

# NUMERICAL STUDY ON FLOW PAST AN OSCILLATING PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION UNDER CHEMICAL REACTION

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An numerical study on an unsteady flow past an oscillating semi-infinite vertical plate with variable temperature and uniform mass diffusion under the influence of a first order chemical reaction has been carried out. The dimensionless governing equations are solved by an unconditionally stable and fast converging implicit finite difference scheme. The effects of velocity and temperature for different physical parameters such as the chemical reaction parameter, thermal Grashof number, mass Grashof number and time are analysed. It is observed that due to the presence of first order chemical reaction, the velocity increases during a generative reaction and decreases in a destructive reaction.

**Key words:** chemical reaction, oscillating vertical plate, heat transfer, mass diffusion.

## 1. Introduction

The effect of a chemical reaction depends on whether the reaction is homogenous or heterogeneous. This depends on whether the reaction occurs at an interface or as a single phase volume reaction. In well mixed systems, the reaction is heterogeneous if it takes place at an interface and homogenous if it takes place in the solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. In many chemical engineering processes, the chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware and food processing. Oscillating plates are used as heat exchangers in cooling systems.

Apelblat (1980) analysed the effects of mass transfer coupled with an irreversible chemical reaction of the first order in plug flow, in Couette flow, in Couette flow with a moving interface and in fully developed boundary layer flow analytically. A first order chemical reaction in the neighbourhood of a horizontal plate was studied by Chambre and Young (1958). Transient laminar free convection along a vertical, isothermal flat plate arising from buoyancy forces created by both temperature and concentration gradients was analysed by Callahan and Marner (1976). Cai and Zhang (2003) studied the heat and mass transfer effects of an infinite vertical plate. Some new algebraic explicit analytical solutions were derived without special functions for the linear and nonlinear unsteady one-dimensional partial differential equation set describing the drying process of an infinite plate. Das *et al.* (1994) presented the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Mass transfer effects on a moving isothermal vertical plate in the presence of a chemical reaction were studied by Das *et al.* (1999).

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Gebhart and Pera (1971) presented similarity solutions for steady laminar flows adjacent to vertical surfaces and in plumes. An interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and chemical species were considered. The coupled non linear partial differential equations were solved numerically by an explicit finite-difference scheme. The effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate in the presence of constant heat flux were studied by Gokhale *et al.* (2003). Hossain *et al.* (1998) considered an unsteady free convection flow of a viscous incompressible fluid along a semi-infinite vertical plate when the surface heat flux of the plate oscillated with a small amplitude about a mean flux which itself varied as the power of  $n$  of the distance from the leading edge. The heat transfer arising due to the movement of a continuous moving plate in manufacturing processes such as hot rolling and extrusion was studied by Karwe and Jaluria (1988). Mass transfer effects on an isothermal vertical oscillating plate in the presence of a chemical reaction were solved analytically by Muthucumarasamy *et al.* (2008).

The response of a laminar boundary layer flow past a semi-infinite vertical plate to harmonic oscillations in the plate temperature, in the form of a traveling wave convected in the direction of the free-stream was studied by Pop *et al.* (1982). Temperature distributions, when an infinite thin plate is exposed to the environment were determined by Rama Murthy and Sairam (1997). The flow of an incompressible, viscous fluid past an infinite vertical plate, either impulsively started in its own plane in the vertically upward direction with oscillating plate temperature or oscillating harmonically in its own plane with an oscillating plate temperature was considered by Revankar (2000).

Soundalgekar (1977) presented an exact solution of Stokes' problem for the flow of a viscous incompressible fluid past an impulsively started infinite isothermal vertical plate in its own plane using the Laplace transform technique. The fluid considered in this study was pure air and water. An exact analysis of Stokes' problem for an infinite vertical plate with variable temperature was made by Soundalgekar and Patil (1978). The temperature of the plate varied linearly with time. Soundalgekar (1979) presented an exact solution of the flow past an infinite isothermal vertical plate with mass transfer by taking into account the free convection current. It was observed that velocity and skin-friction increased due to a greater cooling of the plate and decreased due to a greater heating of the plate in the presence of foreign mass.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar (1979). Soundalgekar and Akolkar (1983) derived an exact solution of flow caused by an oscillating vertical plate in the presence of free-convection currents and foreign mass. It was found that the tendency of instability at  $\omega t = \pi/2$  was more when the Schmidt number was less than one and the Grashof number larger than zero.

The numerical solutions for the induced flow field and temperature field for the material emerging from the die or rollers were obtained. Soundalgekar *et al.* (1994) derived an exact solution for the effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux. Again, an exact solution to the flow of a viscous incompressible fluid past an infinite vertical oscillating plate, in the presence of foreign mass was derived by Soundalgekar *et al.* (1995).

