Larry M. Silverberg*

The anatomy of the bank shot in men’s basketball

Abstract: A detailed study of the bank shot trajectory was performed with a focus on breaking it down into its significant features. The bank shot’s effectiveness was examined in terms of the player’s ability to accurately impart launch speed, back spin, pitch angle, and side angle. It was shown that the bank shot is more forgiving than the direct shot in launch speed and launch angle and has about the same level of forgiveness in side angle at particular locations on the court. This paper concludes that the bank shot can be extremely advantageous over the direct shot to the shooter who learns how to accurately impart side angle.

Keywords: basketball, bank shot, trajectory.

Introduction

Increased attention placed on physical training has strongly impacted many collegiate sports, men’s basketball among them. The strength and athleticism of the collegiate basketball player led to the dominance of an inside-outside style of play in which most shots are either taken close to the hoop or around the perimeter. The bank shot opens up this style of play, bridging the gap between the inside shot and outside shot. The probability of a successful bank shot is about 20% greater than the probability of a direct shot in the intermediate ranges toward the sides of the court (Silverberg, Tran and Adams 2011). A player down low (close to the hoop) can step back and shoot a bank shot over an opponent or dish the ball to the perimeter. A player at the perimeter can pivot around a screen with the option of going low or pulling up and taking a bank shot.

One of the challenges with utilizing the bank shot is that many players are unskilled in it. The bank shot trajectory is relatively unintuitive since the player needs to aim to the side. Once mastered, the bank shot can be extremely advantageous, which was shown previously (Silverberg et al. 2011). This raises the basic question of why it can be so advantageous. This question has not been answered and requires the bank shot to be broken down into parts. More specifically, the following parameter ranges are examined: (1) the alpha-beta plot, (2) the backboard imprint, (3) the reflection hoop, and (4) the multi-trajectory plot. The alpha-beta plot shows the range of pitch angles (alpha) and side angles (beta) at launch for which a shot is successful. The backboard imprint shows the locations on the backboard associated with a range of pitch angles and side angles for which the shot is successful. The reflection hoop is a visual way to compare the bank shot and the direct shot. Finally, multi-trajectory plots are a way to see how varying parameters affect trajectories. Overall, the paper scrutinizes how launch conditions and court location provide advantages and compare with the direct shot. The method section describes each of the studies, the results section describes the findings, and the paper ends with a summary and conclusions.

Methods

A basketball is launched from a particular position with a particular velocity and angular velocity. Early studies were analytical (for example, Brancazio 1981) whereas more recent studies are numerical and enable a large amount of data to be collected. Using the same approach that was adopted in a previous study (Silverberg, Tran and Adcock 2003), the launch position is expressed in terms of distance \( r \), polar angle \( \theta \), and launch height \( h \) (see Figure 1). As shown, the launch velocity is expressed in terms of its speed (magnitude) \( v \), pitch angle \( \alpha \), and side angle \( \beta \). The magnitude of the angular velocity is the back spin \( \omega \) and it is assumed to act about the horizontal axis perpendicular to the plane of the trajectory. These launch conditions are regarded as statistical quantities; each is the sum of a nominal quantity and a statistical error. The nominal quantities are desirable coordinates, that is, coordinates that are chosen by the shooter and for which corrections are made. The errors are taken to be normally distributed and statistically independent (Silverberg et al. 2003).

It is convenient to think of the act of launching a ball as aiming and then shooting. The aiming parameters are pitch angle \( \alpha \) and side angle \( \beta \), and the shooting
parameters are launch speed $v$ and back spin $\omega$. Of course, no shooter releases the ball the same way every time. There is a level of forgiveness present in each shot because there is a range of pitch angles, side angles, launch speeds, and back spins that are successful – but how much range does the shooter have? This paper studies the ranges of these aiming and shooting parameters for different positions on the court and how they compare with the direct shot.

