Application of the BEM approach for a determination of the regional marine geoid model and the mean dynamic topography in the Southwest Pacific Ocean and Tasman Sea

Research Article

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Abstract:
We apply a novel approach for the gravimetric marine geoid modelling which utilise the boundary element method (BEM). The direct BEM formulation for the Laplace equation is applied to obtain a numerical solution to the linearised fixed gravimetric boundary-value problem in points at the Earth’s surface. The numerical scheme uses the collocation method with linear basis functions. It involves a discretisation of the Earth’s surface which is considered as a fixed boundary. The surface gravity disturbances represent the oblique derivative boundary condition. The BEM approach is applied to determine the marine geoid model over the study area of the Southwest Pacific Ocean and Tasman Sea using DNSC08 marine gravity data. The comparison of the BEM-derived and EGM2008 geoid models reveals that the geoid height differences vary within -25 and 18 cm with the standard deviation of 6 cm. The DNSC08 sea surface topography data and the new marine geoid are then used for modelling of the mean dynamic topography (MDT) over the study area. The local vertical datum (LVD) offsets estimated at 15 tide-gauge stations in New Zealand are finally used for testing the coastal MDT. The average value of differences between the MDT and LVD offsets is 1 cm.

Keywords:
boundary element method • geoid • gravity • mean dynamic topography • mean sea surface

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1. Introduction

Various methods for modelling the mean dynamic topography (MDT) have been developed and applied (Hernandez et al., 2001). These methods utilise the historical hydrographical evidence (e.g., Levitus and Boyer, 1994; Levitus et al., 1994), circulation models (Semtner and Chervin, 1992; Stammer et al., 1996; Castruccio et al., 2008), climatic models (Levitus, 1982; Glenn et al., 1991; Qiu et al., 1991; hikawa et al., 1995), and altimetry and gravity data (Ekman and Mäkinen, 1996; Blinken and Koch, 2001; Gourdeau et al., 2003; Vannier et al., 2007; Fenoglio et al., 2007; Forsberg et al., 2007; Foreman et al., 2008; Andersen and Knudsen, 2009; Karimi and Ardalan, 2010, and others). Rio and Hernandez (2004), Rio et al. (2005, 2007), and Maximenko et al. (2009) combined various models and data for modelling MDT. Vossepoel (2007) summarised and compared various existing MDT solutions. In this study, we apply a novel approach based on the boundary element method (BEM) to compute the marine geoid model and to determine MDT from altimetry and gravity disturbance data. The
The main advantage of this method is in using the gravity disturbance data for the gravimetric marine geoid modelling compared to the traditional methods based on using the gravity anomaly data. This is particularly important in cases when airborne gravity data become available over large areas of the ocean. The mathematical formulation of the BEM approach is reviewed in Section 2. The new marine geoid model and the altimetry-derived mean sea surface (MSS) data are used to validate our results. The input data description is given in Section 3. The results are presented, validated, and discussed in Sections 4–6. The summary and conclusions are given in Section 7.

2. Methodology

The linearised fixed gravimetric boundary-value problem represents an exterior oblique derivative problem for the Laplace equation. It is defined as (Roch and Pope, 1972; Grafarend, 1989)

\begin{align}
\Delta T(x) &= 0, \quad x \in \mathbb{R}^3 - \Omega \\
\langle \nabla T(x), s(x) \rangle &= -\delta g(x), \quad x \in \Gamma \\
T(x) &= O(|x|)^{-1} \quad x \to \infty
\end{align}

where \( T \) is the disturbing potential (i.e., difference between the actual gravity potential \( W \) and the normal gravity potential \( U \)) at the point \( x \), and \( \delta g \) is the gravity disturbance. The domain \( \Omega \) represents the body of the Earth with its boundary \( \Gamma \) given by the Earth’s surface. \( \langle \nabla T, s \rangle \) is the inner product of two vectors \( \nabla T \) and \( s \). The vector \( s \) is defined as

\[ s(x) = -\frac{\nabla U(x)}{|\nabla U(x)|}, \quad x \in \Gamma \]

The expression in eq. (2) represents the oblique derivative boundary condition as the normal to the Earth’s surface \( \mathbf{n}_\Gamma \) does not coincide with the vector defined in eq. (4). The direct BEM formulation of the Laplace equation leads to a boundary integral equation that can be derived using Green’s third identity or through the method of weighted residual (Brebis et al., 1984; Schatz et al., 1990). A main advantage arises from the fact that only the boundary of the solution domain requires a subdivision into its elements. Thus, the dimension of the problem is effectively reduced by one.

