Chen Haoxiang, Chengzhi Qi*, Liu Peng, Li Kairui and Elias C. Aifantis

Modeling the zonal disintegration of rocks near deep level tunnels by gradient internal variable continuous phase transition theory

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Abstract: The occurrence of alternating damage zones surrounding underground openings (commonly known as zonal disintegration) is treated as a “far from thermodynamic equilibrium” dynamical process or a nonlinear continuous phase transition phenomenon. The approach of internal variable gradient theory with diffusive transport, which may be viewed as a subclass of Landau’s phase transition theory, is adopted. The order parameter is identified with an irreversible strain quantity, the gradient of which enters into the expression for the free energy of the rock system. The gradient term stabilizes the material behavior in the post-softening regime, where zonal disintegration occurs. The results of a simplified linearized analysis are confirmed by the numerical solution of the nonlinear problem.

Keywords: deep-level tunnels; deformation patterning; gradient plasticity; Landau’s phase transition theory; zonal disintegration.

1 Introduction

Deformation patterning in the form of thin regions where shear strain localizes (shear bands) or micro cracking intensifies (damage zones) is routinely observed in metallic and geological materials when the local stress (shear, tensile or a combination of them) exceeds a certain threshold value. This threshold is usually identified with the critical state where the material undergoes a “phase transition” from stable hardening to unstable softening behavior. In the “hardening” regime, the material modulus measuring local strength or mechanical integrity (e.g. elastic or plastic modulus) is positive, while in the “softening” regime this modulus becomes negative. The situation is reminiscent to the classical case of liquid-vapor phase transition where a negative compressibility regime occurs in the pressure (p) vs. density (ρ) diagram, as described by Van der Waals’ classical non-monotone equation of state [1]. To determine the density patterns generated by the instability of homogeneous states in the negative slope regime of Van der Waals equation of state, Aifantis and Serrin [2] introduced high-order density gradients in the constitutive equation for the stress describing the mechanical state (material integrity) in the liquid-vapor inhomogeneous region; i.e. the liquid-vapor interface. By solving analytically in one dimension the corresponding nonlinear differential equation resulting from equilibrium, they found that such “diffuse interfaces” appear in one of the following three forms: (i) a single interface separating the liquid from the vapor phase; (ii) a double (symmetric) interface confining a “liquid drop” in a vapor environment or a “vapor droplet” in a liquid environment; (iii) multiple “liquid drops” or “vapor droplets” filling the infinite one-dimensional space in a periodic fashion, i.e. periodic solutions. The extra dividend gained from such type of mechanical analysis was that Maxwell’s “equal area rule” resulting from thermodynamic and statistical mechanics considerations [3, 4] of inhomogeneous fluids does not hold in general, but in replaced by another integral condition involving the material parameters appearing in the gradient constitutive expression for the interfacial stress. Such generalization of Maxwell’s “equal area rule” condition and the use of level set theory led to the result that radially symmetric solutions in three dimensions can only be concentric cylinders or concentric spheres.

The aforementioned work on fluid interfaces was the direct motivation of Aifantis [5–9] to introduce higher-order gradients of accumulated equivalent shear strain.
in the yield condition of plasticity theory and the Laplacian of Hookean stress in the standard constitutive relation of classical elasticity, as well as diffusive terms in the evolution equations of internal variables such as those modeling damage nucleation/growth in metals and geomaterials. The Laplacian of the stress/strain term is multiplied (for dimensional consistency) with the square of an internal length which is a “configuration” rather than a strictly “material” parameter, signifying the effect of underlying substructure on the overall macroscopic response. This type of internal length gradient (ILG) material mechanics approach and its variants has been used to address a large number of shear/damage localization (shear bands/fracture zones) and size-dependent problems (size effects) in solid mechanics ranging from macro/meso scales to micro/nano scales and from metals/polymer to soils/rocks and concrete (e.g. [10–12]). Moreover, it has been used to address spatio-temporal pattern forming material instabilities [13] focusing mainly on dislocation patterning in metals [14, 15], and to a much lesser extent on geomaterials [11, 16]. In this connection, it is pointed out that in [16] the Walgraef-Aifantis model for dislocation patterning (i.e. a reaction-diffusion ILG model) was convincingly translated to interpret fracture pattern formation in frictional cohesive granular materials.