It was inferred that in the presence of viscous dissipative heat, more time was required to reach the steady state temperature than that in the absence of viscous dissipative heat for a large value of buoyancy parameter. Zhang *et al.* (2004) studied the effects of laminar natural convection on a periodically oscillating vertical flat plate heated at a uniform temperature. The governing equations were solved by the finite-volume method. It was observed that the heat transfer performance of an oscillating plate depended significantly on the dimensionless oscillation velocity. The dimensionless governing equations were solved by the Laplace-transform technique and the solutions are valid only at lower time level.

Numerical work on a transient flow past an oscillating semi-infinite vertical plate under the combined buoyancy effects of heat and mass diffusion in the presence of a chemical reaction has not received attention of any researcher. Hence, the present study is to investigate the flow past an oscillating semi-infinite vertical plate with variable temperature and uniform mass under a homogeneous first order chemical reaction by an implicit finite-difference scheme of Crank-Nicolson type.

## 2. Formulation of the problem

Consider a laminar, unsteady natural convection flow of a viscous incompressible fluid past an oscillating semi-infinite vertical plate which is at rest and surrounds the plate with temperature  $T'_\infty$  and concentration  $C'_\infty$ . It is assumed that there is a first order chemical reaction between the diffusing species and the fluid. Here, the  $x$ -axis is taken along the plate in the vertically upward direction and the  $y$ -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time  $t' > 0$ , the plate starts oscillating in its own plane with frequency  $\omega'$  and the temperature of the plate and the concentration level are also raised linearly with time. Then by Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$\rho C_p \left( \frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} \right) = \frac{\partial^2 T'}{\partial y^2}, \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} - K_l (C' - C'_\infty). \tag{2.4}$$

The initial and boundary conditions are

$$\begin{aligned} t' \leq 0 \quad u = 0, \quad v = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \\ t' > 0: \quad u = u_0 \cos \omega' t', \quad v = 0, \quad T' = T'_\infty + (T'_w - T'_\infty)x^n, \quad C' = C'_w \quad \text{at} \quad y = 0, \\ u = 0, \quad v = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{at} \quad x = 0, \\ u \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as} \quad y = \infty. \end{aligned} \tag{2.5}$$

On introducing the following non-dimensional quantities

$$\begin{aligned} X = \frac{xu_0}{\nu}, \quad Y = \frac{yu_0}{\nu}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad \omega = \frac{\omega' \nu}{u_0^2}, \\ t = \frac{t'u_0^2}{\nu}, \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{u_0^3}, \quad Pr = \frac{\nu}{\alpha}, \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gc = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{\nu K_l}{u_0^2}. \end{aligned} \tag{2.6}$$

Equations (2.1) to (2.4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (2.7)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \text{Gr } T + \text{Gc } C + \frac{\partial^2 U}{\partial Y^2}, \quad (2.8)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\text{Pr}} \frac{\partial^2 T}{\partial Y^2}, \quad (2.9)$$

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial Y^2} - KC. \quad (2.10)$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} t \leq 0: \quad U &= 0, \quad V = 0, \quad T = 0, \quad C = 0, \\ t > 0: \quad U &= \cos \omega t, \quad V = 0, \quad T = X^n, \quad C = 1 \quad \text{at} \quad Y = 0, \\ U &= 0, \quad T = 0, \quad C = 0, \quad \text{at} \quad X = 0, \\ U &\rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0, \quad \text{as} \quad Y \rightarrow \infty. \end{aligned} \quad (2.11)$$

A reaction is said to be of the order  $n$ , if the reaction rate is proportional to the  $n^{\text{th}}$  power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself. Such a study is found useful in chemical processing industries such as fibre drawing, crystal pulling from the melt and polymer production.

### 3. Mathematical solution by numerical technique

In order to solve these unsteady, non-linear coupled Eqs (2.7) to (2.10) under the conditions Eqs (2.11), an implicit finite difference scheme of Crank-Nicolson type has been employed. The finite difference equations corresponding to Eqs (2.7) to (2.10) are as follows

$$\begin{aligned} & \frac{[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^n - U_{i-1,j-1}^n]}{4\Delta X} + \\ & + \frac{[V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^n - V_{i,j-1}^n]}{2\Delta Y} = 0, \end{aligned} \quad (3.1)$$

$$\begin{aligned} & \frac{[U_{i,j}^{n+1} - U_{i,j}^n]}{\Delta t} + U_{i,j}^n \frac{[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n]}{2\Delta X} + \\ & + V_{i,j}^n \frac{[U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n]}{4\Delta Y} = \frac{\text{Gr}}{2} [T_{i,j}^{n+1} + T_{i,j}^n] + \frac{\text{Gc}}{2} [C_{i,j}^{n+1} + C_{i,j}^n] + \\ & + \frac{[U_{i,j-1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j+1}^{n+1} + U_{i,j-1}^n - 2U_{i,j}^n + U_{i,j+1}^n]}{2(\Delta Y)^2}, \end{aligned} \quad (3.2)$$

