Digital Images with 3D Geometry from Data Compression by Multi-scale Representations of B-Spline Surfaces

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Abstract:
To build up a 3D (three-dimensional) model of the surface of an object, the heights of points on the surface are measured, for instance, by a laser scanner. The intensities of the reflected laser beam of the points can be used to visualize the 3D model as range image. It is proposed here to fit a two-dimensional B-spline surface to the measured heights and intensities by the lofting method. To fully use the geometric information of the laser scanning, points on the fitted surface with their intensities are computed with a density higher than that of the measurements. This gives a 3D model of high resolution which is visualized by the intensities of the points on the B-spline surface. For a realistic view of the 3D model, the coordinates of a digital photo of the object are transformed to the coordinate system of the 3D model so that the points get the colors of the digital image. To efficiently compute and store the 3D model, data compression is applied. It is derived from the multi-scale representation of the dense grid of points on the B-spline surface. The proposed method is demonstrated for an example.

Keywords:
3D model • two-dimensional B-spline surface • multi-scale representation of signals • lofting method • laser scanner

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1. Introduction
Computing 3D (three-dimensional) models of natural objects or man-made constructions and visualizing them is a task which frequently needs to be solved. In reverse engineering, for instance, an analytical model of a manufactured object is determined by a free-form surface and then modified by computer aided design Yang and Qian (2007). Models of buildings or cities are set up for planning, preservation or navigation through the city. The models should fulfill two tasks: to provide 3D information with a specified resolution and to visualize the geometry.

Laser scanners or similar instruments provide rapidly the geometry of objects. The 3D coordinates of points on the surface of an object are measured together with the intensities of the reflected laser beam of the points being observed. The intensities give a range image of the point cloud of the laser scanner either as a monochromatic picture or as a colored one by choosing different colors for different intensities. The result is a 3D model represented by a digital image from which 3D information can be extracted for all measured points in the coordinate system of the laser scanner or in a chosen system.

Range images of the facades of buildings can be used to extract information about the buildings, especially about windows and entrances. Becker (2009), for instance, uses a bottom-up, i.e. data driven strategy, for the reconstruction of facades. The results serve as a knowledge base for inferring by a formal grammar facade structures, where no sensor data is available. Thus, the
bottom-up strategy is complemented by a top-down approach. A
top-down method is applied by Schmittwilken and Plümer (2010)
for the reconstruction and classification of parts of a facade in
a point cloud of a laser scanner. The authors start from a data
base containing shape parameters with their probability density
functions for stairs, doors and windows. They use scoring functions
for the prediction of parts of the facade. A review on automatic 3D
building reconstruction is given by Haala and Kada (2010).

Extracting features of a building from point clouds of a laser scanner
uses only a part of the geometric information which is available
in the measurements of a laser scanner. To make full use of
the information, it is proposed here to fit a free-form surface
to the heights of an object measured by a laser scanner. The
surface is represented by the tensor product of B-splines, and its
fit is efficiently computed by the lofting method Koch (2009a). It
has been shown by Monte Carlo simulations that such a fit can
be determined with an uncertainty which does not surpass the
uncertainty of the measurements Koch (2009b). By computing
points on the fitted B-spline surface with a density higher than
that of the measurements, the geometric information of the laser
scanning is preserved and is made available for points which have
not been observed. To visualize the 3D model established by the
points of high density, the B-spline surface is fitted in addition
to the intensity values of the reflected laser beam so that these
values are also available for the points of high density. Because
of the high resolution, the image does not need to be interpreted
like in the references cited above, the image speaks for itself. The
points of high density, which form the 3D model and which are
represented as range image, allow the extraction of geometric
information from any point of the model. Thus, a digital image of
high resolution containing 3D information is derived.

To obtain more expressive images, it is proposed that the points
of high density on the fitted B-spline surface get the colors of the
image of a digital camera. A digital photo of the object being
observed by the laser scanner is therefore needed. Because of
the high density of the points computed on the B-spline surface,
points of the range image can readily be determined which match
the points of the digital image of the camera. The coordinates of
the digital image are then transformed to the coordinates of the
range image by a translation, a rotation, a scale factor and possibly
by introducing parameters for the lens distortions, cf. Amiri Parian
and Gruen (2010). This gives the colors of the digital image for
each point of the 3D model. The color is introduced for pixels with
sizes which are equal to the distances between the points of the
model. Thus, a 3D model is obtained which is visualized by a digital
image with colors of a digital photo.