Another interesting case for applying the ILG approach to interpret deformation patterning phenomena in geomaterials is the occurrence of diffuse damage in the form of circular-like zones surrounding underground tunnels. It turns out that the ILG formulation applied to internal variable theory with diffusive transport, may be viewed as a special case of Landau’s continuous phase transition theory which we will use here to discuss the phenomenon of multiple damage zone formation in underground rock openings. The phenomenon of “zonal disintegration” near deep level openings – first addressed by Russian geoscientists in the middle 1980s [17–20], as well as in more recent years [21–25] – has attracted increasing attention by South African [26] and Chinese [27–32] researchers on theoretical and experimental rock mechanics due to its frequent appearance in mining excavations and deep level tunneling constructions. A schematic of this phenomenon is given in Figure 1. Various mechanisms and theoretical models were suggested in the above works such as zonal disintegration due to rock mass splitting along the direction of maximum tangential stress [17–20], loss of stability due to material softening [21], or slip line plastic yielding [22, 23]. In [28] the point of view was advanced that under the action of high axial compression, rupture near the tunnel wall will occur when the radial displacement exceeds a certain threshold value. If the geostatic pressure is sufficiently large, a second and third failure will emerge leading to sequential damage layer formation and progressive zonal disintegration of the rock mass surrounding the borehole opening. Correspondingly, the energy stored in the rock mass during straining, is released abruptly when critical conditions are established, in order to be subsequently dissipated through the formation of tensile microcracks in the form of circular-like damage zones concentric to the perimeter of the opening. The main characteristic features of zonal disintegration phenomenon were revealed by reproducing in the laboratory [28, 29] the scenario outlined above, which was further confirmed experimentally in [30] through carefully designed experiments in iron-crystal/rock-like specimens showing the occurrence of regular alternating zonal failure around the tunnel.

As far as theoretical modeling is concerned, of particular interest to the present article is the approach of Qi and co-workers [33–35] (see also [36–38]). These authors have adopted the Landau-Ginzburg (L-G) expansion of free energy [39, 40] which, in spirit, is similar to a variational

Figure 1: (A) The phenomenon of rock mass zonal disintegration near deep level tunnel. (B) Zonal fracture structure of rock mass near tunnel in planar model test.
formulation of the ILG approach. In this application of L-G theory, the order parameter is identified with an irreversible (plastic) strain quantity, the gradient of which enters the expression for the free energy of the rock system in a similar way as in Van der Waals thermodynamic theory [1–4] of liquid-vapor transitions, as well as in the Cahn-Hilliard [41] theory of spinodal decomposition and the thermodynamic counterpart of Aifantis [42, 43] and Gurtin [44, 45] gradient theories. Other general or more specific references that may provide physical insight into the problem and technical tools for addressing it can be found in [46–56]. It turns out that in the linear counterpart of the continuous phase transition theory, the spatial distribution of damage can be described by Bessel functions as in the case of gradient elasticity. But analytical solutions are usually possible for linear problems; thus the corresponding non-linear continuous phase transition equation for the problem at hand, is numerically investigated in this paper. The numerical analysis confirms the occurrence of alternating damage zones that form during the zonal disintegration process, as observed in deep level tunnels.