Aiming and shooting parameter ranges are studied by looking at successful shots that surround the optimal centered shot. A successful shot is defined in this paper to be a bank shot or direct shot that passes through the hoop without making contact with it. This definition omits shots that enter the hoop after making contact with the hoop. In other words, success is restricted to swish direct shots and swish bank shots. The centered shot is the shot whose vertical plane passes through the center of the hoop and the optimal centered shot is the centered shot that has the highest probability of success. There is an optimal centered shot for each position on the court. The optimal centered shots were determined in a previous numerical study that considered swish and non-swish shots. In that study and in the study here we only consider one representative set of conditions and do not carry out an exhaustive study. The ball undergoes 3 revolutions per second (Hertz) of back spin and is launched 7 ft (2.13 m) above the floor. The shooter was assumed to successfully make the free throw 70% of the time and to shoot other shots with the same statistics (see Appendix). Note that although the optimal centered shots were determined numerically, considering complex interactions with the hoop and backboard, the subsequent restriction to swish shots allowed ranges of parameters to be determined analytically. The errors associated with omitting non-swish shots were observed to be relatively small (Tran and Silverberg 2008). In a later paper it was shown that the shooter can select a bank shot over a direct shot with as much as a 20% advantage. The distribution over the court of preferences of the bank shot over the direct shot was determined. The bank shots in the distribution were optimal (Silverberg et al. 2011).

### Aiming parameters

The aiming parameters are pitch angle and side angle. The aiming parameters will be studied using the alpha-beta plot, the backboard imprint, and the reflection hoop (Figure 2). As shown, each set of data is associated with a critical stage in the trajectories of successful shots – from initial launch, to the point at which the ball makes contact with the backboard and finally as it passes through the hoop and the reflection hoop. The optimal centered shot is shown as a black dot.

#### Alpha-beta plots

The vertical axis of the alpha-beta ($\alpha$-$\beta$) plot is pitch angle deviation $\Delta \alpha = \alpha - \alpha_0$ and the horizontal axis is side angle deviation $\Delta \beta = \beta - \beta_0$, in which $\alpha_0$ is the pitch angle of the optimal centered shot and $\beta_0$ is the side angle of the optimal centered shot (see Figure 3). The $\alpha$-$\beta$ plot graphs successful shots over a range of pitch angle deviations $\Delta \alpha$ and side angle deviations $\Delta \beta$. The boundary lines on the $\alpha$-$\beta$ plots are color-coded to correspond to the quadrants in which the ball is located at launch. The boundary points corresponding to launch from the upper right quadrant are shown as a blue line, from the upper left quadrant as a red line, from the lower left quadrant as a green line,
and from the lower right quadrant as a black line. In the results section, the reader will see that the locations of the boundary points in the $\alpha$-$\beta$ plots change over the basketball's trajectory, as the ball contacts the backboard and passes through the hoop or the reflection hoop.

**Backboard imprints**

The backboard imprint is a map of successful shots of points where the ball makes contact with the backboard (see Figure 4). The points shown on these figures are aim points. An aim point is the point on the backboard in the same horizontal plane as a corresponding contact point produced when the vertical plane of the trajectory is extended toward the backboard. These are the points that the shooter aims toward when shooting the basketball. The use of aim points corrects the visual misalignment problem that a shooter would otherwise experience with contact points (Silverberg et al. 2011).

**Reflection hoop**

The reflection hoop graphs successful shots behind the backboard (see Figure 5). Each point on the reflection hoop is the landing point of the imaginary trajectory that passes through the backboard and ends in the plane of the hoop.

**Shooting parameters**

The shooting parameters are launch velocity and back spin. During flight, the ball's velocity changes only slightly, but changes abruptly during impact with the backboard. The way in which the backboard changes in-flight velocity and back spin is central to the advantages gained by the bank shot. It is well known that a basketball loses energy when it hits the backboard. The $x$, $y$, and $z$ components of the ball's velocity decrease over the extremely short duration of the impact. In general terms, the loss of energy during impact causes the ball to stay closer to the hoop, which increases the chances of success. Over the extremely short duration of the impact, the ball's back spin also changes. The change in back spin causes the downward component of the ball's velocity to increase thereby making the ball bounce off of the backboard at a steeper angle. This, also, causes the ball to stay closer to the hoop, increasing the chances of success.

The range of the shooting parameters for the bank shot are studied for different court positions and compared with the direct shot. The range of launch velocities is presented graphically; the figure shows the range of bank shot velocity versus polar angle overlaid with range of direct shot velocity versus polar angle (see Figure 6). The effect of energy loss is presented by observing how the decrements in the $x$, $y$, and $z$ components of the ball's

---

**Figure 3** Alpha-beta plots (comparing ranges of $\alpha$ and $\beta$).

**Figure 4** Backboard imprints (comparing sizes of backboard imprints of successful shots).

**Figure 5** Hoops and corresponding reflection hoops (comparing sizes of landing regions).
velocity during impact with the backboard change the ball’s trajectory. Imaginary trajectories associated with $x$, $y$, and $z$ decrements “turned off” are calculated. The points where the trajectories land in the plane of the hoop produce hoop plots that show how each of the decrements keeps the ball closer to the center of the hoop (see Figure 7).