$$\begin{aligned} & \frac{[T_{i,j}^{n+1} - T_{i,j}^n]}{\Delta t} + U_{i,j}^n \frac{[T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^n - T_{i-1,j}^n]}{2\Delta X} + \\ & + V_{i,j}^n \frac{[T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j+1}^n - T_{i,j-1}^n]}{4\Delta Y} = \frac{I}{Pr} \frac{[T_{i,j-1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j+1}^{n+1} + T_{i,j-1}^n - 2T_{i,j}^n + T_{i,j+1}^n]}{2(\Delta Y)^2}, \end{aligned} \tag{3.3}$$

$$\begin{aligned} & \frac{[C_{i,j}^{n+1} - C_{i,j}^n]}{\Delta t} + U_{i,j}^n \frac{[C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^n - C_{i-1,j}^n]}{2\Delta X} + V_{i,j}^n \frac{[C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^n - C_{i,j-1}^n]}{4\Delta Y} = \\ & = \frac{I}{Sc} \frac{[C_{i,j-1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j+1}^{n+1} + C_{i,j-1}^n - 2C_{i,j}^n + C_{i,j+1}^n]}{2(\Delta Y)^2} - \frac{K}{2} (C_{i,j}^{n+1} + C_{i,j}^n). \end{aligned} \tag{3.4}$$

Here the region of integration is considered as a rectangle with sides  $X_{max}(=1)$  and  $Y_{max}(=3)$ , where  $Y_{max}$  corresponds to  $Y = \infty$  which lies very well outside both the momentum and energy boundary layers. The maximum of  $Y$  was chosen as 3 after some preliminary investigations so that the last two of the boundary conditions Eqs (2.11) are satisfied within the tolerance limit  $10^{-5}$ .

After experimenting with a few sets of mesh sizes, the mesh sizes have been fixed at the level  $\Delta X = 0.05$ ,  $\Delta Y = 0.25$  with the time step  $\Delta t = 0.01$ . In this case, the spatial mesh sizes are reduced by 50% in one direction, and later in both directions, and the results are compared. It is observed that when the mesh size is reduced by 50% in the  $Y$ -direction, the results differ in the fifth decimal place while the mesh sizes are reduced by 50% in the  $X$ -direction or in both directions, the results are comparable to three decimal places.

Hence, the above mesh sizes have been considered as appropriate for calculation. The coefficients  $U_{i,j}^n$  and  $V_{i,j}^n$  appearing in the finite-difference equations are treated as constants in any one time step. Here  $i$ -designates the grid point along the  $X$ -direction,  $j$  along the  $Y$ -direction and  $k$  to the  $t$ -time. The values of  $U, V$  and  $T$  are known at all grid points at  $t = 0$  from the initial conditions.

The computations of  $U, V, T$  and  $C$  at time level  $(n+1)$  using the values at previous time level  $(n)$  are carried out as follows. The finite difference Eq.(3.4) at every internal nodal point on a particular  $i$ -level constitutes a tridiagonal system of equations. Such a system of equations is solved by using Thomas algorithm as discussed in Carnahan *et al.* (1969). Thus, the values of  $C$  are found at every nodal point for a particular  $i$  at  $(n+1)^{th}$  time level. Similarly, the values of  $T$  are calculated from Eq.(3.3). Using the values of  $C$  and  $T$  at  $(n+1)^{th}$  time level in Eq.(3.2), the values of  $U$  at  $(n+1)^{th}$  time level are found in a similar manner. Thus, the values of  $C, T$  and  $U$  are known on a particular  $i$ -level. Finally, the values of  $V$  are calculated explicitly using Eq.(3.1) at every nodal point on a particular  $i$ -level at  $(n+1)^{th}$  time level. This process is repeated for various  $i$ -levels. Thus the values of  $C, T, U$  and  $V$  are known at all grid points in the rectangular region at  $(n+1)^{th}$  time level.

In a similar manner computations are carried out by moving along the  $i$ -direction. After computing the values corresponding to each  $i$  at a time level, the values at the next time level are determined in a similar manner. Computations are repeated until the steady-state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of  $U$ , as well as temperature  $T$  and concentration  $C$  at two consecutive time steps is less than  $10^{-5}$  at all grid points.

The finite difference scheme is unconditionally stable. The local truncation error is  $O(\Delta t^2 + \Delta Y^2 + \Delta X)$  and it tends to zero as  $\Delta t, \Delta X$  and  $\Delta Y$  tend to zero. Hence the scheme is compatible. Stability and compatibility ensure convergence.

### 4. Results and discussion

Numerical results for the variable temperature and uniform mass diffusion will be discussed in this section. The velocity field is analysed for variations in the chemical reaction parameter, phase angle, thermal and mass Grashof number in Figs 1-4. The mass diffusion Eq.(2.10) can be adjusted to meet the one takes (i)  $K > 0$  for the destructive reaction, (ii)  $K < 0$  for the generative reaction and (iii)  $K = 0$  for no reaction.