2 Non-linear continuous phase transition model

For geomaterials (soils, rocks), the typical relationship between the shear stress $T$ and the dilatation $\theta(=\varepsilon_1+\varepsilon_2+\varepsilon_3)$ with the shear strain $\gamma(=\varepsilon_1-\varepsilon_3)$ is shown in Figure 2A [47]. It is noted that in conventional metals, changes in $\theta$ during plastic deformation are usually negligible, whereas the negative “slope” regime is much less pronounced as strain localization and fracture commonly occurs before the material, enters that regime. This is not the case, however, for advanced metallic materials such as amorphous, metallic glasses and nanocrystalline materials which in some sense behave like geomaterials but in much smaller scales. Thus, for geomaterials the shear stress-strain curve can be divided into four different regimes:

OA: The initially “open” microcracks in rocks are gradually closing with compression. The stress-strain curve is concave indicating that in the initial nonlinear deformation stage, the dilatation decreases with the increase of loading.

AB: This stage is linear elastic and the stress-strain curve is approximately straight. The deformation is reversible and independent of time.

BC: Plastic deformation takes place in this regime, and the stress-strain curve is convex. In this stage, stable micro propagation occurs. Crack propagation stops when the loading stops. Point C is a turning point where the volumetric deformation changes sign from negative (contraction) to positive (expansion).

CD: This phase is the softening stage in which stress decreases with the increase of strain. The propagation of micro cracks is unstable. The fracture process is controlled by the speed of crack propagation. Microcracks develop and coalesce rapidly, and macroscopic damage forms. These localizations emerge and are gradually intensified. Then localizations evolve into shear bands which divide intact rock masses into pieces with certain sizes. Relative sliding and rotation between rock pieces caused by the applied shear stress leads to a rapid volume expansion.

DE: In this stage the rock sample is divided into separate blocks. The stress in the rock sample remains constant and is determined by the friction between the rock blocks.

The kinetic process of crack formation is graphically depicted in Figure 2B, where $N/N_{\text{max}}$ denotes the ratio of the current number of cracks to the maximum number of cracks at the moment of failure [47]. The hypothesis is advanced herein to view the aforementioned multiscale crack evolution as a kinetic process of multistage continuous phase transition. To this end, we introduce dimensionless parameter ($\psi$) to describe the irreversible deformation of rock, as follows

![Figure 2](drive://path/to/image.png)

Figure 2: (A) The stress-strain diagram for a rock sample. (B) The dependence of cracking on the relative deformation ($\xi=\gamma/\gamma_c$).
\[
\psi = \frac{\gamma' \psi^*}{\gamma_c \psi_s}
\]

(1)

where \(\gamma'_c\) is the elastic shear strain limit, \(\gamma_c\) is the shear strain limit at failure, and \(\gamma\) is the current shear strain. This quantity will play the role of the order parameter in Landau’s phase transition theory [39, 40].

When \(\psi = 0\), energy dissipation does not take place. When \(\psi > 0\), dissipation sets in and a new dissipative structure forms. This is due to cooperative or synergetic phenomena between the nonlinear and spatio-temporal terms in the differential equation describing the evolution of the system. Such differential equation may be derived in a generic form by using the Ginzburg-Landau formulation and introduce the gradient \((\nabla \psi)\) in the nonlinear part of the energy density \((\varphi)\) describing its dependence on \((\psi)\), as follows

\[
\varphi = -\frac{1}{2} V_s \psi^2 + \frac{1}{4} V_s \psi^4 + \frac{1}{6} V_s \psi^6 + \frac{1}{2} C (\nabla \psi)^2
\]

(2)

where \(C\) is the gradient coefficient \((C > 0)\) and \((V_s, V_c, V_e)\) are material parameter, determining the nonlinear dependence of the homogeneous part of \(\varphi\). It should be noted that, in general, the coefficients \(V_s\)'s may depend on the state of stress (or elastic strain) and the parameters \((\gamma'_c, \gamma_c)\). Moreover, the elastic strain energy density contribution to \(\varphi\) has been omitted for simplicity, as we focus on the dissipative cracking/damage process.

