4 (class IVm rod)
0.05	0.128375	0.01425	0.729	0.128375

the proportions analytically. He mentioned the possibility of drawing the fragment dimensions from a non-uniform distribution, tacitly inviting comments on whether such a distribution would be productive. Accordingly this paper will develop the Monte Carlo process for the exponential distribution of particle dimension, again confirming the results analytically.

If L_1, L_2 and L_3 are the lengths of the sides of the hypothetical Zingg box [7] that encloses the particle, the 4 Zingg shape categories were:

class Im $L_1 = L_2 > L_3$ disc
class IIm $L_1 = L_2 = L_3$ sphere
class IIIm $L_1 > L_2 > L_3$ blade
class IVm $L_1 = L_2 < L_3$ rod.

In reality any 2 dimensions would hardly ever be exactly equal so, to be meaningful, the equalities and inequalities in the simplified definitions above must be subject to numerical tolerances. In [6] a "definable accuracy" of 10% on any length was employed. Here a parameter, r, will represent the maximum difference between any 2 dimensions if they are to be deemed equal. The total range of values for each linear dimension is scaled to lie in the range [0,1] and r will be a small proportion of this total range – typically 0.1 or 0.05 (not a percentage, but representable as a percentage by multiplying it by 100).

With r defined thus, the Zingg categories can be expressed more formally as follows:

if the sides of the box, arranged in increasing order of size, are $a \le b \le c$,

class Im $a \le b - r$, $b > c - r$ disc
class IIm $a > b - r$, $b > c - r$ sphere
class IIIm $a \le b - r$, $b \le c - r$ blade
class IVm $a > b - r$, $b \le c - r$ rod

With a class IIm particle (sphere) it is quite possible for c to exceed a by more than r (but not more than 2r), provided b is within r of both a and c. Apart from this, the definitions are intuitive. It can readily be seen that for any values whatsoever of a, b and c, one and only one of the 4 categories will apply.

Choosing the value $r = 0.05$, after Rogers and Smalley [6], Howarth [8] has shown that the theoretical proportions of the particles lying in each category were:

where generally, for $r < 0.5$, $P_1 = P_4 = r\,(7r^2 - 9r - 3)$, $P_2 = 6r^2(1 - r)$, $P_3 = (1 - 2r)^3$ [8].

2. The Selection of a non-uniform spatial Distribution

Up to this point, the dimensions a, b and c may be thought of as values drawn from an arbitrary cumulative probability distribution function – thus r could simply refer to a difference in cumulative probability. From here on, however, we will be dealing in actual dimensions, so the explicit assumption of a spatial probability distribution becomes necessary. The uniform distribution is the easiest and perhaps the most natural one. It does have the obvious limitation that the dimension cannot be greater than a preset value. This problem can be ameliorated by assuming that the linear dimension of the loess particle is taken from an unbounded distribution, but one with a finite mean value. For these reasons the exponential distribution is worthy of consideration, as it is relevant to models that seek to describe the separation between random events, spatial or temporal: for example telephone calls arriving at the exchange. The probability density function is usually expressed as a function of parameter s (time or position), where $f(s) = m^{-1}e^{-s/m}$ and m is the mean separation between the random events and 1/m is the expected number of events per unit of s, that is to say, the mean separation between 2 events will be m. It has the crucial property that the chance of an event taking place in any interval is completely independent of events that may have occurred in any other interval: such a distribution is hence often referred to as 'memoryless'. It can easily be shown that the chance of 2 such random events being more than km apart is e^{-k}. Thus the chance of their being less than m apart is about 63%, between m and 2m apart is about 23%, between 2m and 3m apart is about 9% and more than 3m apart is about 5%.