The forgiveness of back spin is presented using a graph of bank shot back spin range versus polar angle (see Figure 8). The effect of back spin on the steepness of the trajectory coming off of the backboard is presented using a multi-trajectory graph that shows trajectories with different levels of backspin (see Figure 9).

The equations that were used in the mathematical analysis performed throughout this paper are given in the Appendix.

### Results

The results below first consider the aiming study and then the shooting study. The aiming study examines the range of pitch angles $\alpha$ and the range of side angles $\beta$ at launch that produce a successful shot. During the aiming study, the shooting parameters (initial speed $v$ and back spin $\omega$) are set to their optimal values. Then, the shooting study is considered. The shooting study examines the range of initial speeds $v$ and the range of back spins $\omega$ at launch that produce a successful shot. During the shooting study, the aiming parameters (pitch angle $\alpha$ and side angle $\beta$ at launch) are set to their optimal values.

### Aim study results

Figure 3 shows a matrix of $\alpha$-$\beta$ plots at three distances ($r_1=3.281$ ft, $r_2=5.905$ ft, and $r_3=11.15$ ft) and three polar angles ($\theta_1=10^\circ$, $\theta_2=50^\circ$ and $\theta_3=80^\circ$). The ranges of $\alpha$ and $\beta$ at launch are shown as solid gray regions for direct shots and as unfilled regions for bank shots. The centered shot is indicated by a dot and the perimeters of the unfilled regions are red in the upper left quadrant, blue in the upper right, green in the lower left, and black in the lower right. The colors remind the reader of the quadrant from which the ball is launched.
Let’s now compare the bank shot (unfilled regions) and the direct shot (gray regions), beginning with the range of pitch angles $\alpha$. For the bank shot the range of $\alpha$ does not change significantly with polar angle $\theta$ and distance $r$ to the basket (except at $r=3.281$ ft, $\theta=10^\circ$). On the other hand, the range of $\alpha$ tends to be smaller for the direct shot than for the bank shot at smaller and moderate distances $r$ and then about the same for larger distances. This means that the shooter will find the bank shot to be more forgiving (wider range) in $\alpha$ at small and moderate $r$ and to have about the same forgiveness in $\alpha$ at larger $r$. Turning to the side angle $\beta$, the range of $\beta$ tends to be the same for different polar angles $\theta$ and decreases with $r$ for both the bank shot and the direct shot. However, the range of $\beta$ tends to be smaller for the bank shot than for the direct shot at smaller $r$. This means that the shooter will find the direct shot to be more forgiving in $\beta$ at small $r$ and to have about the same forgiveness in $\beta$ at moderate and larger $r$.

This figure shows that at smaller distances the direct shot can be preferred over the bank shot, that at moderate distances the bank shot is preferred over the direct shot, and that at larger distances neither is preferred. Pitch angle $\alpha$ is always more forgiving than side angle $\beta$ so these observations are influenced greatly by the shooter’s ability to prescribe side angle.

Figure 4 shows a matrix of backboard imprint plots. The inner (red) boxes are the regulation rectangles. The color bands on the perimeters of the imprints correspond to the color bands on the quadrants of the $\alpha$-$\beta$ plots. The same blue and green colors on the left (red and black on the right) quadrants of the imprint are on the right (left) quadrants of the $\alpha$-$\beta$ plots and the same green and black colors on the top (bottom) quadrants of the imprint are on the top (bottom) quadrants of the $\alpha$-$\beta$ plots except for the imprint at $\theta=10^\circ$ and $r=3.281$ ft. The switching of this imprint occurs because the distance that a ball travels decreases when the pitch angle increases in this case for which the pitch angles are largest.

The backboard imprint shows the region where the shooter must aim the ball for a successful bank shot.

Unlike the $\alpha$-$\beta$ plot and the hoop and reflection hoop described later, during play the shooter sees where the ball strikes the backboard. The shooter then makes future adjustments, gaining a sense of the backboard imprint.

