Geodetic use of global digital terrain and crustal databases in gravity field modeling and interpretation

Research article

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Abstract:
The release of global digital databases for the description of the Earth's topography and the shape of the Earth's crust in terms of consistency and geometry initiates a new era in the interpretation and analysis of the observed gravity field of our planet. The permanent increase in resolution of these databases permits furthermore the identification of high frequency gravity field components, a feature that is of special interest in applications of local or regional scales. The derivation of topographic/isostatic gravity models is the tool which reveals the gravity content of terrain and crustal databases in the spectral domain. We review the significance of some current global digital models in the frame of this analysis by computing distinct spectral gravity quantities and compare them against the Kaula rule of the gravity signal decay and the recently released reference gravity model EGM2008. The different isostatic hypothesis that can be applied in the derivation of a topographic/isostatic model as well its dependency with the increasing harmonic degree is demonstrated and quantified in terms of geoid heights and gravity anomalies. It is shown that the two fundamental compensation mechanisms, namely Airy and Pratt, act complementary in terms of their compensation effect to the uncompensated topography spectrum. The Airy mechanism reduces the uncompensated topography in the longer and medium wavelength part of the spectrum (up to degree 400), while Pratt acts in a compensating manner only for the high to very high frequencies, from degree 100 and onwards.

Keywords:
CRUST 2.0 • gravity field • EGM2008 • topographic/isostatic gravity models

1. Introduction

Current terrestrial and satellite observation methods and the corresponding data analysis procedures lead to the construction of global digital databases for the description of the earth's crust and topography with an increasing resolution and accuracy. Today global digital terrain models are available, which can reach a spatial resolution of up to 30 × 30 meters (Farr et al. 2007). However, such a resolution is neither global nor homogeneous and can only be achieved over continents. The question of mapping the ocean topography still remains mainly a methodological task with the major contribution being the inversion of altimetric data and altimetry-derived surface gravity data to produce the corresponding interface, i.e. the relief of the oceanic bottom separating water from oceanic crust (Smith and Sandwell 1997). The production of global crustal models on the other hand is based on the exploitation of active seismic data, the compilation of available geological information and the generalization that physical properties of certain tectonic types have global character and can be assigned to similar tectonic settings. Using this methodological approach the model CRUST 5.1 and its follow-up CRUST 2.0 provide a global representation of the geometry and consistency of six (seven, if one includes ice thickness) distinct crustal layers starting from the visible topographic relief and expanding down to the crust-mantle boundary with a unified resolution of 5° × 5° and 2° × 2° respectively.

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tively (Mooney et al. 1998, Bassin et al. 2000). The geometry of the last layer of a global crustal model provides directly a global estimate of the Mohorovicic discontinuity, information that can be opposed either regionally or globally to other independent sources of Moho data. A further example of exploiting the global crustal data is for local or regional applications of gravity field modeling, where the shape and density data emerging from the database can be used in the frame of some forward or inverse modeling procedure (Blakely 1995). However, the main asset of global digital databases with direct relation to gravity field analysis is the spatial resolution of the respective datasets. The exploitation of this information is linked to the spectral analysis of the global geographical grids and leads to the computation of a particular class of gravity field coefficients, the so-called topographic/isostatic (t/i) spherical harmonic coefficients. The highest degree and order up to which these coefficients can be evaluated depends in this case solely on the spatial analysis of the input information, i.e. the global terrain or crustal data. The denser these global grids are, the higher the maximum degree and order of the corresponding t/i spherical harmonic sets. Thus, the incorporation of dense global digital terrain and crustal data in the appropriate spherical harmonic analysis scheme enables the retrieval and therefore interpretation of the high and very high frequencies of the observed gravity field.

2. Theoretical background

The computation of t/i models is based on standard spherical harmonic analysis of equidistant spherical grids combined with the adaptation of some compensating mechanism that should describe the mass equilibrium between crust and mantle. Sünkel (1985, 1986) provided a mathematically thorough definition of an isostatic Earth model elaborating a formal description of an Airy/Heiskanen type topographic/isostatic potential. The definition, numerical computation and assessment of t/i models have been used since then in a variety of studies dealing either with gravity field modeling and interpretation or with the construction of synthetic Earth models (e.g., Rummel et al 1988, Pavlis and Rapp 1990, Haagmans 2000, Claessens 2003). The fundamental quantities from the theoretical and modeling point of view are the geometry of the boundary surface separating crust from mantle and the numerical values defining the corresponding density contrasts. The geometry of this discontinuity surface defines a corresponding density contrast which can be modeled rigorously and can lead to the computation of the corresponding t/i earth gravity model. The computation of the t/i spherical harmonic coefficients is based on the fundamental expression of the earth induced gravitational potential in spherical harmonics (Tsoulis 2001)

\[ V_{P} = \frac{GM}{R} \sum_{l=0}^{\infty} \frac{R_{l}^{l+1}}{R^l} \sum_{m=-l}^{l} \sum_{n=0}^{1} Y_{lm}^{n}(P) C_{lm}^{n} \]  