The application of the direct BEM to the linearised fixed gravimetric boundary-value problem (eqs. (1)–(3)) yields the boundary integral equation in the following form (Cunderlik et al., 2008)

\[ \frac{1}{2} T(x) + \int_\Gamma T(y) \frac{\delta G}{\delta \mathbf{n}_\Gamma}(x, y) dy = \int_\Gamma \frac{\delta T}{\delta \mathbf{n}_\Gamma}(y) G(x, y) dy, \quad x, y \in \Gamma \]

where \( x \) and \( y \) are the geocentric position vectors of the computation and moving (integration) points, respectively, and \( \mathbf{n}_\Gamma \) is the normal to the boundary \( \Gamma \). The kernel function \( G \) in eq. (5) represents the fundamental solution to the Laplace equation

\[ G(x, y) = \frac{1}{4\pi|x-y|}, \quad x, y \in \mathbb{R}^3 \]

In order to handle the oblique derivative problem we used the same simplification as proposed by Cunderlik et al. (2008). According to the oblique derivative boundary condition in eq. (2), the negative value of the gravity disturbance \( \delta g \) is defined as a projection of the vector \( \nabla T(x) \) onto the vector \( s(x) \). The normal derivative term \( \delta T/\delta \mathbf{n}_\Gamma \) on the right-hand side of the boundary integral equation in eq. (5) approximately equals \( \delta T/\delta \mathbf{n}_\Gamma \approx -\delta g(x) \cos \mu(x) \), where \( \mu(x) \) is the angle \( \angle(n(x), s(x)) \). This term represents the projection of the vector \( \delta g(x)s(x) \) onto the normal \( \mathbf{n}_\Gamma(x) \). In this way the oblique derivative boundary condition in eq. (2) is incorporated to the direct BEM formulation in eq. (5). The boundary integral equation in eq. (5) is discretised by means of using the collocation method. It involves a discretisation of the Earth’s surface by a triangulation of the topography and approximations of the boundary functions by linear functions on each triangular panel using linear basis functions \( \{ \psi_j : j = 1, 2, ..., N \} \). This is realised by the piece-wise linear polynomials defined on the planar triangular panels, where vertices of this triangulation represent the collocation points. The boundary integral equation in eq. (5) is then rewritten to the following discrete form (Cunderlik et al., 2008)

\[ c_i T_i \psi_i + \sum_{j=1}^{N} T_j \delta G_{ij} \delta \mathbf{n}_\Gamma \psi_j d\Gamma_j = \sum_{j=1}^{N} \delta g_j \delta G_{ij} \psi_j d\Gamma_j, \quad (i = 1, 2, ..., N) \]

where \( c_i \) represents the spatial segment bounded by the panels joined at the \( i \)-th collocation point, and \( N \) is the number of nodes. The discretised boundary integral equations in eq. (7) form the linear system of observation equations

\[ M \mathbf{t} = L \delta g, \]

where \( \mathbf{t} \) is the vector of unknown disturbing potential values \( T \) at the collocation points, and \( \delta g \) is the vector of observed gravity disturbances \( \delta g \). The coefficients of matrices \( M \) and \( L \) represent the integrals of the discrete form of the boundary integral equations in eq. (7). The discretisation of the integral operators is constrained by the weak singularity of kernel functions. The integrals with regular integrands approximated by the Gaussian quadrature and non-regular integrands (singular elements) require a special treatment.
For more details we refer readers to Cunderlik et al. (2008). In case of the oblique derivative boundary condition in eq. (2), or the Neumann boundary condition using the aforementioned projection, the matrix \( M \) represents a system matrix, and the known vector \( f = \Delta g \) is given on the right-hand side of eq. (8).