On the other hand, note that the overall size of the imprint does not necessarily correlate with the size of the $\alpha$-$\beta$ plot, which does correspond to the level of forgiveness in the aiming parameters. An $\alpha$-$\beta$ plot of relatively small area can correspond to a small or large backboard.

Keeping in mind that backboard imprint size does not necessarily correlate with shooter forgiveness, observe that the overall imprint size increases with both polar angle $\theta$ and distance $r$. Like imprint size, the increase in the imprint’s thickness with increasing $\theta$ should not be interpreted as corresponding to an increase in the range of $\beta$ with increasing $\theta$ because the imprint acts on the backboard – a plane that is not perpendicular to the vertical plane of the trajectory. For example, imagine that the shooter shines a flashlight onto the backboard from the launch position. The flashlight will shine a spot on the backboard that increases in width as the polar angle increases (holding $r$ constant) even though its range of $\beta$ is constant. Indeed, notice that the range of $\beta$ did not increase with $\theta$ in Figure 3.

Figure 5 shows a matrix of hoops and reflection hoops. Each element of the matrix shows solid circular regions of direct swish shots (surrounded by the hoops) and solid reflection hoop regions of swish bank shots corresponding to the same launch $r$ and $\theta$. As shown, the reflection hoops allow you to visually compare the forgiveness of the bank shot and the direct shot. As shown, the reflection hoops are elliptical-like in shape and their long axes are aligned with the directions of the trajectories. Also, note that the distances that the centered shots travel are near-maximal so deviations in $\alpha$, whether positive or negative, can cause the ball to travel a shorter distance. For example, if $\alpha=55^\circ$ is the optimal centered pitch angle, releasing the ball at either $\alpha=50^\circ$ or $\alpha=60^\circ$ will cause the ball to travel a shorter distance in the $x$-$y$ plane. This effect becomes more pronounced as the distance increases. The result is the colorless regions on the boundaries of the reflection hoops and the blue and red colors on the top (green and black on the bottom) to switch to the bottom (top) of the reflection hoops.

First observe that the reflection hoops are considerably longer than wide and the lengths increase with $\theta$. This figure, more than the $\alpha$-$\beta$ plot and the backboard imprint, shows that the bank shot is extremely forgiving in pitch angle. The widths of the reflection hoops are about the same as the widths of the hoops, so the bank shot and
the direct shot have similar levels of forgiveness in side angle. Reinforcing the trends found in the $\alpha$-$\beta$ plot and the backboard imprint, the reflection hoop shows that the advantages of the bank shot over the direct shot are significant in $\alpha$ but highly dependent on the shooter’s accurate launch of $\beta$.

**Shooting study results**

The shooting study begins in Figure 6 by studying the ranges of launch velocity $v$ as functions of the polar angle $\theta$ for different $r$.

First observe that the axes on the left (bank shot $v$) are not aligned with the axes on the right (direct shot $v$). The axes of the bank shot velocities are shifted up relative to the axes of the direct shot velocities, while not changing the 0.5 ft/s (15.24 cm/s) increments. This was done to make it easier to compare the ranges of the direct shots and the bank shots. Observe that the ranges of $v$ of banks shots are higher than the ranges of $v$ of direct shots (vertical distances on Figure 6) and that the ranges of $v$ of bank shots increase with $\theta$. Finally observe as $r$ increases that the range of $v$ of the bank shots approaches the range of $v$ of the direct shots. During play, when close to the basket, the bank shot has several advantages over the direct shot. The required launch velocity is relatively low and well within the shooter’s capability. Also, the range of velocities for the bank shot is relatively high; the bank shot is more forgiving in $v$ than the direct shot. When moving farther from the basket, the required launch velocity can become a limiting factor. Since the launch velocity of the direct shot is lower than the launch velocity of the bank shot, the direct shot becomes more attractive. Also, recall that the range of velocities for the bank shot approaches the range of velocities for the direct shots at larger distances.

The high forgiveness of $v$ for the bank shot is explained by the energy loss of the ball when it impacts the backboard. The effect of energy loss is examined in detail in Figure 7.