The total potential energy for the whole sample is expressed as follows

\[
\Phi = \int \left[ -\frac{1}{2} V_s \psi^2 + \frac{1}{4} V_s \psi^4 + \frac{1}{6} V_s \psi^6 + \frac{1}{2} C (\nabla \psi)^2 \right] d\Omega
\]

(3)

The corresponding evolution equation for the deformation field may be obtained through the usual variational postulate (see, for example, [47])

\[
\frac{\partial \psi}{\partial t} = \frac{\partial \Phi}{\partial \psi} = \frac{\partial \psi}{\partial \psi} = \Gamma \left[ -V_s \psi + V_s \psi^3 + V_s \psi^5 - \nabla (C \nabla \psi) \right]
\]

(4)

where \(\Gamma\) is a dynamic coefficient \((\Gamma > 0)\), and the associated boundary condition reads

\[
(\nabla \psi) \bar{n} |_{\text{wall}} = 0
\]

(5)

where \(\bar{n}\) denotes the unit outward normal vector on the surface of the underground opening (Figure 1) of radius \(a\).

For axial symmetry, Eq. (4) becomes

\[
\frac{\partial \psi}{\partial t} = \Gamma \left[ C \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial r} \right) + V_s \psi - V_s \psi^3 - V_s \psi^5 \right]
\]

(6)

i.e. a nonlinear diffusion equation for the damage (internal variable) \(\psi\). It should be noted that Eq. (4) or Eq. (6) is an evolution equation for the internal variable \(\psi\) with diffusive transport. Thus the Ginzburg-Landau formalism that we followed here, leads to analogous results with the internal variable theory with diffusive transport, as advanced by Aifantis [7]. This latter approach may have some advantages for material mechanics formulation, while the first approach which we followed here is more familiar to physicists, the attention of which we wish to attract to this geotechnical problem.

The physical meaning of \(V_s, V_c, V_e\) is crucial for model development and for deducing their dependence on the external field strength. Beyond the limit of elasticity and in the softening regime, in particular, the one-to-one correspondence between stress and strain disappears. The internal variable \(\psi\) related to irreversible strain is thus introduced to describe the state of the medium and we assume that damage, as a measure of crack density, is proportional to this irreversible deformation. Then, we choose one dimensionless parameter \(K\), the ratio of the average distance \(d\) to the average length \(l\) of the cracks, for describing the effect and intensity on the evolution of the internal strain variable \(\psi\); i.e.

\[
K = d/l
\]

(7)

Physical reasoning and related experimental investigations suggest that if \(K\) is sufficiently large, the interaction between cracks can be neglected. When \(K\) is approaching a critical value \(K_c = 3-5\), the material enters the rapid growth damage stage. The critical value \(K_c\) is taken to be almost independent of the properties of the material and may be used to formulate a density damage-like criterion for material failure. To be more specific, we assume that when \(K = K_c = \pi (\approx 3.14)\) the stress reaches its maximum value and the material enters the rapid damage stage, whereas when \(K = K_c = 1\) the rock will totally fracture without any residual strength. With these assumptions, we propose a specific dependence of \(V_s\)'s on \(K\), as shown below by rewriting Eq. (6) as follows

\[
\frac{\partial \psi}{\partial t} = \Gamma \left[ \nabla (C \nabla \psi) - A \left(1 - \frac{K}{K_c}\right) \psi - V_s \psi^3 - B \left(1 - \frac{K}{K_c}\right) \psi^5 \right]
\]

(8)

In actuality, the parameter \(K\) may also depend on other deformation characteristics as described by the order parameter \(\psi\). In order to simplify the analysis and reduce the order of non-linearity in the governing equation, \(K\) is selected as an independent parameter here.
3 Properties of solutions

The properties of solutions of the non-linear continuous phase transition model at hand can be obtained by considering the stationary state of Eq. (8) and studying the corresponding “phase” curves of the resulting equation, i.e.