In order to ascertain the accuracy of the numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for  $K = 0.2$ ,  $(Sc = 0.16, 0.2)$ ,  $(Gr = 2, 5)$ ,  $(Gc = 2, 5)$  and  $Pr = 0.71$  (corresponding to  $\eta = \frac{Y}{2\sqrt{t}}$ ) are compared with the available exact solution of Das *et al.* (1999) at  $t = 0.2$  in Fig.1 and they are found to be in good agreement.

The velocity profiles for different phase angles ( $\omega t = \pi/6, \pi/4, \pi/3, \pi/2$ ),  $Gr=5$ ,  $Gc=5$ ,  $Sc=0.6$ ,  $Pr=0.71$  and  $t=0.2$  are shown in Fig.2. It is observed that the velocity increases with decreasing phase angle. This shows that velocity decreases in the presence of high oscillation.

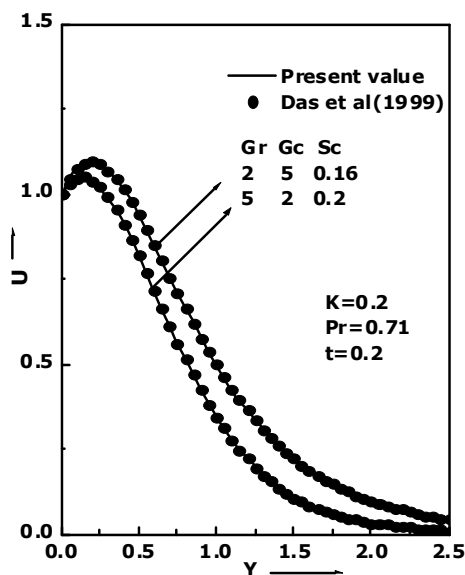


Fig.1. Comparison of velocity profiles.

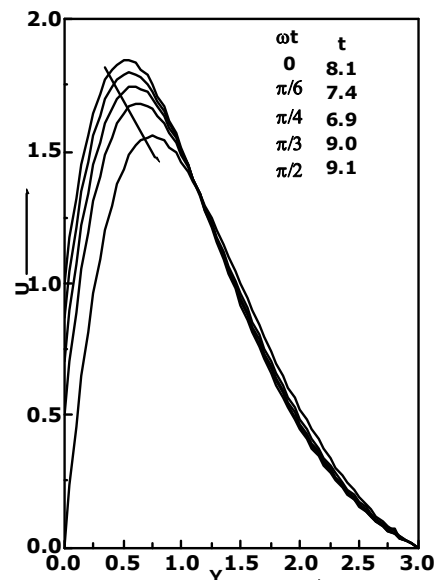


Fig.2. Steady state velocity profiles for different  $\omega t$ .

The velocity for different values of the chemical reaction parameter ( $K=-5, -2, 0, 0.2, 2$ ),  $\omega t = \pi/6$ ,  $Gr=5$ ,  $Gc=5$ ,  $Pr=0.71$ ,  $Sc=0.6$  and  $Pr=0.71$  is shown in Fig.3. It is observed that the velocity increases with a decreasing chemical reaction parameter. This shows that the velocity increases during the generative reaction and decreases in the destructive reaction. This shows that the contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly. The steady state is reached slowly during the generative reaction.

The steady state velocity profiles for different values of the thermal Grashof number, mass Grashof number are shown in Fig.4. It is observed that the velocity increases with an increasing thermal Grashof number or mass Grashof number. As the thermal Grashof number or mass Grashof number increases, the

buoyancy effect becomes more significant, it implies that more fluid is entrained from the free stream due to the strong buoyancy effects as  $Gr$  or  $Gc$  increases. Greater cooling of the plate enhances the velocity.