(1)

where \( P \) denotes the computation point, \( G \) the gravitational constant, \( R \) a mean earth radius, \( M \) the earth’s mass in a spherical approximation and \( Y_{lm}^{n}(P) \) stands for the abbreviation

\[ Y_{lm}^{n}(P) = P_{lm}(\cos \sigma \partial_{\sigma}) \begin{cases} \cos m \lambda \rho \text{ for } a = 0 \\ \sin m \lambda \rho \text{ for } a = 1 \end{cases} \]  

(2)

of the products of the fully normalized associated Legendre functions \( P_{lm} \) with the trigonometric functions of the longitude of the computation point \( \lambda \partial_{\lambda} \) and \( m \) and \( n \) denoting degree and order respectively. The parameter \( C_{lm}^{n} \) in equation (1) expresses finally the dimensionless potential harmonic coefficients \( \tilde{C}_{lm} \) and \( \tilde{S}_{lm} \) defined as (ibid.)

\[ C_{lm}^{n} = \left\{ \frac{\tilde{C}_{lm}}{\tilde{S}_{lm}} \right\} = \frac{3}{2l+1} \frac{1}{\tilde{\rho} R^l} \frac{1}{4\pi} \int_{e} \left( \int_{r} \rho(Q) \frac{r_{0}}{R} d\sigma_{0} \right) Y_{lm}^{n}(Q) d\sigma_{0} \]  

(3)

with \( \tilde{\rho} \) denoting a mean density value for the earth’s masses, \( Q \) the integration point and \( \sigma \) the surface of the sphere with radius \( R \). Equation (3) is the basic expression for the evaluation of t/i earth gravity models. The coefficients defined by this formula imply integration over the entire earth masses. The fundamental unknown is quantity \( \rho(Q) \), the exact knowledge of which would, theoretically, permit the straightforward numerical computation of (3). As a rigorous definition of the actual density distribution in the whole range of the earth’s interior, is not available and apparently not possible, the computation of the potential harmonic coefficients of the geopotential is based today either on the analysis of tracking and accelerometry data of dedicated satellite gravity missions, such as CHAMP or GRACE (Reigber et al. 2002, Tapley et al. 2005), or on the combined processing of satellite and terrestrial data that leads to the evaluation of so-called Earth Gravity Models, more widely known as EGMs (Lemoine et al. 1998, Pavlis et al. 2012). Equation (3), though not implemented in a rigorous manner, provides the underlying definition for all these different approaches. However, in the case of EGMs and t/i models this expression becomes embedded in the evaluation procedure and is employed on real data. The source of information that enables such an evaluation is the availability of global datasets of terrain or crustal structure. If we go again to Eq. (3) we can observe that although not applicable over the whole range of the earth’s masses it can be restricted to that part of the earth where the mass distribution is given. At first glance we think of a global digital elevation model, which by describing the variations of the visible topography and the bottom bathymetry gives a good account on the outside boundary of the uppermost layer of the earths’ crust. Then, a standard idealized isostatic model can provide the structure of the second boundary layer (crust-mantle boundary) by using the height and depth values of the global DEM and applying some theory for the description of the mass equilibrium between crustal and
mantle masses. In this way Eq. (3) can be applied only to a layer of the entire earth masses and, in relation to some reference spherical earth, lead to a set of t/i spherical harmonic coefficients computed according to the general expression (Rummel et al. 1988, Tsoulis 2001, Claessens, 2003)

\[
C^{t/i}_{lm} = \frac{3}{\rho R(2l+1)} \frac{1}{4\pi} \int_0^1 \left[ A^t(Q) - A^i(Q) \right] Y^{\ast}_{lm}(Q) dQ
\]

(4)

The terms \( A^t \) and \( A^i \) denote here the topographic and isostatic contributions respectively and express the one-dimensional integration along the radial direction \( r, \) i.e. the quantity inside the parentheses in Eq. (3). Depending on the applied compensation mechanism the definite limits of this integration will vary, while their definition differs also between continental and ocean parts inside the same isostatic model (Tsoulis 2001). Finally, it is important to stress that the present discussion focuses solely on spherical harmonics. Discussion on computational aspects of ellipsoidal harmonics or the discrepancies between ellipsoidal and spherical harmonic fields can be found respectively, for example, in Sebera et al (2012) and Holmes and Pavlis (2007).