\[ A\left(1 - \frac{K}{K_*}\right)\psi + V_\psi \psi^3 + B\left(1 - \frac{K}{K_*}\right)\psi^5 - V(C\nabla \psi) = 0 \]  

(9)

The corresponding potential energy function is

\[ U(\psi) = \int_0^\infty A\left(1 - \frac{K}{K_*}\right)\psi + V_\psi \psi^3 + B\left(1 - \frac{K}{K_*}\right)\psi^5 \, d\psi \]

\[ = -\frac{A}{2}\left(1 - \frac{K}{K_*}\right)\psi^2 - \frac{V}{4}\psi^4 - B\left(1 - \frac{K}{K_*}\right)\psi^6 \]  

(10)

When \(K > K_*\), the potential energy, the phase plane curves and the form of the solution are depicted in Figure 3. It is obvious that the potential energy function is an even function and \(\psi = 0\) is a critical point at which the potential energy reaches a local maximum value. In our mathematical modeling approach, the mechanical properties of the rock mass are determined by the type of the governing differential equation, the solutions of which also depend on the initial and boundary conditions. In the present case, the governing differential equation is an elliptic. Solutions of the governing equation exhibit spatial periodicity, and spatial sizes depend on the magnitude of external loading. This kind of situation corresponds to a stable spatial phenomenon of zonal disintegration.

The potential energy and phase curves shown in Figure 4 correspond to the case \(K < K < K_*\). We notice that if the initial condition corresponds to the curve \(S\), the solution will change from a stable into a solitonic form \(\psi = \psi(r-\nu t)\). This solution corresponds to a “strain wave” which propagates into the surrounding rock mass. This soliton solution describes the phenomenon of the incremental strain sign-change in the radial direction.

Another new form of solution occurs when \(K < K_*\). In this case the material enters the rapid damage growth stage, and defects nucleate explosively. Microcracks nucleate, develop and coalesce rapidly, and macroscopic damage surfaces emerge. In rock masses near deep-level tunnels, such explosive generation of defects may result in catastrophic rock bursting (Figure 5).

4 Analytical solution of the governing linear equation

Equation (6) is a non-linear differential equation, analytical solutions of which are generally difficult or not possible to obtain. Nevertheless, in one dimension, analytical results for traveling wave solutions are available and for steady states general solutions in infinite domains are readily derived (see, e.g. [2, 7]). If we neglect higher order terms of \(\psi\) in Eq. (6), we will obtain the following linear differential equation

\[ \frac{\partial \psi}{\partial t} = K \left( C \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + V_\psi \psi \right) \]  

(11)

Equation (11) may be simplified into

\[ \frac{\partial \psi}{\partial t} = \lambda \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + D^2 \psi \]  

(12)

where \(\lambda = 1/KC\) is a new coefficient, and \(D^2 = V/JC\).

Assuming \(\frac{\partial \psi}{\partial t} = e^{\nu t}/r^2\), a solution of Eq. (12) was obtained in [33] as follows

![Figure 3: Potential energy, phase curves and form of solution when \(K > K_*\).](image-url)
\[ \psi = \left[ N_0 (D_r) f_1 (D_r) - J_0 (D_r) N_1 (D_r) \right] \frac{\pi}{2} D_r (-0.046) e^{i\pi t} \]
\[ + 1.862 \left[ J_0 (D_r) - 0.705 N_0 (D_r) \right] \left[ 1 - 0.937 e^{i\pi t} \right] \]

(13)

where \( D_{r_0} = 3 \), and \( r_0 \) is radius of the tunnel.

For a tunnel with radius 5 m, when \( t \) approaches to infinity, the deformation of the surrounding tunnel rock mass will approach a stable state. Assuming that the tunnel wall is in the limit state and \( \psi = 1 \), the spatial distribution of the order parameter \( \psi \) in the radial direction may be obtained and is plotted in Figure 6.