3. Input data

The geocentric positions of collocation points at the sea surface were determined using the DNSC08 MSS data (Andersen and Knudsen, 2009). The gravity data used for a geoid determination were extracted from the DNSC08 marine gravity database (Andersen et al., 2009). The DNSC08 marine gravity database and the DNSC08 MSS data are made available by the Danish National Space Centre (DNSC). Within New Zealand, we used the terrestrial gravity data from the GNS Science gravity database. The geocentric positions of onshore collocation points were determined from the topographical heights of detailed local and global elevation models and from the quasi-geoid heights. In our numerical study we used the \( 30 \times 30 \) arc-sec SRTM30PLUS_V5.0 global elevation data (Becker et al., 2009) and the \( 1 \times 1 \) arc-sec detailed digital terrain model of New Zealand (Columbus et al., 2011). The quasi-geoid heights at the collocation points were evaluated using the Earth Gravitational Model 2008 (EGM2008) complete to spherical harmonic degree 2160 (Pavlis et al., 2008).

The conversion of the gravity anomalies \( \Delta g \) to the gravity disturbances \( \delta g \) at the collocation points was realised using the following well-known formula (Heiskanen and Moritz, 1967)

\[
\delta g = \Delta g - \frac{\delta \gamma}{\delta h} \zeta, \tag{9}
\]

where \( \frac{\delta \gamma}{\delta h} \) is the linear normal gravity gradient computed based on the parameters of the GRS80 reference ellipsoid (Moritz, 1980), and \( \zeta \) is the height anomaly. The height anomalies \( \zeta \) in eq. (9) were calculated using the EGM2008 coefficients complete to spherical harmonic degree 2160. The gravity disturbances at the study area (bounded by the parallels of 20 and 70 arc-deg southern latitude and the meridians of 150 and 210 arc-deg eastern longitude) are shown in Fig. 1. The gravity disturbances vary from -306.4 to 317.3 mGal with the mean of 2.0 mGal, and the standard deviation is 37.3 mGal. The extreme values of marine gravity disturbances and the largest horizontal gravity gradients are located to the north of New Zealand along the boundary between the Pacific and Australian tectonic plates, with the largest positive values along the side of the Australian continental plate, while the largest negative values are situated at the side of the Pacific oceanic tectonic plate. The large spatial gravity variations in this area are explained by the subduction of the Pacific plate under the Australian plate. Under the South Island of New Zealand these two tectonic plates are colliding as oblique strike slip, while further south of New Zealand the Australian plate subducing under the Pacific plate. Here, the boundary between the oceanic and continental tectonic plates is less pronounced in the gravity signal.

![Figure 1. The gravity disturbances at the study area. The units are in milligals.](image)

### Table 1. Statistics of the BEM-derived and EGM2008 geoid models and their differences

<table>
<thead>
<tr>
<th>Geoid Models</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Mean [m]</th>
<th>STD [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>-63.97</td>
<td>65.57</td>
<td>4.30</td>
<td>31.06</td>
</tr>
<tr>
<td>EGM2008</td>
<td>-63.92</td>
<td>65.45</td>
<td>4.34</td>
<td>31.01</td>
</tr>
<tr>
<td>BEM—EGM2008</td>
<td>-0.25</td>
<td>0.18</td>
<td>-0.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

This area is characterised mainly by positive values of the gravity disturbances along the side of the Pacific oceanic plate. Elsewhere over areas of the open sea, the gravity disturbances are mostly negative.