3. High degree t/i computations

The critical quantity behind the evaluation of a t/i gravity model, i.e. the integration limits in terms \( A^t \) and \( A^i \) of Eq. (4), is the choice of the compensation model, or equally the definition of the density distribution defining the compensating masses. From the computational point of view, the derivation requests the availability of global stratified data on a sphere in terms of equidistant geographical grids (topographical heights and depths with respect to sea level, crustal density) and their processing through a standard procedure of spherical harmonic analysis (Sneeveu 1994, Pavlis and Rapp 1990). In this context the maximum degree and order of the obtained t/i models is defined essentially by the spatial resolution of the input data in the latitude direction. For example, a global terrain or crustal database with a 2 degrees times 2 degrees global resolution can produce a t/i model of \( L_{\max} = 90, \) a 5 min times 5 min global dataset corresponds to a \( L_{\max} = 2160 \) t/i gravity model. When a t/i model is then used in a forward modeling fashion, for example for the synthetic construction of selected gravity field functionals on the earth’s surface or at satellite altitude, it possesses the same attributes as any other spherical harmonic model. The general relation connecting \( L_{\max} \) and the half-wavelength spatial resolution \( D \) (expressed in km) of the corresponding functional is given namely by the rule of thumb \( D = 20000 / L_{\max}, \) with 20000 expressing in kilometers the half earth circumference at equator. We now proceed to the computation of several typical t/i models and we perform their spectral assessment by comparing them to the combined global gravity model EGM2008 and Kaula’s rule. The latter emerged from autocovariance analysis of terrestrial gravimetry data and provides a rough empirical estimate for the course of the gravitational spectrum. It attempts to describe roughly the gravitational signal decay implying, that for a given degree the quadratic mean over all orders of the harmonic coefficients of the earth’s gravity field decreases approximately like \( 10^{-5} / l^2, \) denoting degree. Kaula’s formal statement that leads to this degree-dependent rule of thumb reads (Kaula 1966)

\[
s^2_t = \frac{1.6 \times 10^{-10}}{l^2}
\]

(5)

with \( s^2_t \) denoting the degree variance of a model, merely expressing the power spectrum of its coefficients as \( s^2_t = \sum_{l=0}^{l_{\max}} C^{t/i}_{lm} \) and \( s^{2}_o. \) By taking the square root of the degree variances one computes the corresponding degree RMS values. Departing from these dimensionless quantities the degree RMS values for other gravity field quantities can be obtained by taking advantage of the fact that the different gravity field functionals are connected in the spectral domain by means of the corresponding eigenvalues (Rummel and van Gelderen 1992). For geoid heights and gravity anomalies these eigenvalues read respectively \( R \) and \( \langle GM (l-1) / R^4 \rangle \cdot 1e5, \) with the corresponding quantities expressed in m and mGal respectively. Multiplication of these quantities with the square root of the degree variance of a model provides the corresponding degree RMS values.

Figures 1 and 2 depict the RMS geoid height and gravity anomaly curves of three different t/i models, the uncompensated topography, the combined gravity model EGM2008 and the rule of Kaula. The term uncompensated topography is used here to describe the direct effect of global topography and bathymetry on the corresponding gravity field quantities. Hence, it represents the high frequency signature of the terrain without applying some sort of compensation at the crust-mantle boundary. This means that such a model is not well suited to be used as an earth gravity model. However, it permits a qualitative assessment of different isostatic mechanisms, in terms of the actual signal power dumping of the uncompensated topography spectrum that they represent. Models ‘Airy’ and ‘Pratt’ express the application of an Airy/Heiskanen and a Pratt/Hayford isostatic compensation mechanism respectively as applied to ETOPOS data, a global topography/bathymetry dataset at a 5’ resolution (NGDC 1988), and are available up to degree and order 1082, while model ‘t/i’ expresses a topographic/isostatic model available up to degree and order 2160, obtained from the application of a standard Airy compensation mechanism to the digital terrain model DTM2002, a global terrain and bathymetry database available in a 2’ and a 5’ global resolution (Saleh and Pavlis, 2003). Finally, ‘Topo’ denotes the uncompensated topography obtained from the elevation data of the ETOPOS database. It is interesting to observe how all of the aforementioned models, though to a different degree, comply generally with the Kaula-decay. The main remark however refers to the direct comparison of the two compensating mechanisms evaluated here. Thus, while all t/i models, including EGM2008, tend to converge to the power of the uncompensated topography for higher degrees (threshold for this behavior being degree 100), the Pratt model while presenting...
almost identical spectral power for the low degrees as the uncompensated topography. It has significantly less power than EGM2008 for the higher degrees. This is due to the fact that the topographic masses (height $> 0$) always have a smaller density in the Pratt model than for the reference density used for the uncompensated topography. Thus, the Pratt mechanism fails to isostatically compensate the crustal masses in the lower degrees of the spectrum, and commences to retrieve its compensating feature with increasing degree. Of course, this tends to reveal at the end and unrealistic behavior at the very high degrees, which complies generally with the fact that the Pratt theory expresses a very rough and artificial compensating mechanism which is of little use when applied in a global sense. The Airy model behaves in an opposite manner showing significantly less power than EGM2008 for the lower degrees (up to degree 100) and approaching the spectral power of EGM2008 as well as the uncompensated topography for the higher degrees. Here, the Airy mechanism provides a significant compensation in the power of the uncompensated topography at the lower degrees, gradually perishing its isostatic effect for increasing degree, thus its gradual convergence with the uncompensated topography. Generally, the Airy model behaves almost in the same way as the t/i model where the small differences merely express the different global terrain and bathymetry data used.

