GPS baseline configuration design based on robustness analysis

Research Article

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Abstract:
The robustness analysis results obtained from a Global Positioning System (GPS) network are dramatically influenced by the configuration of the observed baselines. The selection of optimal GPS baselines may allow for a cost effective survey campaign and a sufficiently robust network. Furthermore, using the approach described in this paper, the required number of sessions, the baselines to be observed, and the significance levels for statistical testing and robustness analysis can be determined even before the GPS campaign starts. In this study, we propose a robustness criterion for the optimal design of geodetic networks, and present a very simple and efficient algorithm based on this criterion for the selection of optimal GPS baselines. We also show the relationship between the number of sessions and the non-centrality parameter. Finally, a numerical example is given to verify the efficacy of the proposed approach.

Keywords:
GPS network • Reliability • Robustness • Statistical testing • Type I and Type II errors

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1. Introduction

The Global Positioning System (GPS) networks are used for various types of surveys, such as topographic surveys, construction surveys and deformation surveys. Like traditional surveying, GPS networks are analyzed according to some design criteria. The main design criteria for surveying networks are precision, reliability, robustness and cost (Kuang 1996, Simkooei 2001a, Simkooei 2001b).

Precision refers to the quality of the network in terms of random errors. The precision of a GPS result is evaluated using the measures derived from the covariance matrix of the estimated parameters. Reliability is the ability of the network to sense and identify blunders in the observations; specifically, it is a measure of the maximum magnitudes of blunders that cannot be detected via statistical tests and the effect of undetected blunders on the estimated parameters (Baarda 1968). For blunders that are not detected by Baarda’s approach, the influence of these undetected errors on the network can be evaluated and solved using an approach by Vaníček et al. (1991). In their approach, traditional reliability analysis is augmented with the geometrical strength analysis using strain in a technique called robustness analysis.

In statistical testing, the probability α (Type I error) is defined as the risk of rejecting a good observation, while the confidence level is 1 − α. The probability β (Type II error) is defined as the risk of accepting a biased observation, while the power of the test is 1 − β. The null hypothesis H0 prevails under the existence of only random errors, while the alternative hypothesis H1 prevails under the existence of a blunder or systematic bias. Systematic biases are therefore characterized by a shift towards the alternative hypothesis H1, and away from H0. This translation is called the non-centrality parameter. The non-centrality parameter is a function of Type I and Type II error levels (Krakiwsky et al. 1993). The displacements of robustness analysis are used to quantitatively describe the effect of systematic biases on the network points. Therefore, robustness analysis results are heavily influenced by the chosen Type I and II error levels.

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Economic considerations also have to be taken into account while designing a geodetic network. An optimal network design should yield both a precise, reliable, and robust network and a cost-effective survey plan. For GPS networks, the number of baselines to be measured and thereby the number of sessions is an important cost criterion. A greater number of sessions may increase the robustness, but it leads to an expensive network. Furthermore, the selection of GPS baselines heavily influences the robustness of the network, which needs to be accounted for. Hence, the selection of optimal baselines is crucial in GPS network planning.

Confidence and power levels used in the statistical analysis of networks can also affect the robustness analysis results. Higher confidence and power levels lead to a larger non-centrality parameter, which in return means larger displacements at network points. Better session planning and high redundancy numbers are required to alleviate this predicament. As a result, it is necessary to maintain a good balance among the number of sessions, Type I and II error levels, and network robustness.

The selection of optimal baselines has already been examined discussed in the literature (e.g. Kuang 1996, Stopar 2001, Even-Tzur 2002 and Yetkin et al. 2011), as has and robustness analysis of geodetic networks has been investigated (e.g. by Vaníček et al. 1991, Krakiwsky et al. 1993, Vaníček et al. 2001, Berber et al. 2006, Vaníček et al. 2008 and Berber et al. 2009). In this paper, we propose an easily applicable algorithm for classical GPS network design, as well as a special robustness criterion that is suitable for optimal network design based on robustness analysis of a surveying network.