Objects with discontinuities, like edges of windows in a facade
of a building, have to be scanned with a high density of the
measurements. Most of the facade is smooth where the high
resolution is not needed, but it is fixed during the measurements
of laser scanners. Thus, there is a redundancy of information in
the measurements so that data compression is applied here by
the multi-scale representation of signals. Starting from the original
signal, a signal with a lower resolution is computed by a low-pass
filter. The difference in information with respect to the higher
resolution is called the detail signal and is obtained by a wavelet
representation. It results from a band-pass filter. The low-pass
filtered signal plus the detail signals give the original signal. Some
of the coefficients determining the detail signals might be small so
that they can be neglected, thus achieving data compression.

The multi-scale representation of signals goes back to Mallat (1989).
Quak and Weyrich (1994) based on Chui and Quak (1992) intro-
duced spline wavelets, Stollnitz et al. (1995a,b) Stollnitz, DeRose,
and Salesin used the spline wavelets together with the tensor
product for a multi-scale representation of higher dimensions.
Schmidt (2007) and Zeilhofer (2008) applied the data compres-
sion for ionospheric signals in two and three dimensions. The
n-dimensional multi-scale representation was presented by Zeil-
hofer (2008) and Schmutz (2010). Koch (2011) gave the details of
deriving the decomposition equations and the n-dimensional
multi-scale representation of points with high density on the B-
spline surface. The derivations use the Kronecker product which
does not lead to efficient computations if its definition is used.
However, B-spline surfaces of two dimensions are needed to set
up 3D models. It is shown here that for such a case the Kronecker
products can be replaced by matrix products. Efficient methods
exist to compute them, for instance, by the basic linear algebra
subroutines (BLAS) Demmel (1997) [p. 66].

To demonstrate the proposed method, a part of a facade with a
window was observed by a laser scanner. Because of the edges
caused by the window-sill and the deeper than the facade lying
window-frames and window-panes, a high density of the points
for the scan had to be chosen. This density is fixed during the
measurements of the laser scanner. As mentioned above, points on
the B-spline surface are re-computed with a density higher than that
of the measurements to make the geometric information of the laser
scanning available for a dense grid of points. Data compression is
applied. It should be mentioned that the information to compute
the detail signals is compressed, not the measurements of the
laser scanner. It turns out for the example that a large number
of wavelet coefficients which determine the detail signals can be
neglected.

The paper is organized as follows: Section 2 defines the two-
dimensional B-spline surface and derives the simultaneous estima-
tion and the lofting method for fitting it to the measured heights
and intensities of points. Section 3 covers the important part of
the paper: the multi-scale representation of signals with a density
higher than that of the measurements by the two-dimensional
B-spline surface. Section 4 presents the example and Section 5 the
conclusions.
2. Two-dimensional B-spline surface and its fit

A two-dimensional B-spline surface depends on two parameters which will be called $\xi_1, \xi_2$. It is expressed by the tensor product of the two B-spline basis functions $N_{1,q_1}(\xi_1), N_{2,q_2}(\xi_2)$ of degrees $q_1, q_2$ with, cf. Piegl and Tiller [17, p. 34],

$$s(\xi_1, \xi_2) = \sum_{i_1=0}^{l_1-1} \sum_{i_2=0}^{l_2-1} N_{1,i_1} q_1(\xi_1) N_{2,i_2} q_2(\xi_2) p_{i_1 i_2} \tag{1}$$

and

$$s(\xi_1, \xi_2) = \begin{bmatrix} x_1(\xi_1) \\ x_2(\xi_2) \\ H_1(\xi_1, \xi_2) \\ H_2(\xi_1, \xi_2) \end{bmatrix} \tag{2}$$

where the $4 \times 1$ vector $s(\xi_1, \xi_2)$ denotes a point on the B-spline surface with two-dimensional rectangular or curvilinear coordinates $x_1$ depending on $\xi_1$ and $x_2$ on $\xi_2$. The third and the fourth coordinates $H_1$ and $H_2$ are the two quantities to be represented by the surface. In the following example, $H_1$ will be the height of the surface with respect to the $x_1$, $x_2$-plane and $H_2$ the intensity of the reflected laser beam of a point measured by a laser scanner. The points

$$p_{i_1 i_2} = [x_{i_1}, x_{i_2}, H_{1,i_1 i_2}, H_{2,i_1 i_2}]^T \quad \text{with} \quad i_1 \in {0, \ldots, h-1}, i_2 \in {0, \ldots, l_2-1} \tag{3}$$

are the unknown control points, which the B-spline surface approximately follows.

The B-spline basis functions $N_{i,q}(\xi)$ are efficiently computed by a recursion formula derived by Cox (1972) and de Boor (1972). So-called knots are introduced as a sequence of nondecreasing real numbers in the interval $[0,1]$. The numbers $l_1, l_2$ of unknown control points depend on the numbers of knots for $\xi_1, \xi_2$ and on the degrees $q_1, q_2$.