4. Marine geoid model

The BEM approach requires the integration over the whole globe with applying the regional refinement (Cunderlik et al., 2008). In our numerical experiment the global rough triangulation over the whole Earth’s surface with the resolution of about 0.2 arc-deg was successively refined until the detailed resolution of 1.5 arc-min at the study area. The large-scale parallel computations were performed on the cluster with 16 processors and 128 GB of distributed internal memory using the standard MPI (Message Passing Interface) subroutines (Aoyama and Nakano, 1999). In order to reduce large memory requirements we eliminated the far-zones interactions using the ITG-GRACE03S satellite geopotential model (Mayer-Gürr, 2007). The arisen long-wavelength error surface was reduced after using four iterations (Cunderlik and Mikulčik, 2009). The regional geoid model compiled at the study area is shown in Fig. 2. The result exhibits large variations in the computed values of the geoid heights varying from -63.97 to 65.57 m (cf. Table 1). We further computed the EGM2008 geoid model within the study area with a spectral resolution complete to spherical harmonic degree 2160 in the zero-tide permanent system (for a definition of tidal systems we refer readers to e.g., IAG...
The differences between the BEM-derived and EGM2008 geoid models are shown in Fig. 3. Statistics of the BEM-derived and EGM2008 geoid models and their differences are given in Table 1. As seen in Fig. 3, the largest differences between the BEM and EGM2008 geoid models are found around the coast of the South Island. These large discrepancies are primarily explained by a low resolution of terrestrial gravity data from the GNS Science gravity database used in the BEM solution. When disregarding the coastal marine area up to 100 km offshore, the standard deviation of these differences improves from 6.0 cm (see Table 1) to 3.2 cm. Additional large differences (seen, for instance, along the boundaries between the Pacific and Australian tectonic plates) are spatially correlated with the input gravity data (cf. Fig. 1). A possible presence of these differences is due to using a different spatial resolution of the geoid solutions as well as due to numerical inaccuracies of BEM. The BEM solution exhibits some details in the geoid undulations which might not be realistic. The application of an optimal filtering method for removing these artefacts in the geoid signal is, therefore, crucial and will be investigated in forthcoming study.

5. MDT

The study area of the Southwest Pacific Ocean and Tasman Sea is characterised by the presence of two distinct upper water masses. Sub-Antarctic water mass in the south and the Subtropical South-West Pacific water mass in the north (Stewart, 2007). These two water masses merge in the Subtropical Convergence Region which is often characterised by comparatively rapid fluctuations in temperature and salinity between the warmer, more saline subtropical seawater and the cooler, less saline Sub-Antarctic seawater. The
Table 2. The comparison of the LVD offsets with the altimetry-gravimetric MDT solutions at 15 tide-gauge (TG) stations in New Zealand

<table>
<thead>
<tr>
<th>LVD (TG)</th>
<th>LVD offset [m]</th>
<th>MDT [m]</th>
<th>Differences [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DNSC08-EGM2008</td>
<td>DNSC08-BEM</td>
<td>LVD offset-DNSC08</td>
</tr>
<tr>
<td>One Tree Point 1964 (AAUV)</td>
<td>0.37</td>
<td>0.34</td>
<td>0.36</td>
</tr>
<tr>
<td>Auckland 1946 (ADNU)</td>
<td>0.12</td>
<td>0.30</td>
<td>0.35</td>
</tr>
<tr>
<td>Tararu 1952 (AF7G)</td>
<td>0.21</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Port Waikato 1954 (AFGQ)</td>
<td>0.32</td>
<td>0.28</td>
<td>0.34</td>
</tr>
<tr>
<td>Moturiki 1953 (AB4N)</td>
<td>0.19</td>
<td>0.21</td>
<td>0.25</td>
</tr>
<tr>
<td>Gisborne 1926 (AHQE)</td>
<td>0.10</td>
<td>0.26</td>
<td>0.20</td>
</tr>
<tr>
<td>Taiaiai 1970 (AGM1)</td>
<td>0.12</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Napier 1962 (B3YP)</td>
<td>0.24</td>
<td>0.32</td>
<td>0.19</td>
</tr>
<tr>
<td>Taikohe (ABRQ)</td>
<td>0.23</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>Nelson 1955 (AC4T)</td>
<td>0.20</td>
<td>0.12</td>
<td>0.01</td>
</tr>
<tr>
<td>Wellington 1953 (ABPC)</td>
<td>0.01</td>
<td>0.03</td>
<td>-0.08</td>
</tr>
<tr>
<td>Lyttelton 1937 (B4OV)</td>
<td>0.13</td>
<td>0.14</td>
<td>-0.01</td>
</tr>
<tr>
<td>Deep Cove 1960 (AEQ2)</td>
<td>0.30</td>
<td>0.06</td>
<td>0.38</td>
</tr>
<tr>
<td>Dunedin 1958 (AFE)</td>
<td>0.07</td>
<td>0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>Bluff 1955 (ABCC)</td>
<td>0.17</td>
<td>0.08</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