2. Robustness analysis

A component of the maximum undetectable error vector \( \delta \Delta \) among the observations, which would not be detected by the data snooping procedure of Baarda (1968), is given as

\[
\Delta \Delta_i = \frac{\sqrt{\lambda_i} \sigma_i}{\sqrt{r_i}}
\]

(1)

where \( \sqrt{\lambda_i} \) is the noncentrality parameter and is a function of Type I and Type II errors, \( \sigma_i \) and \( r_i \) are the standard deviation and redundancy number of the \( i \)th observation, respectively (Baarda 1968). The displacements \( \Delta x \) caused by the maximum undetectable errors \( \delta \Delta \) in the observations are estimated by

\[
\Delta x_i = \left( A^T P A \right)^{-1} A^T P c_i \delta \Delta
\]

(2)

where \( A \) is design matrix, \( P \) is weight matrix and \( c_i \) is a vector containing zeros except identity at the \( i \)th position. The problem with external reliability criterion is its datum dependency, but robustness must be described based on the network geometry and accuracy of the observations. Because of this, the strain technique is incorporated as it is independent of adjustment constraints and reflects only the network geometry and the accuracy of the observations (Vaníček et al. 1991, Krakiwsky et al. 1993, Vaníček et al. 2001, Berber 2006, Berber 2008, Berber et al. 2008). The combination of reliability analysis with strain technique is named robustness analysis. Only a brief summary of this method is given below; interested readers will find more detail in Berber et al. (2009).

The displacement of a point \( P \), can be expressed as

\[
\Delta x_i = \left[ \begin{array}{c} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{array} \right] = \left[ \begin{array}{c} u_i \\ v_i \\ w_i \end{array} \right]
\]

(3)

where \( u \) is the displacement in the \( x \) direction, \( v \) is the displacement in the \( y \) direction and \( w \) is the displacement in the \( z \) direction. The strain matrix is therefore

\[
E_i = \left[ \begin{array}{ccc} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{array} \right]
\]

(4)

To evaluate the displacement field of point of interest, \( P \_i \), the following equations can be used,

\[
\begin{align*}
a_i + \frac{\partial u}{\partial x} (X_i - X) + \frac{\partial u}{\partial y} (Y_i - Y) + \frac{\partial u}{\partial z} (Z_i - Z) &= u_i \\
b_i + \frac{\partial v}{\partial x} (X_i - X) + \frac{\partial v}{\partial y} (Y_i - Y) + \frac{\partial v}{\partial z} (Z_i - Z) &= v_i \\
c_i + \frac{\partial w}{\partial x} (X_i - X) + \frac{\partial w}{\partial y} (Y_i - Y) + \frac{\partial w}{\partial z} (Z_i - Z) &= w_i
\end{align*}
\]

(5)

where all the partial derivatives as well as the absolute terms \( a_i, b_i, \) and \( c_i \), and the coordinates \( X_i, Y_i, \) and \( Z_i \) refer to point \( P_i \) and point \( P \_i \) is connected by observations to the point of interest, \( P \_i \). In order to obtain the strain matrix, the system of equations given above is solved using the least squares method. This procedure is explained in depth in Berber (2006).

The displacements can then be computed using the elements of strain matrix. However, the initial conditions \( X_0, Y_0, \) and \( Z_0 \) must first be determined, representing the coordinates obtained by minimizing the norm of displacement vector at all points in the network. To calculate \( X_0, Y_0, \) and \( Z_0, \) then, the displacements in the network points should first be minimized (Berber et al. 2009). Once \( X_0, Y_0, \) and \( Z_0 \) have been determined, \( u_i, v_i, \) and \( w_i \) are calculated from:

\[
\begin{bmatrix}
\Delta x_i \\
\Delta y_i \\
\Delta z_i
\end{bmatrix} = \left[ \begin{array}{ccc}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}
\end{array} \right] \begin{bmatrix}
X_i - X_0 \\
Y_i - Y_0 \\
Z_i - Z_0
\end{bmatrix}.
\]

(6)

After computing the displacements \( u, v, \) and \( w \) for each point in the network, the amount of total displacement at each point is calculated from the equation

\[
d_i = \sqrt{u_i^2 + v_i^2 + w_i^2}.
\]