In Figure 6, the oscillatory nature and periodicity of \( \psi \) along the radial direction is apparent. Meanwhile, the amplitude of the strain wave decays along the radial direction. The regions between two adjacent fractured zones are weakly damaged, a fact which is consistent with in-situ observations. Even though the linear analytical solution may describe the main features of the stable zonal disintegration quite well, some disadvantages are apparent. For example, the distance between two adjacent wave crests is constant, namely, the spatial period of the order parameter is constant. But in-situ observations show that the distance between two neighboring fractured zones increases in the radial direction. Therefore the non-linear equation should be considered.

5 Numerical solution of non-linear governing equation

For steady-states, Eq. (8) can be rewritten as

\[ \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial r} \right) + a\psi + b\psi^3 + c\psi^5 = 0 \]

(14)

where, \( a = V/JKC \), \( b = -V/JKC \), \( c = -V/JKC \). Analytical solutions of Eq. (14) is difficult to obtain, thus, a numerical analysis is performed. It is well known that in numerical simulations, the size of the computational step (or mesh size) has a significant effect on the stability, accuracy and convergence properties of the solution. In order to guarantee reliable solutions, a small step is necessary. But a small step may slow down the rate of convergence and increase the computational time required for convergence. Therefore, a proper calculating step should be chosen. In order
to analyze the step sensitivity, the following values of the coefficients $a=0.4096$, $b=-1.6$, $c=2.4$ are used. Solutions for step sizes $h=0.25$, 0.025, 0.0025 m are depicted in Figure 7. Computation results by using these three steps satisfy the need for stability requirements. The accuracy of the result with step $h=0.25$ m is less than that with steps 0.025 m and 0.0025 m. Computation results for steps 0.025 m and 0.0025 m are very close to each other. Thus, for economizing computation time the value of $h=0.025$ m is selected as the proper calculating step.

The distance from the first fractured zone to the opening wall approximately equals to the radius of the opening as shown by in-situ observations. Based on this and the linear solution given by Eq. (13) as explained in [33], three groups of coefficients are used: (1) $a=0.4096$, $b=-1.6$, $c=2.4$; (2) $a=0.4096$, $b=1.0$, $c=1.5$; (3) $a=0.4096$, $b=-2.0$, $c=3.0$. The radius of the opening is taken as $r_0=5$ m.

The finite difference method is used to solve Eq. (14), and the results for the three sets of coefficients are shown in Figure 8.

It is clear from Figure 8 that the solution of the non-linear equation exhibits quasi-periodicity. The amplitude and decay rate are remarkably affected by the higher order terms of $\psi$. The magnitude of the obtained quasi-periodicity which may be identified as the distance between two neighboring fractured zones, increases with the distance in the radial direction and is significantly influenced by the higher order terms. The behavior, exhibited by the numerical solution of the nonlinear equation agrees with the in-situ observations much better than that of the linear equation. The values of the coefficients ($a$, $b$, $c$) obviously affect the solution of the equation. For comparison purposes, only one coefficient is varying at the time, such that to clearly identify the effect of each coefficient.

(i) First the effect of coefficient $a$ of the first order term is analyzed with the coefficients $b$ and $c$ taken equal to zero. In comparison with the coefficients of higher order terms ($b$, $c$), the coefficient $a$ has greater impact on the properties of solution. Four solutions are obtained with four different values of the coefficient $a$: 0.2, 0.4096, 0.6, 0.8, as shown in Figure 9. It is clear from Figure 9 that with the increase of coefficient $a$ the period of fractured zone pattern decreases and the thickness of the fractured zones also decreases. It is noticed that the rate of decay of the order parameter $\psi$ in the radial direction does not change with the coefficient $a$.