The B-spline surface is fitted to the coordinates of points measured, for instance, by a laser scanner. Let the rectangular or curvilinear coordinates $x_1, x_2$ together with $H_1$ and $H_2$ of $e_1 \times e_2$ points $s(\xi_{1_{a_1}}, \xi_{2_{a_2}})$ be given, where $\xi_{1_{a_1}}$, with $a_1 \in \{1, \ldots, e_1\}$ and $\xi_{2_{a_2}}$ with $a_2 \in \{1, \ldots, e_2\}$ denote the location parameters which shall be known. Eq. (1) then leads to a linear relation between the unknown control points $p_{i_1 i_2}$ and the given points $s(\xi_{1_{a_1}}, \xi_{2_{a_2}})$. The observation equations for estimating $p_{i_1 i_2}$ in a linear model are therefore given by

$$\sum_{i_1=0}^{l_1-1} \sum_{i_2=0}^{l_2-1} N_{1,i_1} q_1(\xi_{1_{a_1}}) N_{2,i_2} q_2(\xi_{2_{a_2}}) p_{i_1 i_2} = s(\xi_{1_{a_1}}, \xi_{2_{a_2}}) + e(\xi_{1_{a_1}}, \xi_{2_{a_2}}), \quad a_1 \in \{1, \ldots, e_1\}, a_2 \in \{1, \ldots, e_2\} \tag{4}$$

where $e(\xi_{1_{a_1}}, \xi_{2_{a_2}})$ denotes the vector of errors of $s(\xi_{1_{a_1}}, \xi_{2_{a_2}})$. There are $e_1 \times e_2$ linear equations for determining $l_1 \times l_2$ unknown control points so that $e_1 \times e_2 \geq l_1 \times l_2$ must hold.

As shown in the following, the tensor product of the observation equations (4) can be expressed by the Kronecker product

$$(N(\xi_2) \otimes N(\xi_1)) \vec{D} = \vec{S} + \vec{E} \tag{5}$$

where the $e_1 \times l_1$ matrix $N(\xi_1)$ of B-spline basis functions is given by

$$N(\xi_1) = \begin{bmatrix} N_{0,q_1}(\xi_{11}) & \cdots & N_{h-1,q_1}(\xi_{11}) \\ N_{0,q_2}(\xi_{1e_1}) & \cdots & N_{h-1,q_2}(\xi_{1e_1}) \end{bmatrix} \tag{6}$$

and correspondingly the $e_2 \times l_2$ matrix $N(\xi_2)$. The $l_1 \times l_2$ matrix $D$ of unknown control points is defined by

$$D = \begin{bmatrix} p_{00} & \cdots & p_{0,l_2-1} \\ \vdots & \ddots & \vdots \\ p_{l_1-1,0} & \cdots & p_{l_1-1,l_2-1} \end{bmatrix} \tag{7}$$

and with

$$s(\xi_{1_{a_1}}, \xi_{2_{a_2}}) = s_{a_1 a_2} \tag{8}$$

the $e_1 \times e_2$ matrix $S$ of given points by

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1e_2} \\ \vdots & \ddots & \vdots \\ s_{e_11} & \cdots & s_{e_1 e_2} \end{bmatrix} \tag{9}$$

The $e_1 \times e_2$ matrix $E$ of errors is obtained with replacing $s$ by $e$ in (8) and (9).

Eq. (5) can be obtained by writing (4) in matrix notation

$$(N(\xi_2) \otimes N(\xi_1))D^T \vec{S} = S + E \tag{10}$$

and by transforming this matrix equation for $D$ to the system (5) of linear equations Koch (1999) [p. 41]. The simultaneous estimate $\vec{D}$ of $\vec{D}$ for the unknown control points follows from (5) by

$$\vec{D} = [(N(\xi_2) \otimes N(\xi_1))^T(N(\xi_2) \otimes N(\xi_1))]^{-1} \times (N(\xi_2) \otimes N(\xi_1))^T \vec{S} \tag{11}$$

If one expands this result by the rules of the Kronecker product, one obtains the least-squares method for estimating the unknown control points Koch (2009a)

$$\vec{D} = (N(\xi_2)N(\xi_1)^T)^{-1} N(\xi_1)^T \vec{S} \times (N(\xi_2)N(\xi_1)^T)^{-1}. \tag{12}$$
Instead of solving for the simultaneous estimation the \((I_1 \times I_2) \times (I_1 \times I_2)\) system of linear equations (11), the two \(I_1 \times I_1\) and \(I_2 \times I_2\) systems (12) of linear equations have to be solved for the lofting method which saves a considerable amount of computing time. By generalizing the derivations of Koch (2010a), it was shown by Koch and Schmidt (2011) that the lofting method and the simultaneous estimation of the control points give identical results also in case of \(n\)-dimensional B-spline surfaces if the measured points are arranged in regular grids.