The red cross symbols in Figure 7 are the locations of the ball centers when it makes contact with the backboard for different $\theta$ and $r$. The black zero symbols are the locations of the ball centers when it lands in the plane of the hoop for different $\theta$ and $r$. The other symbols (diamonds, squares, and pluses) are fictitious locations of the ball centers, also, when it lands in the plane of the hoop for different $\theta$ and $r$. The locations become fictitious by “turning off” changes in ball velocity components during impact with the backboard (see Figure 1 showing the convention used for the $x$, $y$, and $z$ directions). The green diamond symbols are the fictitious locations resulting from artificially turning off the change in the $x$ component of velocity during hoop impact. The pink square symbols are the fictitious locations resulting from artificially turning off the change in the $y$ component of velocity during impact. The blue plus symbols are the fictitious locations resulting from artificially turning off the change in the $z$ component of velocity during impact.

First, notice that the diamonds are closer to the zeros than the squares or the pluses. Therefore the $x$ component of velocity has a smaller effect on where the ball lands than the $y$ and $z$ components of velocity. Next, notice that the squares become farther from the zeros as $\theta$ and $r$ increase. Finally, notice that the pluses become farther from the zeros as $r$ decreases. This suggests that the ball stays close to the basket, and hence has a higher likelihood of entering the hoop, as a result of the loss in the $y$ component of velocity more than any other component and that this effect increases in significance as $\theta$ and $r$ increase.

Figure 8 shows the range of back spins $\omega$ as functions of $\theta$ for different $r$. The dashed line is the nominal value of 3 Hz of back spin. The upper ranges of $\omega$ occur at values that are higher than would be found during play so the important information to focus on is the minimum values.

Observe that the minimum $\omega$ decreases with the polar angle $\theta$. At low $\theta$, it becomes important to provide at least 3 Hz of backspin (these are the court locations from which bank shots are less common). At moderate and high $\theta$ the minimum $\omega$ decreases and the bank shot becomes forgiving in $\omega$. The high forgiveness of $\omega$ is explained by the velocity change $\Delta v$ (upward direction) that results from $\omega$ when the ball impacts the backboard, as shown in Figure 9. Figure 9 is a multi-trajectory figure showing the effect of $\omega$. The black region of the hoop is crossed when there is no contact with the rim and the red regions extend to the edges of the rim. As shown, the ball stays closer to the hoop when $\omega$ increases.

**Combined results**

The results in this section were divided into aim study results and shooting study results. Recall in the aim study that the ball is launched at its optimal speed $v$ and optimal back spin $\omega$ while the pitch angle $\alpha$ and side angle $\beta$ are varied whereas in the shooting study that the ball is launched at its optimal pitch angle $\alpha$ and side angle $\beta$ while the launch speed $v$ and back spin $\omega$ are varied.
Considering the results from both of these studies, one arrives at several new observations.

Again, recall that the key parameters are launch speed $v$, back spin $\omega$, pitch angle $\alpha$, and side angle $\beta$. We found in Figure 3, and it was reinforced in Figures 4 and 5, that the bank shot is more forgiving than the direct shot in pitch angle while the level of forgiveness of the bank shot and the direct shot are about the same in side angle. We found in Figure 6 that the bank shot is more forgiving in launch speed than the direct shot. Also, we found in Figure 8 that back spin, which is notably important for the bank shot, only serves to increase the forgiveness of the bank shot. In other words, the in-plane parameters ($v$, $\omega$, and $\alpha$) have relatively high forgiveness in the bank shot and the out-of-plane parameter – the side angle $\beta$ – has about the same level of forgiveness in the bank shot and the direct shot. Therefore, during play if the shooter can learn how to impart the side angle with the same level of accuracy as for the direct shot, the advantages of the bank shot over the direct shot become very significant at key spots on the court; a 70% direct shot corresponds to about a 90% bank shot from these spots. Several of these key spots are shown in Figure 10. The significance of the increase in shooting percentage from 70% to 90% becomes more apparent when one recognizes that this corresponds to a decrease from missing a direct shot 30% of the time to missing the bank shot just 10% of the time – a three-fold decrease.

**Imparting side angle and the layup**

The side angle is difficult to impart because of the lack of a reference toward which to aim. Whereas the shooter lines up his shot with the center of the hoop in the case of the direct shot, the shooter has no such reference in the case of the bank shot. Any increase in the utilization of the bank shot in play may very well require some level of training. One training approach, first suggested by Silverberg et al. (2011), places a V-shaped mark on the backboard together with a vertical pole behind the backboard. It replaces the backboard rectangle during training, to show the player precisely in what direction to aim and where the ball needs to hit the backboard.