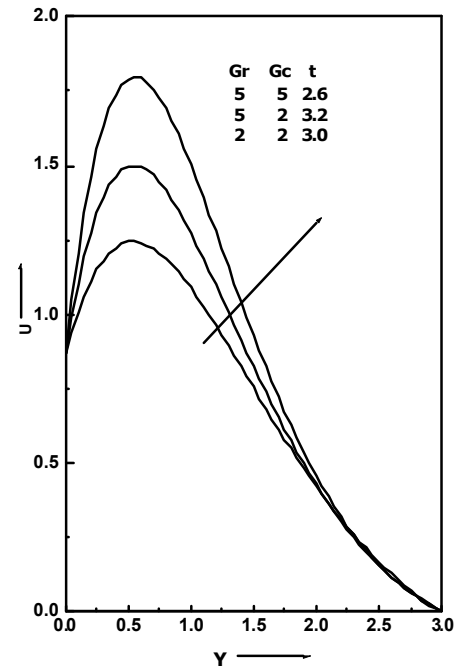
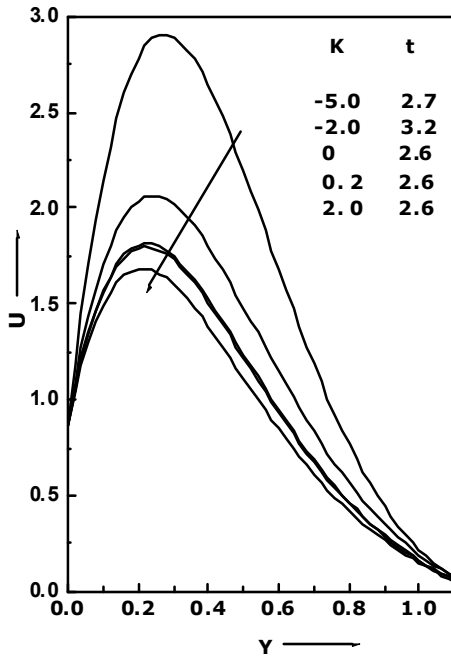


Fig.3. Steady state velocity profiles for different  $K$ . Fig.4. Steady state velocity profiles at  $X=1.0$  for different  $Gr$ ,  $Gc$ .

Figure 5 presents the temperature profiles for different values of the Prandtl number. The effect of the Prandtl number plays a significant role in the temperature field. It is observed that there is a fall in temperature with increasing the Prandtl number and  $n$ . Physically, it indicates that when the Prandtl number is small heat diffuses away from the plate more rapidly than for higher values of the Prandtl number.

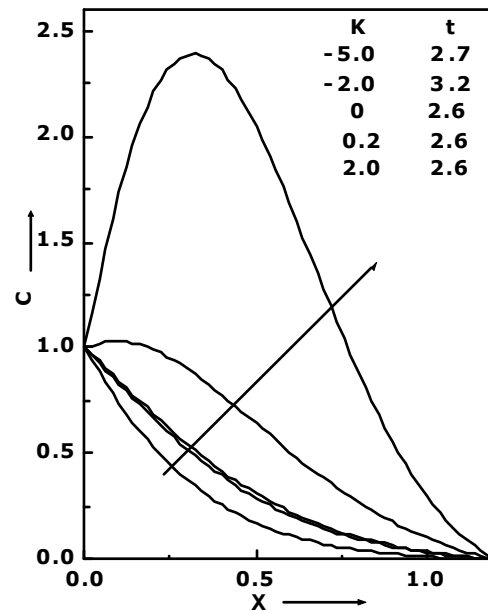
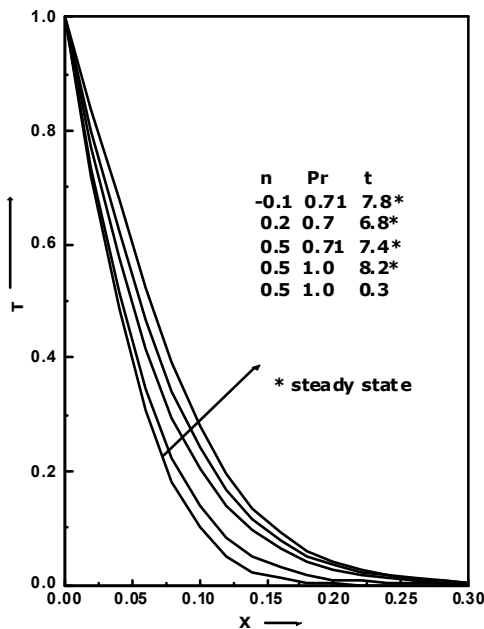


Fig.5. Transient temperature profiles at  $x=1.0$  for different  $n$  and  $Pr$ .

Fig.6. Concentration profiles at  $X=1.0$  for different  $K$ .

The concentration profiles for different values of the chemical reaction parameter ( $K=-5,-2,0,0.2,2$ ,  $Sc=0.6$ ) are exhibited in Fig.6. The wall concentration increases during the generative reaction and decreases during the destructive reaction. It is observed that there is a fall in concentration due to increasing the values of the chemical reaction parameter.

The skin-friction, the rate of heat transfer and the rate of concentration in their transient and steady-state conditions are studied. The dimensionless local as well as average values of the skin-friction, the Nusselt number and the Sherwood number are given by the following expressions

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0}, \quad (4.1)$$

$$\bar{\tau} = -\int_0^l \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX, \quad (4.2)$$

$$\text{Nu}_X = -X \left[ \frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}} \right], \quad (4.3)$$

$$\bar{\text{Nu}} = -\int_0^l \left[ \frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}} \right] dX, \quad (4.4)$$

$$\text{Sh}_X = -X \left(\frac{\partial C}{\partial Y}\right)_{Y=0}, \quad (4.5)$$

$$\bar{\text{Sh}} = -\int_0^l \left(\frac{\partial C}{\partial Y}\right)_{Y=0} dX. \quad (4.6)$$

The derivatives involved in Eqs (3.5) to (4.6) are evaluated using a five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula. The local skin-friction values are evaluated from Eq.(4.1) and plotted in Fig.7 as a function of the axial coordinate  $X$ . The local skin-friction decreases as  $X$  increases. The local wall shear stress increases with decreasing values of phase angles  $\omega t$ . The local Nusselt number for different values of the Prandtl number and  $n$  is shown in Fig.8. The rate of heat transfer decreases with increasing the Prandtl number and decreasing  $n$ .