The exponential distribution can reasonably be applied spatially to predict the spacing between imperfections in the crystal from which the loess is believed to originate, if each imperfection occurs randomly, and independently of the presence of other imperfections. In the model to be detailed here it is assumed that the original crystal is quartz, but a similar analysis could be applied (but with different parameters) to feldspar and other crystalline materials which also have random imperfections Moss [9] postulated defects (now called Moss defects) in quartz sand grains due to randomly occurring imperfections in the original crystalline structure. These can be fatigue sites or inclusions of foreign material. Whilst the initial ablation of the crystal gives rise to coarse sandy material of diameter 100 -1000 μm, secondary aeolean abrasion can mobilise the internal crystalline (Moss) defects giving a comminuted diameter of 20 - 60 μm (Kumar et al [10]). Subsequent aeolean abrasion gives diameters in the range 20 - 30 μm

(Jefferson et al [11]) which is the mean dimension for loessic deposits (Smalley [3] and Smalley et al. [12]). Moss [9] also suggests that the larger particles can be more susceptible to breakage than the smaller ones which is consistent with the reduction of the above upper limits from 60 to 30 µm. If the sole trigger for the splitting of a larger crystal during comminution is the presence of such an imperfection then the linear dimension of the Loess particle will be exponentially distributed. This argument is, of course, simplistic in that it neglects preferential aeolean transport of smaller particles, and preferential comminution of larger particles due to the greater sheer forces they may experience. Nevertheless, the assumption of an exponential distribution will be more justifiable than that of the uniform distribution.

3. Average Dimensions for a Blade Shaped (tabular) Particle

The parameter m becomes the mean separation between adjacent imperfections and, following Smalley [3] who gives a mode size of 30 µm for loessic material, it may be tentatively equated to this same value. The mean dimensions for exponentially distributed particles can be calculated by integration, similarly to the method used in [8], except that the linear dimension must now be weighted according to the cumulative exponential distribution, thus we now have:

A_{exp} = average smallest dimension a of tabular particle =

$$-\frac{6m}{P_3}\left\{\int_{2r}^{1}\int_{r}^{c-r}\int_{0}^{b-r}\log_e(1-a)\,da\,db\,dc\right\} \quad (1)$$

B_{exp} = average middle dimension b of tabular particle =

$$-\frac{6m}{P_3}\left\{\int_{2r}^{1}\int_{r}^{c-r}\int_{0}^{b-r}da\,\log_e(1-b)\,db\,dc\right\} \quad (2)$$

C_{exp} = average greatest dimension c of tabular particle =

$$-\frac{6m}{P_3}\left\{\int_{2r}^{1}\int_{r}^{c-r}\int_{0}^{b-r}da\,db\,\log_e(1-c)\,dc\right\} \quad (3)$$

where $P_3 = (1-2r)^3$ and m = mean separation between successive supposed imperfections.

It was asserted in [8] that Monte Carlo modelling would probably be the best approach for non-uniform distribution. Hence, choosing values r= 0.05 and 0.1, Monte Carlo processes were run for 1,000,000 replications. With each replication, 3 random numbers were first chosen from a uniform distribution to represent the cumulative probability for the 3 particle axes, then those particles that did not satisfy the class IIIm (blade) criteria were rejected. The successful replications were weighted exponentially with m=30µm, then averaged to give Table 2.

Table 2. Average Dimensions for a class IIIm Particle, m=30µm (Monte Carlo).

r	A_3 (average smallest dimension in class IIIm)	B_2 (average middle dimension in class IIIm)	C_3 (average largest dimension in class IIIm)
0.0	10.01µm	25.05µm	55.02µm
0.05	8.61µm	23.87µm	58.21µm
0.1	7.37µm	23.07µm	61.62µm

Table 3. Average Dimensions for a class IIIm Particle, m=30 µm (analytical).

r	A_3 (average smallest dimension in class IIIm)	B_2 (average middle dimension in class IIIm)	C_3 (average largest dimension in class IIIm)
tending to 0.0	10.00 µm	25.00 µm	55.00 µm
0.05	8.61 µm	23.87 µm	58.16 µm
0.1	7.37 µm	23.07 µm	61.69 µm

However, extensive, but elementary, manipulation of the above equations (1)-(3) yields the following closed forms (4)-(6):

$$A_{exp} = m(1-2r)^{-3}\{8r^3\log_e(2r)+(1-2r)(1-7r-22r^2)/3\} \quad (4)$$

$$B_{exp} = m(1-2r)^{-3}\{r^2(4r-3)\log_e(r) - (1-r)^2(1-4r)\log_e(1-r)+(1-2r)(5-32r+32r^2)/6\} \quad (5)$$

$$C_{exp} = m\{-\log_e(1-2r)+11/6\} \quad (6)$$

Application of these formulae yields the results in Table 3, which, it will be seen, show excellent agreement with the earlier Monte Carlo results shown in Table 2.