During play, although the shooter has no reference for lining up the side angle for most bank shots, there is one exception – the layup. In the case of the layup, since the shot is taken at close range, the shooter is guided by two references – the edge of the hoop on one side and the backboard on the other side. When shooting a layup, the shooter imparts an aim angle that divides the angle made by the edge of the rim and the backboard. The superiority of the layup stems from this advantage, as well as several others. When close to the hoop, a small amount of launch speed is needed. This relatively small launch speed can be used either to make the ball pass over the hoop and then pass through the horizontal plane of the hoop for a direct shot, or to bounce off of the vertical plane of the backboard for the bank shot. Whereas the horizontal plane of the hoop is obstructed by the rim (for a direct shot), the backboard is unobstructed and therefore easier to direct the ball toward. Secondly there is the matter of speed sensitivity. When close to the hoop, having to make a ball pass through the horizontal plane of the hoop by imparting a low level of speed is a more delicate act than imparting the higher level of speed for a bank shot from the same location.

**Summary and conclusions**

This paper began by studying bank shot parameter relationships – expressing them graphically to assist with understanding the bank shot. In particular, we developed $\alpha$-$\beta$ plots, backboard imprints, reflection hoops, and multi-trajectory plots. The important parameters being considered were the launch speed $v$, the back spin $\omega$, the pitch angle $\alpha$, and the side angle $\beta$. We found, at particular regions on the court, that the launch speed $v$ and the pitch angle $\alpha$ of the bank shot have a high level of forgiveness.
(wider acceptable range) relative to the direct shot and that the level of forgiveness of the side angle $\beta$ was the same for the bank shot and the direct shot. Also, it was shown how back spin improves forgiveness. The aiming parameters $\alpha$ and $\beta$ are geometric variables whereas the shooting parameters, $v$ and $\omega$, are kinematic variables. Although not verified, it is likely that kinematic variables are more difficult to learn how to prescribe accurately than geometric variables because kinematic variables rely heavily on kinesthetic memory and lack the benefit of geometric markers that allow for real-time adjustments. It follows that the high level of forgiveness of $v$ in the bank shot is likely of particular advantage. On the other hand, the equivalent levels of forgiveness of the side angle $\beta$ for the bank shot and the direct shot does not imply that the shooter can impart the side angle with the same level of accuracy for the bank shot as for the direct shot. Without a reference, imparting side angle accurately relies heavily on kinesthetic memory even though it is a geometric variable.

During play, situations frequently arise in which the bank shot offers significant advantages over the direct shot. The unskilled shooter cannot capitalize on the bank shot, however. This paper showed that the critical skill the shooter must learn is to accurately prescribe side angle; the shooter who learns this skill will be able to capitalize on the bank shot.

**Appendix**

As stated in the paper, the shooter was assumed to successfully make the free throw 70% of the time and to shoot other shots with the same statistics. The optimal shots were selected from a set of free throws having a range of launch velocities $v$ and launch angles $\alpha$ for a ground height of 7 ft. The launch speed $v$ varied between 22.70 ft/s (6.92 m/s) and 28.45 ft/s (8.67 m/s) in 144 steps and $\alpha$ varied from 22° to 82° in 300 steps (43,200 cases). Each of the trajectories is a highly accurate trajectory that accounts for ball contact with the hoop and the backboard (Silverberg et al. 2003). The free throws were launched with 3 Hz of back spin which was determined to be optimal (Tran and Silverberg 2008). As an illustration, one set of data is shown in Figure A1.

Notice that the successful shots appear in bands. The large left band is associated with direct shots, the two thin middle bands are associated with bank shots, and the large right band is associated with bank shots. Our interest lies in the large left band. The probability of success depends on the means and standard deviations of $v$ and $\alpha$, in which the mean values of $v$ and $\alpha$ are the values that the shooter seeks to prescribe and where the standard deviations correspond to shooter consistency. Improvements in shooting depend on the values of $v$ and $\alpha$ that the shooter prescribes (understanding the nature of the highest probability shot) and consistency. The standard deviations used in this paper were associated with a 70% shooter (Tran and Silverberg 2008). The mean values were determined using the segment bisection shown in Figure A1. In previous work it was verified that the best mean $v$ and $\alpha$ can be accurately determined by bisecting the line segment between the point of the smallest $v$ on the left boundary of the band and the point of the smallest $v$ on the right boundary of the band. The $v$ and $\alpha$ of the bisecting point are the best mean values.