The local Sherwood number for different values of the chemical reaction parameter is shown in Fig.9. The local Sherwood number increases with increasing the chemical reaction parameter.

The average values of the skin-friction, Nusselt number and Sherwood number are shown in Figs 10, 11, 12 and 13 respectively. The effects of the phase angles  $\omega t$  on the average values of the skin-friction are shown in Fig.10. The average skin-friction decreases with increasing phase angles  $\omega t$ . The effects of the phase angles  $\omega t$ , Prandtl number and  $n$  on the average values of the Nusselt number are shown in Figs 11, 12 respectively. The average Nusselt number decreases with increasing phase angles  $\omega t$  or decreasing Prandtl number and  $n$ .



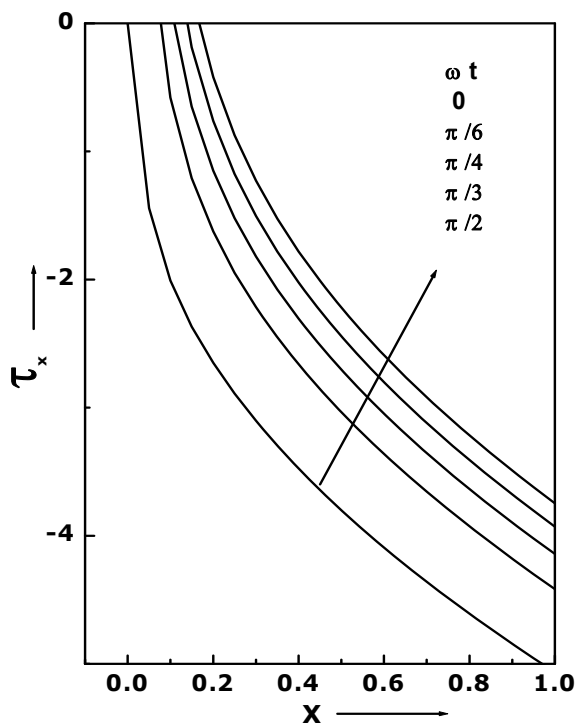


Fig.7. Local skin friction for different  $\omega t$ .

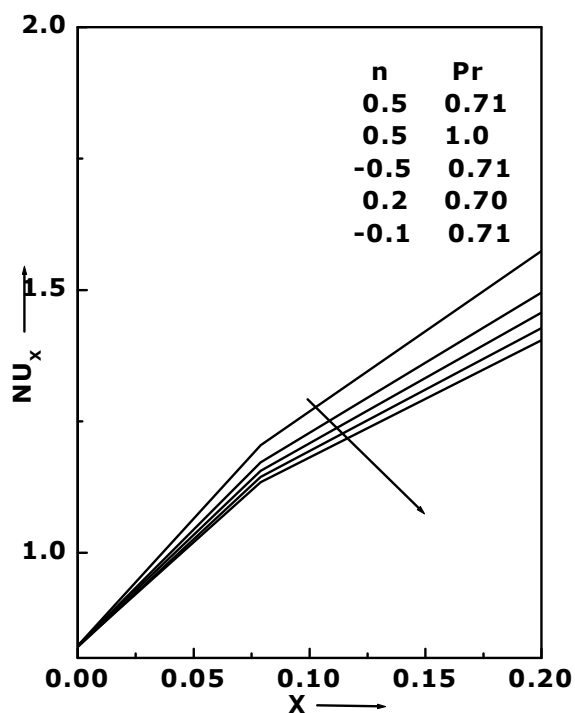


Fig.8. Local Nusselt number.