As shown in Tables 2 and 3, a particularly interesting case occurs when r tends to zero. This effectively makes all particles tabular, since in the limiting case dimensions can only be deemed equal if they are numerically identical, a situation which can only happen with zero probability.

The formulae then become A_{exp} = m/3, B_{exp} = 5m/6, C_{exp} = 11m/6, which yields a ratio of precisely 2:5:11. The existence of these simple integer ratios, independent of the choice of parameter m, was quite unexpected. The lower ratio is the same as for the integral ratios 2:5:8 postulated by Rogers and Smalley [6], and this analysis has fortuitously provided a parameter-free way of deriving it. As pointed out by Smalley [3] blade-shaped particles of mode size 30 μm and relative dimensions in this ratio are likely to give rise to a soil of a very open structure, subject to precisely the hydroconsolidation properties mentioned earlier. As discussed previously, the long dimension is likely to be the most susceptible to fracture by further comminution or aeolean abrasion, ultimately giving fragments close to the 2:5:8 ratio. The average smallest and largest dimensions will now be 10 and 40 μm respectively, giving an overall diameter that comfortably bounds the 20 - 30 μm values asserted by Jefferson et al [11]. It might even be argued that the predominance of 2:5:8 particles, rather than the 2:5:11 that would be consistent with a purely exponential distribution of cracks, implies the existence of a secondary fracturing mechanism.

4. Summary

As well as the natural Monte Carlo approach, it has proven perfectly feasible to derive analytical forms for the average dimensions of blade shaped particles, assuming an exponential distribution for each dimension of the particle. The ratio of the smallest to the middle dimensions becomes close to the Zingg Box ratio of 2:5 as r becomes close to zero, tending to precisely 2:5 as r tends to zero. Because the exponential distribution is theoretically unbounded, it is to be expected that the greatest dimension will be larger when an exponential, as opposed to a uniform, distribution is assumed. As considered earlier, in reality, the greatest dimension will probably be limited by what can be borne by the wind, and what particles can escape further comminution because of their small size. These processes can only mean that the ratio of the middle to the largest dimension will move in the direction of 5:8, the higher Zingg Box ratio, which shape is especially susceptible to hydroconsolidation, with the implications alluded to earlier for civil engineering projects.

References

[1] Scott C., Smalley I.J., The original shapes of quartz sand grains, Area, 1991, 23, 353-355

[2] Smalley I.J., Possible formation mechanisms for the modal coarse-silt quartz particles in loess deposits, Quatern. Int., 1990, 7/8, 23-28

[3] Smalley I.J., Formation of quartz sand, Nature, 1966, 211, 476-479

[4] Smalley I., Making the material: the formation of silt-sized primary mineral particles for loess deposits, Quarternary Sci. Rev., 1995, 14, 645-651

[5] Dijkstra T.A., Smalley I.J., Rogers C.D.F., Particle packing in loess deposits and the problem of structure collapse and hydroconsolidation, Eng. Geol., 1995, 40, 49-64

[6] Rogers C.D.F., Smalley I.J., The shape of loess particles, Naturwissenschaften, 1993, 80, 461-462

[7] Smalley I.J., The expected shapes of blocks and grains, J. Sediment. Petrol., 1966, 36, 626-629

[8] Howarth J. J., The shape of loess particles reviewed, Cent. Eur. J. Geosci., 2010, 2, 41-44

[9] Moss A.J., Fatigue effects in quartz sand grains, Sediment. Geol., 1973, 10, 239-247

[10] Kumar R., Jefferson I.F., O'Hara-Dhand K., Smalley I.J., Controls on quartz silt formation by crystalline defects, Naturwissenschaften, 2006, 93, 185-188

[11] Jefferson I.F., Jefferson B.Q., Assallay A.M., Rogers C.D.F., Smalley I.J., Crushing of quartz sand to produce silt particles, Naturwissenschaften, 1997, 84, 148-149

[12] Smalley I.J., Kumar R., O'Hara-Dhand K., Jefferson I.F., Evans R.D., The formation of silt material for terrestrial sediments: Particularly loess and dust, Sediment. Geol., 2005, 179, 321-328