The estimate \(\hat{\sigma}^2\) of the vector \(\sigma^2\) of variance factors for each of the four coordinates \(x_1, x_2, H_1, H_2\) follows from cf. Koch (2007) [p. 96],

\[
\hat{\sigma}^2 = \text{vec} E^T \text{vec} \hat{E}/(e_1, e_2 - I_1 I_2)
\]

with the matrix \(\hat{E}\) of residuals from (10) by

\[
\hat{E} = N(\xi_1) \hat{D} N^T(\xi_2) - S.
\]

Thus, by summing the squares of the residuals of \(x_1, x_2, H_1\) or \(H_2\) and by dividing by the degrees of freedom, the variances of these quantities are obtained Koch (2010b). It can be shown by Monte Carlo simulations that fitting a B-spline surface can be accomplished with an uncertainty which agrees with the uncertainty of the measurements Koch (2009b).

Surfaces with discontinuities such as edges have to be scanned by a dense grid of points. In order to preserve the high resolution of the measurements when extracting geometrical information from the B-spline surface, points on the surface are computed with a density higher than that of the measurements. Let the points on the fitted surface have the given location parameters \(\xi_{1w}, \xi_{2w}\) with \(w_1 \in \{1, \ldots, v_1\}, w_2 \in \{1, \ldots, v_2\}\). They are collected in the \(v_1 \times v_2\) matrix \(S_w\) which is defined by replacing \(e_1, e_2\) in (9) by \(v_1, v_2\). The matrix \(S_w\) is computed with (10) by

\[
S_w = N(\xi_{1w}) \hat{D} N^T(\xi_{2w})
\]

where the \(v_1 \times I_1\) matrix \(N(\xi_{1w})\) and the \(v_2 \times I_2\) matrix \(N(\xi_{2w})\) of B-spline basis functions are obtained correspondingly to (6).

3. Multi-scale representation of signals by a two-dimensional B-spline surface

The multi-scale representation of signals by a B-spline surface, for instance points on a B-spline surface, starts with a certain level \(j \in \mathbb{N}_0\) of resolution. Signals of lower levels of resolution are low-pass filtered versions of the signal of the higher level \(j\). The difference in information with respect to the higher level is expressed by the detail signals. They are band-pass filtered versions of the signal of the higher level and obtained by a wavelet representation. The derivation of the \(n\)-dimensional multi-scale representation by Schmidt (2010) and Koch (2011) is specialized here to two dimensions. The Kronecker products can then be replaced by matrix products as mentioned in the introduction. The so-called scaling functions \(\Phi_{j+1,q_1}(\xi_1)\) and \(\Phi_{j+2,q_2}(\xi_2)\) of level \(j_1\) and \(j_2\) for \(\xi_1\) and \(\xi_2\) of a multi-scale representation are identified with the B-spline basis functions in (1), thus

\[
\Phi_{j+1,q_1}(\xi_1) = N_{j+1,q_1}(\xi_1), \Phi_{j+2,q_2}(\xi_2) = N_{j+2,q_2}(\xi_2).
\]

The number \(I_1\) and \(I_2\) of scaling functions are equal to the number of control points for the parameters \(\xi_1\) and \(\xi_2\) in (1). \(I_1\) and \(I_2\) are determined by

\[
I_1 = 2^0 + q_1, I_2 = 2^0 + q_2
\]

and the level \(j\) of resolution by

\[
j = \max(j_1, j_2).
\]

The signal \(s_j(\xi_1, \xi_2)\) of level \(j\) follows from (1) with (5) by

\[
s_j(\xi_1, \xi_2) = (\Phi_{j,q_1}^T(\xi_1) \otimes \Phi_{j,q_2}^T(\xi_2))d_j
\]

where \(\Phi_{j,q_1}^T(\xi_1)\) denotes the \(I_1 \times 1\) vector of scaling functions

\[
\Phi_{j,q_1}(\xi_1) = [\Phi_{j,0,q_1}(\xi_1), \ldots, \Phi_{j,2^0-1,q_1}(\xi_1)]^T
\]

and accordingly \(\Phi_{j,q_2}(\xi_2)\) the \(I_2 \times 1\) vector of scaling functions. The \((I_1 \times I_2) \times 1\) vector \(d_j\) of the so-called scaling coefficients follows with \(d_j = \text{vec} D_j = \text{vec} D\) from (7). By applying (10), the signal \(s_j(\xi_1, \xi_2)\) is obtained by

\[
s_j(\xi_1, \xi_2) = \Phi_{j,q_1}^T(\xi_1) D \Phi_{j,q_2}^T(\xi_2).
\]