(ii) Next, the effect of the third order term, i.e. the coefficient $b$, is investigated with the coefficient $a$ taken equal to 0.4096 and the coefficient $c$ equals to zero. The value of $b$, in particular, should be in the range: $b\geq a=-0.4096$. The values of $b$ are chosen as follows: 1.0, 0.5, -0.2, -0.35. Figure 10A and B correspond to the case of $b>0$. The period of the distribution of the order parameter decreases with the increase of $b$. But for a fixed value of $b$ the period in the radial direction increases. When $b<0$, the period of the distribution of order parameter increases with the increase of absolute value of $b$, as seen from Figure 10C and D. In other words, the difference between these two cases is that when $b>0$, the period in the radial direction increases with the increase of coefficient $b$; but when

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**Figure 6:** Stationary deformation pattern in surrounding tunnel rock based on linear equation.

**Figure 7:** Solutions for different spatial step sizes.

**Figure 8:** Stable formation of rock zones obtained by the non-linear equation.
When \( b < 0 \), the period decreases with the increase of coefficient \( b \). When \( b \) equals to zero, Eq. (14) reduces to a linear equation. When \( b = -a = -0.4096 \), the solution of the nonlinear equation is constant, and \( \psi = 1 \). It is noticed that the decay rate of the order parameter accelerates with the decrease of the algebraic value of \( b \). It is also clear from Figure 10 that the third term has a significant influence on the decay rate of the order parameter.

Finally, the effect of coefficient \( c \) of the fifth order term is studied with \( a = 0.4096 \) and \( b = 0 \). Here four values of \( c \) are considered: 1.0, 0.5, -0.2, -0.35. On comparing Figure 11 with Figure 10, it is noticed that the effect of the coefficient \( c \) on the solution is similar to that of the coefficient \( b \).

When the coefficient \( c \) is negative and its absolute value is large enough, a new temporal-spatial pattern of deformation and failure takes place. Defects occur explosively at some distance from the tunnel wall. In this regime, the rock mass fractures rapidly and rock bursting may occur. When the rock fractures, a new free surface forms which can be regarded as a “false” contour of opening. The aforementioned process may repeat in rock masses behind “false” opening contour until the stress-strain state in rock mass satisfies the condition of stability. The distribution of the order parameter is depicted in
Further discussion on the physical mechanism of the influence of the model parameters

The physical meaning and influence of the model parameters are further discussed in this section. To this end we first note that Eq. (14) may be rewritten as

\[ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \tilde{k} \psi = 0 \]  

(15)

where \( \tilde{k} = a + b \psi^3 + c \psi^4 \).

It is noticed that Eq. (15) is formally similar to the equation of string lying on an elastic foundation. The coefficient \( \tilde{k} \) can be regarded as equivalent to the foundation stiffness and the value of \( \tilde{k} \) may be positive, negative, or zero. The solution is oscillatory when \( \tilde{k} \) is positive. The vibration frequency is proportional to the equivalent foundation stiffness. The term of \( 1/r \) provides a geometric decay mechanism in the radial direction and endows the solution with some properties of Bessel function. We have \( \psi > 1 \) when the equivalent stiffness is negative, while \( \psi = 1 \) when the equivalent stiffness equals to zero. It is obvious that when \( \tilde{k} \leq 0 \), the solutions do not agree with in-situ observations; thus, the equivalent stiffness should be positive.

The equivalent stiffness increases with the growth of any one of the coefficients \( (a, b, c) \). The oscillation frequency increases and the period decreases with the increase of any one of \( (a, b, c) \). The period of order parameter in the radial direction will be discussed below for

Figure 11: The effect of coefficient \( c \) on the behavior of the solution.  
(A) \( c = 1.0 \), (B) \( c = 0.5 \), (C) \( c = -0.2 \) and (D) \( c = -0.35 \).

Figure 12: The effect of coefficient \( c \) on the behavior of the solution.  
(A) \( c = -4.0 \) and (B) \( c = -5.0 \).
the following 4 cases: (1) \( b>0, c>0 \); (2) \( b<0, c<0 \); (3) \( b<0, c>0 \); (4) \( b>0, c<0 \).