Figures 3 and 4 present the corresponding RMS curves for the t/i model obtained from the analysis of all seven crustal layers of model CRUST 2.0. The obtained model is complete up to degree and order 90 (Tsoulis 2004). The poor compensation that the CRUST 2.0 data imply to the uncompensated topography becomes apparent from these results. Although based on real data and not on some theoretical assumption for the exact geometric

4. Concluding remarks

The analysis of global digital terrain and crustal databases leads to the recovery of the medium to high frequencies of the observed gravity field signal. The continuous release of databases with very high resolution enables furthermore the retrieval of the very high frequency part. The significance of the obtained topographic/isostatic models is twofold. On the one hand they provide a direct insight to the corresponding bandwidth of the observed gravity field in terms of their dimensionless potential harmonic coefficients. At the same time they act complementary to other gravity field information, especially when it does not contain information on the small wavelength features of the gravity field, e.g. the currently available satellite-only gravity field models. In this way the geometrical and physical data of the aforementioned databases can assist the challenging task of gravity field analy-
sis and interpretation. With the resolution of the available global databases steadily increasing the t/i approach presents an efficient tool for an advanced band-limited analysis for capturing, interrelating and interpreting the medium, high and very high frequency part of the observed gravity field of the available, and especially forthcoming satellite only and combined Earth gravity models at all computational scales (global, regional and local).

Apart from the resolution of the input data the features of a t/i approach in terms of gravity field recovery are determined by the type of compensation that is applied to the crustal masses. The different compensation mechanisms affect the mathematical formulation of the t/i model and thus the actual computed t/i spectrum. The two standard approaches that have been considered here are the Airy/Heiskanen and the Pratt/Hayford isostatic hypotheses. These two models appear also in the related literature of t/i models, either for the computation of purely isostatic Earth gravity models (Tsoulis 2001), or in the frame of the development of synthetic Earth gravity models (Bagherbandi and Sjöberg 2012). The spectral representation of the t/i models that are produced from these two compensation mechanisms reveals a complementary aspect of the two isostatic hypotheses in the spectral domain. One of the main features of t/i models is their smoothing effect on the observed field, without any loss of physical information that any low-pass filtering would cause (Göttl and Rummel 2009). This smoothing feature can be seen spectrally by the decrease in power of a t/i model when compared to an uncompensated topography spectrum or some observed reference gravity field. The displayed computations showed that the A/H based t/i model acts in a compensating manner for the long wavelength and most of the medium to short wavelength part of the gravity spectrum. Up to degree 400 it reduces significantly the power of the observed field, and starts to converge to the uncompensated topography spectrum for the high and very high frequencies, the part of the spectrum where the uncompensated topography coincides spectrally with the EGM2008 reference field. The P/H model on the other hand performs in a reversed manner, and thus complementary to A/H as far as its compensation effect is concerned. It shows no compensating effect for the long wavelengths, but starts to act as one would expect a t/i model to perform, only for the high and very high frequencies, where it reduces the power of the uncompensated topography and EGM2008 spectra in almost the same order of magnitude as the A/H model does in the lower degrees. This complementary feature is important and can be utilized in current gravity field analysis, as the incorporation of databases with increasing resolution permits the retrieval of even higher frequencies of the observed gravity field.

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