We introduce for level \(j_1 = 0\) the wavelet function \(\Psi_{j_1,q_1}(\xi_1)\) with \(l \in \{0, 1, \ldots, L_{j_1} - 1\}\), \(L_{j_1} = I_1 - j_1\) and the \(L_{j_1} \times 1\) vector \(\Psi_{j_1}(\xi_1)\) of wavelet functions

\[
\Psi_{j_1}(\xi_1) = [\Psi_{j_1,0,q_1}(\xi_1), \ldots, \Psi_{j_1,2^0-1,q_1}(\xi_1)]^T
\]

and accordingly the \(L_{j_1} \times 1\) vector \(\Psi_{j_1}(\xi_2)\) of wavelet functions. The vector \(\Phi_{j_1}(\xi_1)\) from (20) is transformed to the \(I_1 \times 1\) vector \(\Phi_{j_1}(\xi_1)\) of the lower level \(j_1 = 1\) by the \(I_1 \times I_1\) matrix \(P_{j_1}\) of constants

\[
\Phi_{j_1}(\xi_1) = \Phi_{j_1}(\xi_1) P_{j_1}.
\]
The $I_{n-1} \times 1$ vector $\Psi_n^{-1}(\xi_1)$ of wavelet functions is computed by the $I_n \times I_{n-1}$ matrix $Q_n$ of constants

$$\Psi_n^{-1}(\xi_1) = \Phi_n(\xi_1)Q_n.$$  \hfill (24)

Correspondingly, the $I_{n-1} \times 1$ vector $\Phi_n(\xi_1)$ and the $I_{n-1} \times 1$ vector $\Psi_n^{-1}(\xi_1)$ are transformed. Eqs. (23) and (24), which are called two-scale relations, are also applicable for transformations to lower levels than $j_1 - 1$ and $j_2 - 1$. The matrices $P_n$ and $Q_n$ are found for $j_1 \in \{1, 2, \ldots\} $ and $q_1 \in \{1, 2, 3\}$ by Stollnitz et al. (1995a) Stollnitz, DeRose, and Salesin. For determining $Q_n$, special conditions have been imposed.

In case of a two-dimensional multi-scale representation, there are $2^2 - 1$ detail signals so that the signal $s_{j_1}(\xi_1, \xi_2)$ follows from the smoothed signal $s_{j-1}(\xi_1, \xi_2)$ and the three detail signals $g^1_{j-1}(\xi_1, \xi_2)$ to $g^3_{j-1}(\xi_1, \xi_2)$

$$s_{j_1}(\xi_1, \xi_2) = s_{j_1}(\xi_1, \xi_2) + g^1_{j-1}(\xi_1, \xi_2) + g^2_{j-1}(\xi_1, \xi_2) + g^3_{j-1}(\xi_1, \xi_2)$$ \hfill (25)

with $s_{j-1}(\xi_1, \xi_2)$ from (19) and (21)

$$s_{j-1}(\xi_1, \xi_2) = (\Phi_n^{-1}(\xi_1) \otimes \Phi_n^{-1}(\xi_2))d_{j-1} = \Phi_n^{-1}(\xi_1)D_{j-1} \Phi_n^{-1}(\xi_2)$$ \hfill (26)

and with the detail signals

$$g^1_{j-1}(\xi_1, \xi_2) = (\Psi_n^{-1}(\xi_1) \otimes \Phi_n^{-1}(\xi_2))c_{j-1}^1 = \Phi_n^{-1}(\xi_1)C_{j-1} \Psi_n^{-1}(\xi_2)$$
$$g^2_{j-1}(\xi_1, \xi_2) = (\Phi_n^{-1}(\xi_1) \otimes \Psi_n^{-1}(\xi_2))c_{j-1}^2 = \Psi_n^{-1}(\xi_1)C_{j-1} \Phi_n^{-1}(\xi_2)$$
$$g^3_{j-1}(\xi_1, \xi_2) = (\Psi_n^{-1}(\xi_1) \otimes \Psi_n^{-1}(\xi_2))c_{j-1}^3 = \Psi_n^{-1}(\xi_1)C_{j-1} \Psi_n^{-1}(\xi_2).$$ \hfill (27)

The $(I_{n-1} \times L_{n-1}) \times 1$ vector $c_{j-1}^1$ of the so-called wavelet coefficients follows from the $(I_{n-1} \times L_{n-1})$ matrix $C_{j-1}$ by $c_{j-1} = \operatorname{vec} C_{j-1}$. The matrix $C_{j-1}$ contains as elements the $4 \times 1$ vectors of coordinates of the points which determine the detail signal $g^1_{j-1}(\xi_1, \xi_2)$. Correspondingly, the $(I_{n-1} \times L_{n-1}) \times 1$ vector $c_{j-1}^2$ and the $(I_{n-1} \times L_{n-1}) \times 1$ vector $c_{j-1}^3$ of wavelet coefficients are obtained.