(7)

Obtaining the minimum displacement values is desirable because the smaller the displacement at network points, the more robust the network will be at these points.
3. Type I and Type II error levels

Hypothesis testing is an essential part of statistical inference. In statistical hypothesis testing, \( \alpha \) (Type I error), or the “significance level”, is the probability of rejecting the true null hypothesis. The complementary probability \( (1 - \alpha) \) is called the confidence level. \( \beta \) (Type II error) arises when null hypothesis is false and we accept it. The remaining probability \( (1 - \beta) \) is called the “power” of the test. These hypothesis and error levels are depicted in Fig. 1. For more information about statistical testing for geodetic networks, interested readers are referred to Krakiwsky et al. (1999). As mentioned in Section 2, the non-centrality parameter of the postulated distribution in the alternative hypothesis of Eq. (1) is a function of these selected probabilities. Since the maximum undetectable errors are function of the non-centrality parameter, robustness analysis results are heavily influenced by the selection of \( \alpha \) and \( \beta \) values.

Unfortunately, higher confidence \( (1 - \alpha) \) and power levels \( (1 - \beta) \) also give rise to a larger non-centrality parameter. This means larger maximum undetectable errors and therefore larger displacements in the robustness analysis. Nevertheless, higher confidence \( (1 - \alpha) \) and power levels \( (1 - \beta) \) are desirable traits in a statistical analysis. As shown in Table 3, increasing the redundancy of the network by performing additional observations (more sessions) can also yield smaller displacements, but this brings with it an increase in cost that may render the survey uneconomical. Thus, a surveyor must design a GPS session plan (number of sessions, repeat observations of certain baselines, baseline configuration and etc.) by considering reasonable project cost, displacements and significance levels. A good balance among these will aid in the success of a GPS survey.

4. GPS baseline configuration design

It is well known that the displacement value at a network point should be as small as possible [ref]. With this in mind, a special robustness criterion can be of the type

\[
\max (d_i) \rightarrow \min
\]  

In this study, this criterion will be used for GPS baseline configuration design.

Robustness of a network among other things is affected by the design of the network. Therefore, the points that lack robustness in the network may be improved by increasing the number of observations in the network. Increasing the number of observations also increases the redundancy of the network, resulting in smaller displacements. Adding an observation should never deteriorate the solution because the current network will already be in place. In fact, by adding an observation, the current network will be improved due to the improvement in the redundancy of the network (Berber, 2006).

Let us begin with an initial design that consists of all possible baselines. At least, it must have enough observations for robustness analysis and must not have the singularity cases discussed in Vaníček et al. (2001). For the purpose of achieving the largest possible improvement in robustness of a network, we should seek a GPS baseline that provides the largest reduction in the objective function value given in Eq. (8). In order to do that, each baseline is added to the network and its max \( (d_i) \) values are determined individually. The baseline with the minimum \( \max (d_i) \) can then be identified. Next, following the same procedure, the remaining baselines are tested. Naturally, the baseline which allows the highest increase in robustness is considered first. If the new maximum displacement value is still larger than the desired level, further addition of the baselines with the largest improvement is considered. This process is repeated until a sufficiently robust network is obtained.

5. Numerical example

A GPS network example given in Snow (2002) is used to test the proposed approach. The network consists of 6 continuously operating reference stations (see Fig. 2). The datum of the network is provided by a minimum number of constraints, so only DET1 has been considered as a fixed station. The initial network configuration consists of all 15 possible baselines that can be observed among network stations. This initial design does not include any repeated baselines.

Following Section 2, robustness analysis of this network is done and the displacement values are given in Table 1 for \( \alpha = 0.05 \) and \( \beta_0 = 0.05 \). These values were arbitrarily selected at this phase of the analysis.

Table 1. Displacements at network points (mm).