It is easy to conclude that when \( (b>0, c>0) \), the period in the radial direction increases; while when \( (b<0, c<0) \), the period in the radial direction decreases. However, when \( (b<0, c>0) \) or \( (b>0, c<0) \), the properties of the solution are more complex and depend on the relative contributions of these coefficients.

The equivalent stiffness can be regarded as a quadratic function of the term \( \psi^2 \). It has been noticed that the order parameter \( \psi \) decays with the distance in the radial direction. Therefore, the equivalent stiffness may be related to the radius \( r \) via the order parameter \( \psi \). When \( (b>0, c>0) \), the equivalent stiffness curve is a concave-upward parabola of the term \( \psi^2 \). The horizontal coordinate of the symmetry axis is \(-b/2c<0\), which lies outside the range of \([0, 1]\). The value of the order parameter increases monotonically in this range. The order parameter and the equivalent stiffness decrease in the radial direction, but the vibration frequency decreases and the period increases in this direction. When \( (b<0, c<0) \), the equivalent stiffness curve is a concave-downward parabola of the term \( \psi^2 \). The horizontal coordinate of symmetry axis is \(-b/2c<0\) which lies outside the range of \([0, 1]\). The value of the order parameter decreases monotonically in this range. The order parameter and the equivalent stiffness increase in the radial direction, but the vibration frequency increases and the period decreases in this direction.

The equivalent stiffness curve is a concave-upward parabola of the term \( \psi^2 \) when \( b<0, c>0 \). The value of symmetry axis is \(-b/2c>0\). Two cases should be taken into consideration here.

(i) The symmetry axis lies in the range of \([0, 1]\): If the coefficient \( c \) is large, the value of the symmetry axis approaches zero. The value of \( \psi^2 \) monotonically decreases, and the equivalent stiffness decreases in the radial direction, whereas the vibration frequency decreases and the period increases in this direction. If \(-b/2c \) approaches 1, the equivalent stiffness increases first and then decreases with the increase of the order parameter \( \psi \), whereas the period increases first and then decreases in this direction.

(ii) The symmetry axis lies outside the range of \([0, 1]\): The equivalent stiffness decreases with the increase of the order parameter \( \psi \). The equivalent stiffness increases in the radial direction, whereas the frequency decreases and the period increases in this direction. The coefficient \( b \) dominates the behavior in this situation.

When \( (b>0, c<0) \), the solution is similar to the case of \( (b<0, c>0) \) and the details will not be repeated here.

7 Conclusions

Under high-level stressing, such as in underground tunnel excavations, irreversible deformations in the surrounding rock mass take place. Consequently, energy dissipation appears and self-organization phenomena occur leading to the formation of dissipative structures and damage patterns. Classical theories of elasticity and plasticity cannot describe these phenomena. The kinetic process of microcrack formation leading to irreversible deformations of the rock mass can be regarded as a multi-stage process of continuous phase transition; a viewpoint adopted in this paper to simulate the phenomenon of zonal disintegration. The governing nonlinear equation is solved numerically and the advantages over the analytical solution of its linearized counterpart are shown. For instance, the nonlinear solution can describe the variation of the distance between two adjacent fractured zones in the radial direction, and the occurrence of inner rock burst. These features, cannot be captured by the linear solution. The effect of the phenomenological damage coefficients on the solution is discussed. With the help of the elastic foundation string theory, the physical meaning of these coefficients is further clarified. This “continuous phase transition” approach is similar in spirit with the approach of “gradient theory” and “internal variables with diffusive transport”, as proposed earlier by the last author. A combination of the two approaches for this problem of zonal rock disintegration is forthcoming. We will in the following work consider the limit of the region near tunnel within which rock mass enters self-organized state and combine continuous phase transition theory with continuum mechanics to improve current non-linear continuous phase transition model.

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