The vectors of scaling and wavelet coefficients of the lower level $j-1$ result with the vector $d_j$ of scaling coefficients of level $j$ from the so-called decomposition equations

$$d_{j-1} = (P_n \otimes P_n) d_j$$
$$c_{j-1}^1 = (Q_n \otimes P_n) d_j$$
$$c_{j-1}^2 = (P_n \otimes Q_n) d_j$$
$$c_{j-1}^3 = (Q_n \otimes Q_n) d_j.$$ \hfill (28)

The $(I_{n-1} \times I_{n-1})$ matrix $P_n$ and the $(I_{n-1} \times I_{n-1})$ matrix $Q_n$ follow from the matrices $P_n$ and $Q_n$ in (23) and (24) by

$$[P_n] = |P_n, Q_n|^{-1}$$ \hfill (29)

and correspondingly $P_n$ and $Q_n$.

The Kronecker products in (28) are replaced by matrix products, thus with (9) and (10)

$$D_{j-1} = P_n D \xi_1 
C_{j-1} = P_n D \xi_2 
C_{j-1}^{-1} = Q_n D \xi_1 
C_{j-1}^{-1} = Q_n D \xi_2.$$ \hfill (30)

By collecting these four equations in one equation, we find an efficient way of computing the matrices of scaling and wavelet coefficients

$$D_{j-1} C_{j-1}^{-1} = |P_n, Q_n| D |P_n, Q_n|^{-1}.$$ \hfill (31)

If $s_{j-1}(\xi_1, \xi_2)$ is expressed by $s_{j-1}(\xi_1, \xi_2)$ plus detail signals of level $j-2$, if $s_{j-2}(\xi_1, \xi_2)$ is expressed by $s_{j-3}(\xi_1, \xi_2)$ plus detail signals of level $j-3$ and so on and if the results are substituted in (25), we get

$$s_{j_1}(\xi_1, \xi_2) = s_{j-m}(\xi_1, \xi_2) + \sum_{k=1}^{m} (g^1_{j-k}(\xi_1, \xi_2) + g^2_{j-k}(\xi_1, \xi_2))$$ \hfill (32)

with $m$ chosen to be

$$m = \min(j_1, j_2)$$ \hfill (33)

and $s_{j-m}(\xi_1, \xi_2)$ from (26) by replacing $j-1$ by $j-m$ and accordingly $g^1_{j-k}(\xi_1, \xi_2)$ to $g^3_{j-k}(\xi_1, \xi_2)$ from (27). The signal $s_{j_1}(\xi_1, \xi_2)$ is expressed by the low-pass filtered signal $s_{j-m}(\xi_1, \xi_2)$ and by the sum of detail signals which are band-pass filtered versions of $s_{j_1}(\xi_1, \xi_2)$. Some wavelet coefficients in $C_{j-k}, C_{j-k}^{-1}$ might be small so that they can be neglected to compress the data. The multi-scale representation of signals by the B-spline surface starts with the simultaneous estimate of $\hat{D}_j$ of the matrix of control points by (11) or with the estimate $\hat{D}_j$ by the lifting method (12). The points on the B-spline surface arranged in a dense grid and collected in the $v_1 \times v_2$ matrix $S_n$ from (15) shall be determined by a multi-scale representation. The index $j$ is now added, i.e. $S_n$, to indicate the level $j$ of resolution. The given location parameters
are again $\xi_{1w}, \xi_{2w}$ with $w_1 \in \{1, \ldots, v_1\}, w_2 \in \{1, \ldots, v_2\}$. Eq. (32) is applied in connection with (26) and (27) to compute $y$.

$$N_{h}(\xi_{1w}) = \begin{bmatrix} N_{h_1}(\xi_{11}) \cdots N_{h_{1-q_1}}(\xi_{11}) \\ \vdots \\ N_{h_1}(\xi_{1v_1}) \cdots N_{h_{1-q_1}}(\xi_{1v_1}) \end{bmatrix}$$  

where $N_{h}(\xi_{1w})$ is a $v_1 \times l_{h}$ matrix. It is transformed to the lower level $j_1 - 1$ as in (23) by

$$N_{h_{1-1}}(\xi_{1w}) = N_{h}(\xi_{1w})P_{h}$$

where the $v_1 \times l_{h-1}$ matrix $N_{h_{1-1}}(\xi_{1w})$ contains the B-spline basis functions of level $j_1 - 1$. Accordingly, the $v_2 \times l_{h-1}$ matrix $N_{h_{2-1}}(\xi_{2w})$ of B-spline basis functions is obtained.