<table>
<thead>
<tr>
<th>Points</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIL1</td>
<td>0.30</td>
</tr>
<tr>
<td>NLIB</td>
<td>0.60</td>
</tr>
<tr>
<td>SAG1</td>
<td>0.25</td>
</tr>
<tr>
<td>STB1</td>
<td>0.41</td>
</tr>
<tr>
<td>WLC1</td>
<td>0.40</td>
</tr>
</tbody>
</table>
As can be seen in Table 1, the maximum displacement in the network occurs at station NLIB as 0.60 mm. Displacements at network points are small because the standard deviations of observations in this network are small - standard deviations for coordinate difference measurements are $\delta X: 0.2–0.6$ mm; $\delta Y: 1.1–1.9$ mm; and $\delta Z: 1.0–1.7$ mm.

In order to reduce the displacement values at network points, additional GPS baselines must be considered. Since the number of independent baselines for any GPS observation session is one less than the number of observing receivers, only five baselines could be observed from a single 24-hour observation session using the six CORS (Continuously Operating Reference Stations). Thus, the number of sessions is correlated with the number of baselines and the number of operating receivers in the network. Since the number of sessions has a crucial role in determining system cost, only the most optimal baselines should be selected; here, we select the GPS baselines that provide the highest contribution to the robustness of the network. Thus, as given in Eq. (8), we seek baselines that minimize the maximum displacement the most. Hence, we should identify the effect of each baseline to the maximum displacement value in the network. In the end, the baseline contributions to the maximal reduction are determined and the baseline with the maximal reduction is considered first for additional baseline observation. Then, the baseline with the second highest contribution to the maximal reduction is considered, followed by the baseline with the third highest contribution, and so on. This process continues progressively until the maximum displacement value is below the maximum allowable value.

Table 2 presents the contribution of the first 11 baselines to the robustness of the network. As more baselines are added, maximum displacement values get smaller and eventually they converge to 0.26 mm. Thereafter, the addition of more baselines is pointless, as it does not lower the max ($d_\text{max}$).

In practice, the non-centrality parameter $\sqrt{\lambda_0}$ in Eq. (1) is chosen on the basis of reasonable $\alpha$ and $\beta_0$ values. Table 3 shows the $\sqrt{\lambda_0}$ values for different $\alpha$ and $\beta_0$ values and lists corresponding maximum displacement values for 15, 18 and 24 baselines.

As can be seen in Table 3, higher confidence $(1 - \alpha)$ and power levels $(1 - \beta_0)$ lead to a larger non-centrality parameter $\sqrt{\lambda_0}$ and therefore larger displacements. However, the maximum displacement values can be decreased by increasing the number of baselines and the number of 24-hour sessions.

The number of baselines can be tailored to the purpose of the survey project. For example, if a maximum displacement value below 0.50 mm is desired for the network, $\alpha$ and $\beta_0$ should be selected at 0.100 and 0.100 for 15 baselines, 0.025 and 0.025 for 18 baselines, and 0.001 and 0.001 for 24 baselines, respectively. If we look at Table 3 from left to right, we see that increasing the number of baselines allows us to accommodate smaller $\alpha$ and $\beta_0$ values. If we look from top to bottom, as $\alpha$ and $\beta_0$ values get larger, less displacement is experienced. On the other hand, smaller $\alpha$ and $\beta_0$ values are desired for statistical testing. As a consequence, here, the goal is to get a balance among significance level, robustness and cost.

Increasing the number of baselines and therefore the sessions can yield not only lesser maximum displacement values and error levels for statistical testing but also an improved reliability in the network. Minimum redundancy numbers for increasing baseline numbers are given in Table 4.

According to Table 4, if the number of baselines (and number of sessions) in a network is increased, a more reliable network is achieved; in other words, the minimum redundancy number gets larger. This is portrayed in Fig. 3, which shows the effect of increasing the number of baselines on both maximum displacement and minimum redundancy number. Maximum displacement value in a network is inversely correlated with the minimum redundancy number in the network, which in turn confirms that robustness and reliability are closely related.
Table 3. Significance levels, non-centrality parameters and maximum displacement values (mm).