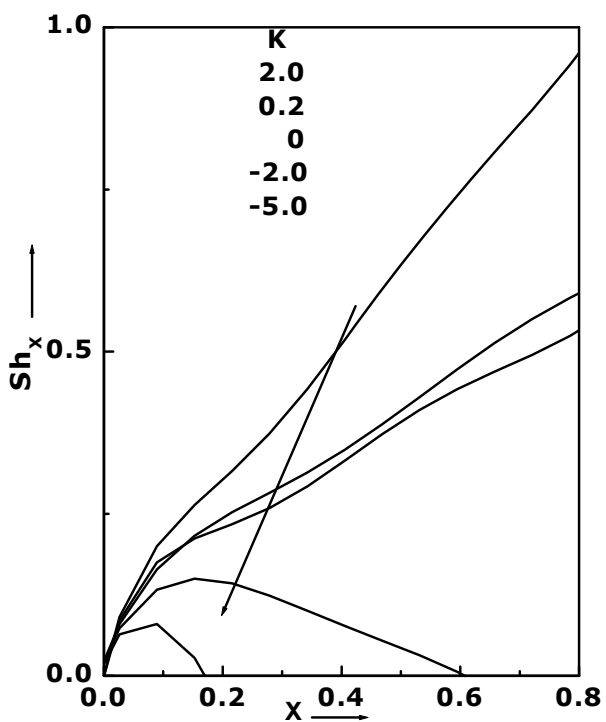


Fig.9. Local Sherwood number for different  $K$ .

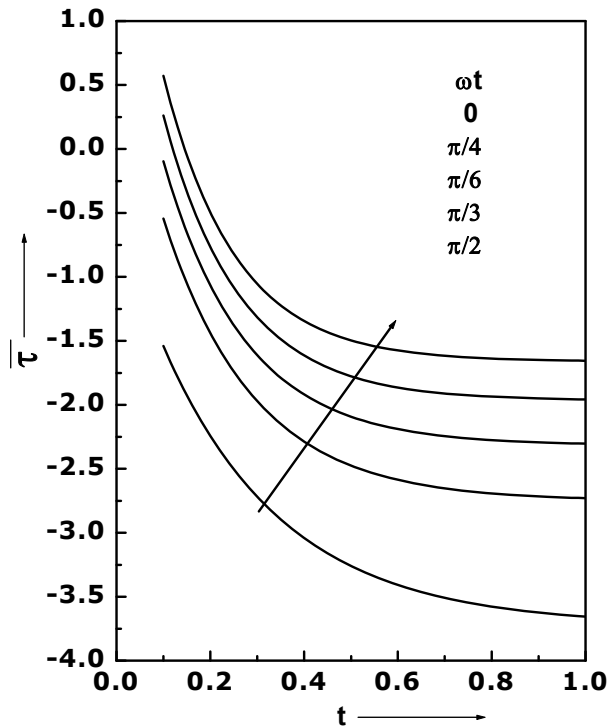


Fig.10. Average skin-friction for different  $\omega t$ .

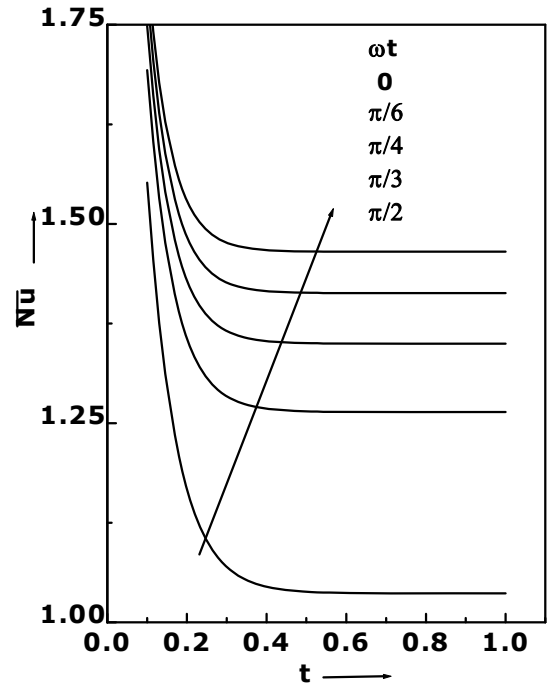
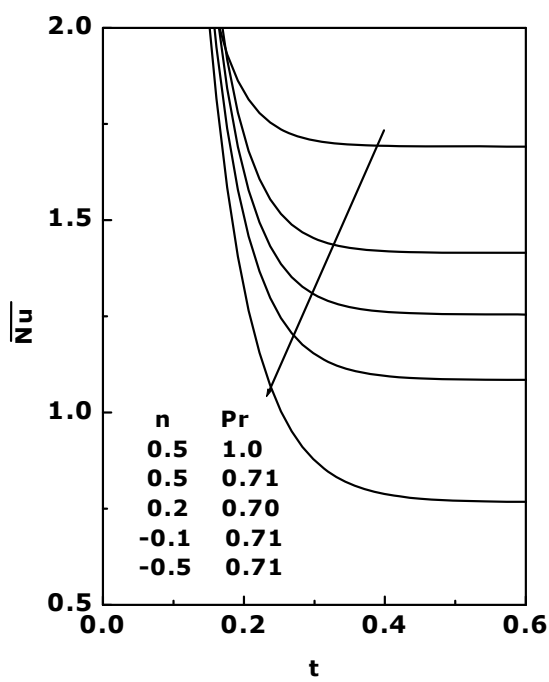


Fig.11. Average Nusselt number for different  $n$  and  $Pr$ . Fig.12. Average Nusselt number for different  $\omega t$ .