Correspondingly, the $v_2 \times l_{j-1}$ matrix $W_{h_{j-1}}(\xi_{2w})$ and the matrices of the lower levels of resolution are obtained. The points of level $j$ on the B-spline surface in the $v_1 \times v_2$ matrix $S_{jw}$ are then computed by

$$S_{jw} = N_{j_{1-1}}(\xi_{1w})D_{j_{1-1}}N_{j_{2-1}}^{T}(\xi_{2w}) + \sum_{k=1}^{m}[N_{h_{1-1}}(\xi_{1w})C_{1-k}^{1}W_{j_{1-1}}^{T}(\xi_{2w}) + W_{h_{1-1}}(\xi_{2w})C_{2-k}^{2}N_{j_{2-1}}^{T}(\xi_{2w}) + W_{h_{1-1}}(\xi_{2w})C_{3-k}^{3}W_{j_{2-1}}^{T}(\xi_{2w})].$$

The matrices of scaling and wavelet coefficients follow according to (31).

4. Example

The coordinates of points of a part of a facade with a window are measured by the laser scanner Leica HDS 3000. Fig. 1 shows a cut of a digital photo of an amateur camera with $967 \times 1962$ pixels which contains the scanned part of the facade. It was taken from the position of the laser scanner. The coordinates are determined in the local coordinate system of the laser scanner. Its origin lies in the center of the instrument, the $z$-axis points to the zenith and the $y$-axis coincides with the center of lines of sight of the instrument which hits the facade of Fig. 1 approximately in the middle of the window-sill. The $x$-axis is perpendicular to $y$ and $z$ and points to the right of Fig. 1. Because of the protruding window-sill and the deeper than the facade lying window-frames and window-panes, a high density of points had to be selected for the scans. Parallel to the $x$-axis 593 points were scanned with $-0.74 \leq x \leq 0.73$ m and along the $z$-axis 1 170 points with $-1.39 \leq z \leq 1.65$ m. This means a point distance of about $2.5$ mm along the $x$-axis and $2.6$ mm along the $z$-axis. The $y$-coordinates vary between $4.04 \leq y \leq 4.25$ m. The glass of the windows reflects the laser beams almost like a plane surface.

A two-dimensional B-spline surface is fitted by (4) to the measured coordinates applying the lofting method (12). The coordinate $x_1$ in (2) is identical with the $x$-coordinate of the coordinate system.
Figure 2. Measured points plus residuals of the fit with colors of the measured intensities plus residuals

of the laser scanner, \( x_2 \) is replaced by the \( z \)-coordinate and the height \( H_1 \) is obtained by \( y_m - y \) with \( y_m \) being the maximum value of \( y \). The coordinate \( H_2 \) results from the intensity of the reflected laser beam of a point measured by the laser scanner. We set \( q_1 = q_2 = 3 \) in (4). The number \( I_1 \) of control points in (4) then follows from (17) by selecting the resolution level \( j_1 = 9 \) for \( \xi_1 \) with \( l_1 = l_m = 515 \) and \( I_2 \) by choosing the resolution level \( j_2 = 10 \) for \( \xi_2 \) with \( l_2 = l_2 = 1 \, 027 \). The level \( j \) of resolution is therefore \( j = 10 \) according to (18). The residuals of (14) from fitting the B-spline surface to \( H_1 \) and \( H_2 \) are added to the measured heights and to the measured intensities so that the points with their fitted intensities on the B-spline surface are obtained. They are shown in Fig. 2 where the colors of the intensities result from the software ‘Cyclone 7.1’ of the Leica HDS 3000. The coordinate system of the laser scanner was rotated for Fig. 2. The points of the upper right corner of the window are depicted as seen from the left in comparison to Fig. 1. The distribution of the points on the window-frame is regular, while the distribution on the window-pane is somewhat irregular because the glass does not reflect the laser beams like an exact plane as mentioned above.

The standard deviation \( \sigma_{H_1} \) of the height \( H_1 \) results from (13) with

\[
\sigma_{H_1} = 2.1 \text{ mm}.
\]

It is of the order of magnitude with which B-spline surfaces are fitted to heights for distances of the laser scanner to the surfaces of about 5 m Koch(2011).

Points and their intensities with a density higher than that of the measurements are computed on the fitted B-spline surface by (15). As an example, a grid of 830 points parallel to the \( x \)-axis with distances of about 1.8 mm and 1 638 points along the \( z \)-axis with distances of 1.9 mm is chosen. Fig. 3 shows the computed points again at the upper right corner of the window. It should be noted that the positions of the computed points differ from the ones of the measured points. A moderate densification was chosen to enable the comparison between Fig. 2 and Fig. 3. The points of higher density of Fig. 3 fit very well between the measured points plus residuals of Fig. 2.