**Average [m]** | 0.00 | 0.01 |

6. Coastal MDT

The altimetry-gravimetric MDT solutions were tested along the coastline of New Zealand using GPS and levelling data. We used the average offsets of local vertical datums (LVDs) estimated by Tenzer et al. (2011) relative to the adopted geoidal geopotential value $W_0 = 626,386,956 \text{m}^2 \text{s}^{-2}$ (Burša et al., 1997; Burša et al., 2007) based on the analysis of GPS and levelling data in New Zealand. These values of the LVD offsets were compared with the MDT solutions at the locations of 15 tide-gauge stations used as the origins for LVDs. The results are summarised in Table 2. The differences between the LVD offsets and MDT solutions are between -18 and 24 cm (for DNSC08 and EGM2008-derived MDT) and between -23 and 23 cm (for DNSC08 and BEM-derived MDT). These large discrepancies between the LVD offsets and the MDT solutions are due to several factors such as a relatively low accuracy of the coastal altimetry results, errors in estimated MSL (caused by short term tide-gauge records), levelling networks realization over different time epoch and the effect of vertical motions (cf. Amos, 2010). Despite the existence of large differences between the LVD offsets and the MDT solutions, their average values are very small: the average of the differences between the LVD offsets and the altimetry-gravimetric MDT solutions, their average values are very small: the average of the differences between the LVD offsets and the DNSC08 and BEM-derived MDT is 1 cm, and the average of the differences between the LVD offsets and the DNSC08 and EGM2008-derived MDT is 0 cm.

7. Summary and conclusions

We have applied a novel approach to compute the marine geoid model over the study area of the Southwest Pacific Ocean and Tasman Sea. This approach utilises the direct formulation of BEM for solving the linearised fixed gravimetric boundary-value problem. The major advantage of this method is in using the...
gravity disturbance data instead of gravity anomalies which are often used as input data when applying traditional approaches. The new regional marine geoid model and DNSC08 MSS data were then used to determine MDT over the study area. The BEM-derived regional marine geoid model at the study area was compared with the EGM2008 global geoid model. The differences between both models are between -25 and 18 cm with the standard deviation of 6 cm. The largest discrepancies were found around the coast of the South Island and explained by a low resolution of terrestrial gravity data. When disregarding the coastal marine area up to 100 km offshore, the standard deviation of these differences decreased to 3.2 cm.

The GPS and levelling data were used for testing the MDT solutions along the coast of New Zealand. Despite the accuracy of the altimetry-derived MSS data close to the coastal areas is typically low and the accurate estimation of LVD offsets is restricted by several factors discussed, for instance, by Amos (2010), the test results revealed that the average values of the differences between the LVD offsets and both altimetry-gravimetric MDT solutions are not more than 1 cm.

The altimetry-gravimetric MDT solutions exhibited major features of the ocean currents and the corresponding surface seawater temperature distribution with a typical south-north horizontal temperature gradient and regional irregularities due to the coastal configuration and the geometry of the ocean bottom relief. These major features are characterised by the presence of two distinct upper water masses. Sub-Antarctic water mass in the south and the Subtropical South-West Pacific water mass in the north. The most characteristic irregularities of MDT to the south-west of New Zealand are explained by the Sub-Antarctic Front and associated cold Antarctic Circumpolar Current flow along the deep ocean floor to the east of the Campbell Plateau and Chatham Rise.

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