Figure13 shows the effect of the chemical reaction parameter  $K$  on the average values of the average Sherwood number. The Sherwood number increases with increasing values of the chemical reaction parameter. It is also observed that the average Sherwood number increases during the destructive reaction and decreases with the generative reaction.

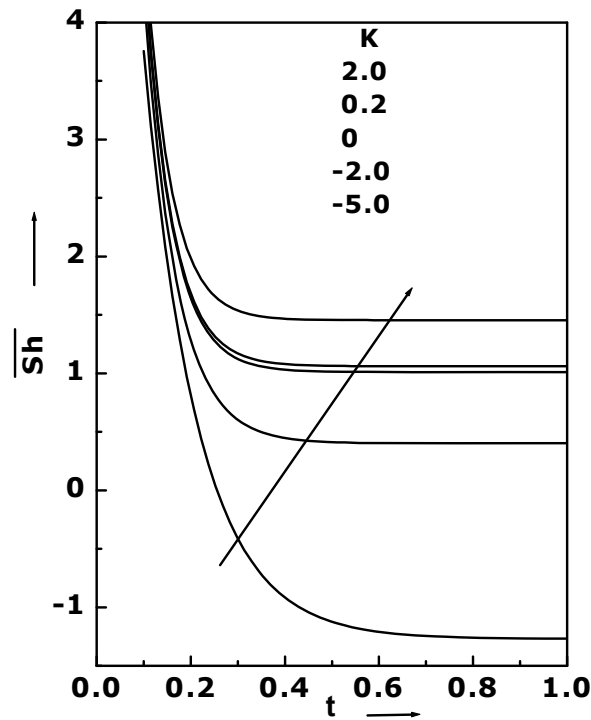


Fig.13. Average Sherwood number for different  $K$ .

## 5. Conclusion

A numerical solution of an unsteady flow past an oscillating semi-infinite vertical plate with variable surface temperature and uniform mass diffusion in the presence of a homogeneous chemical reaction of first order is presented. The dimensionless governing equations are solved by an implicit scheme of Crank-Nicolson type. The effects of velocity, concentration and temperature for different parameters are studied. The local as well as average skin-friction, Nusselt number and Sherwood number are shown graphically.

- Contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly.
- Velocity decreases in the presence of a chemical reaction and phase angle.
- The study shows that the number of time steps to reach the steady-state depends strongly on the chemical reaction parameter and phase angle.
- Velocity increases during the generative reaction and decreases in the destructive reaction
- Heat transfer performance of an oscillating plate is at constant rate.

## Nomenclature

- $A, a$  – constant  
 $C$  – dimensionless concentration  
 $C'$  – concentration  $mol \cdot m^{-3}$   
 $C_p$  – specific heat at constant pressure  $J \cdot kg^{-1} \cdot K^{-1}$   
 $D$  – mass diffusion coefficient  
 $Gc$  – mass Grashof number  
 $Gr$  – thermal Grashof number  
 $g$  – acceleration due to gravity  $m \cdot s^{-2}$   
 $K$  – dimensionless chemical reaction parameter  
 $K_l$  – chemical reaction parameter  $J$   
 $k$  – thermal conductivity  $J \cdot m^{-1} \cdot K^{-1}$   
 $Pr$  – Prandtl number  
 $Sc$  – Schmidt number  
 $T$  – temperature of the fluid near the plate  $K$   
 $t$  – dimensionless time  
 $t'$  – time  $s$   
 $U, V$  – dimensionless velocity components in  $x$  and  $y$ -direction  
 $u, v$  – velocity components in  $x$  and  $y$ -direction  $m \cdot s^{-1}$   
 $u_0$  – velocity of the plate  
 $X$  – dimensionless coordinate along the plate  
 $x$  – spatial coordinate along the plate  
 $Y$  – dimensionless coordinate axis normal to the plate  
 $y$  – spatial coordinate normal to the plate  $m$   
 $\beta$  – volumetric coefficient of thermal expansion  $K^{-1}$   
 $\beta^*$  – volumetric coefficient of expansion with concentration  $K^{-1}$   
 $\eta$  – similarity parameter  
 $\theta$  – dimensionless temperature  
 $\mu$  – coefficient of viscosity  $Pa \cdot s$   
 $\tau$  – dimensionless skin-friction  
 $\rho$  – density of the fluid  $kg \cdot m^{-3}$   
 $\nu$  – kinematic viscosity  $m^2 \cdot s^{-1}$   
 $\omega$  – dimensional frequency of oscillation  
 $\omega'$  – frequency of oscillation  
 $\omega t$  – dimensionless phase angle  
 $\omega t'$  – phase angle in radians

## Subscripts

- $w$  – conditions at the wall  
 $\infty$  – conditions in the free stream

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