To apply a data compression, the 830 \( \times \)1 638 points are also computed by the multi-scale representation (37). As mentioned
above, the B-spline surface is estimated for the resolution level \( j = 10 \). The multi-scale representation therefore starts with \( j = 9 \) and ends with \( j = 1 \) because of \( m = 9 \) from (33). The number of scaling coefficients and wavelet coefficients for the different levels \( j \) are given in Table 1. As can be seen, the number of scaling coefficients plus the number of wavelet coefficients of the same level give the number of scaling coefficients of the next higher level. Thus, the number of scaling coefficients plus the number of wavelet coefficients of the same level plus the number of wavelet coefficients of all higher levels give the number of scaling coefficients of the highest level. This is a consequence of (32).

The maximum absolute values of the wavelet coefficients for the height \( H_1 \) of each level \( j \) are computed. It is assumed that the absolute values below 3% of the maximum value can be neglected. The number of the neglected wavelet coefficients are given for each level in Table 1. The 830 \( \times \) 1638 points are computed by (37) for level \( j = 9 \) to \( j = 1 \) with all wavelet coefficients and with the wavelet coefficients minus the neglected ones. The coordinate differences are formed. The root mean square (rms) differences for the height \( H_1 \) are presented in Table 1. The rms differences for the coordinates \( x \) and \( z \) are less than a factor of 0.23 smaller than the

\[
\text{Table 1. Number of scaling and wavelet coefficients and number of neglected wavelet coefficients with rms difference for decreasing levels}
\]

<table>
<thead>
<tr>
<th>Level ( j )</th>
<th>Scaling coefficients</th>
<th>Wavelet coefficients</th>
<th>Neglected wavelet coefficients</th>
<th>rms difference in [mm]</th>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>395 520</td>
<td>267 974</td>
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<tr>
<td>8</td>
<td>33 929</td>
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</tr>
<tr>
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<td>25 152</td>
<td>20 805</td>
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<tr>
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<td>2 345</td>
<td>6 432</td>
<td>5 108</td>
<td>1.6</td>
</tr>
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<td>1 680</td>
<td>1 342</td>
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<td>1</td>
<td>20</td>
<td>15</td>
<td>5</td>
<td>2.2</td>
</tr>
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</table>
Figure 4. Computed points and their intensities on the fitted B-spline surface after a data compression at level $j = 9$

compression. Fig. 4 shows the results. Almost no change is visible between Fig. 3 without data compression and Fig. 4 with data compression except that the distribution of the points is slightly smoother. As indicated in Table 1, $267,974$ of the $395,520$ wavelet coefficients of level $j = 9$ have been neglected to compute the points of Fig. 4, which is about $50.7\%$ of the $528,905$ scaling coefficients of level $j = 10$. If one would have chosen the data compression at level $j = 8$ with a slightly worse result than Fig. 4, $267,974 + 71,393$ wavelet coefficients would have been neglected which is about $64.2\%$ of the $528,905$ scaling coefficients of level $j = 10$.

The range image of $919 \times 1,814$ points on the fitted B-spline surface is computed by their intensities. It is used to determine points which match the points of the digital photo of Fig. 1. With the distances between the points of about $1.6$ mm along the $x$-axis and $1.7$ mm along $z$, the range image approaches the resolution of $967 \times 1,962$ pixels of the digital photo of Fig. 1 with a size of the pixels of $1.5 \times 1.5$ mm$^2$ expressed by the coordinates of the laser scanner. The coordinates of the digital photo are transformed to the coordinates of the laser scanner and the $919 \times 1,814$ points obtain instead of their intensities the colors of the RGB values of the digital photo. This can be interpreted such that almost each pixel of the digital photo gets a 3D coordinate. The result is shown in Fig. 5. The software “Cyclone 7.1” of the Leica HDS 3000 was used. Fig. 5 does not quite reach the quality of Fig. 1 so that there is room for improving the software. In contrast to Fig. 1, Fig. 5 represents a 3D model from which geometric information can be extracted for points about $1.6$ mm in the $x$-direction and $1.7$ mm in $z$-apart. The coordinates of the points can be rotated, the scale can be enlarged, to obtain images like Fig. 2 to Fig. 4, where the colors of the intensity values are replaced by the colors of Fig. 1.

5. Conclusions

To obtain a digital image of high resolution with 3D geometry of the surface of an object, a two-dimensional B-spline surface is fitted to the measured heights by the lofting method. A dense grid of points together with their intensities is computed on the fitted surface. For a realistic visualization, the points on the surface get instead of the intensities the RGB values of a digital photo. Data compression is applied for an efficient computation and storage of the 3D model. It is based on the multi-scale representation of the dense grid of points on the B-spline surface. It turns out
that more than 50% of the coefficients which determine the fitted B-spline surface can be neglected. Data compression therefore recommends itself especially if different 3D models have to be built up for grids of points with different densities, high resolutions for models with RGB values included. The decision on neglecting wavelet coefficients is based on rms height differences. Additional statistical methods such as Monte Carlo simulations should be applied in further investigations to check this decision.

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References