Referring to Figure A2, the center of the basketball is initially located at $(x_0, y_0, z_0)$, upon contact with the backboard is located at $(x_s, y_s, z_s)$, its contact point is projected onto the backboard at the aim point $(x_a, y_a, z_a)$, the center of the ball enters the plane of the hoop at $(x_f, y_f, 0)$, and is imagined to pass through the backboard and reach the plane of the hoop at $(x_r, y_r, 0)$.

The components of the initial position and the launch velocity are expressed in terms of polar coordinates and in terms of the pitch angle and the side angle by (See again Figure 1)

$$
x_0 = r_0 \cos \theta \quad y_0 = r_0 \sin \theta \quad z_0 = -3 \text{ ft}
$$

$$
v_{x0} = -v_0 \cos \alpha \cos \beta \quad v_{y0} = -v_0 \cos \alpha \sin \beta \quad v_{z0} = v_0 \sin \alpha \quad (A1)
$$

in which the ball is launched 7 ft from the floor (3 ft below the hoop). For the centered shot $(x_f = y_f = 0)$, the side angle...
is related to the physical parameters by (Silverberg et al. 2003)

$$\tan \beta = \frac{\sin \theta}{\cos \theta + \left(1 + \frac{3}{5} \gamma \right) \frac{a-R}{r_0}} \quad (A2)$$

where $\gamma$ is the coefficient of restitution between the basketball and the backboard. The velocity components of the center of the ball, the time of flight $T_1$ to the backboard, and the coordinates $y_b$ and $z_b$ of the center of the ball when it makes contact with the backboard are

$$v_x = v_{x0}, \quad v_y = v_{y0}, \quad v_z = v_{z0} - gT_1,$$

$$R - a = x_0 + v_{x0}T_1, \quad y_b = y_{y0} + v_{y0}T_1, \quad z_b = z_{z0} + v_{z0}T_1 - \frac{g}{2}T_1^2,$$

$$T_1 = \frac{x_0 + a - R}{v_{x0}} \quad (A3)$$

$$\tan \beta = \frac{y_b - y_{y0}}{x_b + a - R} \Rightarrow y_b = y_{y0} - (x_b + a - R) \tan \beta$$

The aim points projected onto the backboard are related to the contact points by

$$x_a = x_b - R, \quad y_a = y_b - R \tan \beta, \quad z_a = z_b \quad (A4)$$

The components of the velocity of the center of the ball immediately after contact are (Silverberg et al. 2003)

$$v_{xb} = -v_{vb}, \quad v_{yb} = \frac{3}{5}v_{vb}, \quad v_{zb} = \frac{3}{5}v_{vb} - \frac{2}{5}R \cos \beta \quad (A5)$$

Next, the time of flight $T_2$ from the backboard to the plane of the hoop and the coordinates $x_F$ and $y_F$ of the center of the ball in the plane of the hoop are determined.

$$0 = z_b + v_{vb}T_2 - \frac{g}{2}T_2^2 \Rightarrow T_2 = \frac{v_{vb}}{g} + \sqrt{\frac{v_{vb}^2}{g^2} + \frac{2}{g} z_b} \quad (A6)$$

$$x_F = x_b + R + v_{vb}T_2, \quad y_F = y_b + v_{vb}T_2$$

Finally, the time of flight $T_3$ from launch to the instant the ball reaches the plane of the reflection hoop and the coordinates $x_R$ and $y_R$ of the center of the ball in the plane of the reflection hoop are determined.

$$0 = z_0 + v_{z0}T_3 - \frac{g}{2}T_3^2 \Rightarrow T_3 = \frac{v_{z0}}{g} + \sqrt{\frac{v_{z0}^2}{g^2} + \frac{2}{g} z_0} \quad (A7)$$

$$x_R = x_0 + v_{x0}T_3, \quad y_R = y_0 + v_{y0}T_3, \quad z_R = 0$$

The range of successful shots is determined by incrementing with respect to pitch angle and side angle and testing whether $(x_F, y_F)$ falls inside the circle $x_F^2 + y_F^2 \leq (R_h - R)^2$ corresponding to a successful shot in which $R$ denotes the radius of the ball and $R_h$ denote the radius of the hoop. The boundary points at launch located in the upper right quadrant are shown as red lines, in the upper left quadrant as blue lines, in the lower left quadrant as green lines, and in the lower right quadrant as black lines (see Figure A3).