<table>
<thead>
<tr>
<th>α</th>
<th>β</th>
<th>$\sqrt{\lambda}$</th>
<th>max ($d_i$) 15 baselines</th>
<th>max ($d_i$) 18 baselines</th>
<th>max ($d_i$) 24 baselines</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.001</td>
<td>6.38</td>
<td>1.05</td>
<td>0.65</td>
<td>0.47</td>
</tr>
<tr>
<td>0.005</td>
<td>0.005</td>
<td>5.38</td>
<td>0.89</td>
<td>0.55</td>
<td>0.39</td>
</tr>
<tr>
<td>0.010</td>
<td>0.010</td>
<td>4.90</td>
<td>0.81</td>
<td>0.50</td>
<td>0.36</td>
</tr>
<tr>
<td>0.025</td>
<td>0.025</td>
<td>4.20</td>
<td>0.69</td>
<td>0.43</td>
<td>0.31</td>
</tr>
<tr>
<td>0.050</td>
<td>0.050</td>
<td>3.61</td>
<td>0.60</td>
<td>0.37</td>
<td>0.26</td>
</tr>
<tr>
<td>0.050</td>
<td>0.100</td>
<td>3.24</td>
<td>0.54</td>
<td>0.33</td>
<td>0.24</td>
</tr>
<tr>
<td>0.100</td>
<td>0.100</td>
<td>2.92</td>
<td>0.48</td>
<td>0.30</td>
<td>0.21</td>
</tr>
<tr>
<td>0.050</td>
<td>0.200</td>
<td>2.80</td>
<td>0.46</td>
<td>0.28</td>
<td>0.20</td>
</tr>
<tr>
<td>0.100</td>
<td>0.200</td>
<td>2.48</td>
<td>0.41</td>
<td>0.25</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 4. Minimum redundancy numbers and the number of baselines in the network. Minimum redundancy $\min (r_i)$ is the lowest redundancy of the network with a given number of baselines.

<table>
<thead>
<tr>
<th>Number of baselines</th>
<th>$\min (r_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.4498</td>
</tr>
<tr>
<td>16</td>
<td>0.4501</td>
</tr>
<tr>
<td>17</td>
<td>0.4502</td>
</tr>
<tr>
<td>18</td>
<td>0.4713</td>
</tr>
<tr>
<td>19</td>
<td>0.4911</td>
</tr>
<tr>
<td>20</td>
<td>0.5114</td>
</tr>
<tr>
<td>21</td>
<td>0.5224</td>
</tr>
<tr>
<td>22</td>
<td>0.5225</td>
</tr>
<tr>
<td>23</td>
<td>0.5226</td>
</tr>
<tr>
<td>24</td>
<td>0.5963</td>
</tr>
<tr>
<td>25</td>
<td>0.5820</td>
</tr>
<tr>
<td>26</td>
<td>0.6350</td>
</tr>
</tbody>
</table>

Figure 3. Maximum displacement value and minimum redundancy number as a function of increasing baseline number.

6. Conclusions

GPS networks have been used for many geomatics engineering projects and engineering projects that demand high quality results, and robustness analysis is the main method used to determine the GPS network’s strength. In this paper, maximum displacement value is used to describe the weakest point in the network. Then, additional observations may be planned to strengthen the network. In order to do that the baselines with the highest contribution to the robustness of the network are found, followed by the baselines that provide the biggest reduction to the maximum displacement value. Furthermore, baselines to be re-observed can also be selected more efficiently. However, while adding new baselines, one has to be cautious with the economy of the network and the significance levels to be used. In GPS networks, increasing the number of sessions improves the redundancy of the network yet it pushes the cost up. Significance levels should be as small as possible from the statistical testing point of view, but this leads to larger non-centrality parameters. As the non-centrality parameter gets larger, maximum undetectable errors get larger and the success of statistical testing decreases.

Standard deviations of GPS baselines should be properly estimated in order to achieve a realistic design. For this purpose, GPS receivers’ standard technical specifications may be used, along with data collected through use of similar receivers in a similar fashion. In fact, over time, a surveyor can create an entire library of ‘typical’ observations, accounting for single or dual frequency data, long or short baselines, varying occupation times, and so on.

It should be noted that a thorough GPS survey plan must include point selection (both new and existing points) and selection of appropriate survey methods and session lengths in addition to the optimal selection of baselines. Therefore, considering all of these factors, an optimal design methodology should be developed for further research.

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